Part III: Relational Logic Homework Problems

Exercise 1:

Rosen, section 6.1 problem 2: For the relation "divides" on the set {1,2,3,4,5,6}:

1.List all the ordered pairs in the relation.

2.Display the relation graphically.

3.Display the relation in tabular form.

Solution:
(solution set will be posted later)

Exercise 2:

Rosen, section 6.1 problem 4: Determine whether, for the domain of all people, the following relations are reflexive, symmetric, antisymmetric, transitive.

1.taller?

2.bornSameDay?

3.sameFirstName?

4.commonGrandparent?

Solution:
(solution set will be posted later)

Exercise 3:

While Rosen discusses binary relations (subsets of pairs), one can also have unary relations (subsets of the domain – e.g. isPrime? for numbers), and ternary relations (subsets of triples – e.g. isAChildOf? over the domain of people.)

Suppose you wanted to encode addition as a ternary relation "addsTo?" over (triples of) numbers – describe briefly what triples [x,y,z] would be in the relation; give an example of a triple in the relation addsTo? and a triple not in the relation addsTo?.

(Note that in general, you can represent any k-ary function as a (k+1)-ary relation.)

Solution:
(solution set will be posted later)

Exercise 4:

Are each of the following formulas true for all interpretations ("valid")? (Remember that the relation-names are just names in the formula; don't assume the name has to have any bearing on their interpretation.) (Don't be misled by the relation-names used in the formula; they can be interpreted as any relation over any domain you specify.)

- For arbitrary a,b in the domain, (atLeastAsWiseAs(a,b)$\vee$atLeastAsWiseAs(b,a))
- For arbitrary a in the domain, (prime(a) $\rightarrow$ (odd(a) $\rightarrow$ prime(a)))

•For arbitrary a,b in the domain, (betterThan(a,b) → ¬betterThan(b,a))

For each, if it is true or false under all interpretations, prove that. (For these small examples, a truth table will probably be easier than using boolean algebra or an inference-system proof.) Otherwise, give and interpretation in which it is true, and one in which it is false.

Solution:

(solution set will be posted later)

Exercise 5:

Suppose we wanted to represent the count of neighboring pirates with a binary relation, such that when location A has two neighboring pirates, piratesNextTo(A,2) will be true (and presumably, piratesNextTo(A,1) would not be true in that particular relation, analagous to some the propositional WaterWorld domain axiom A2.)

If we only allow binary relations to be subsets a domain crossed with itself, then what must the domain be for this new relation piratesNextTo?

If we further introduced another relation, isNumber?, what is a formula that would help distinguish intended interpretations from unintended interpretations? That is, give a formula that is true under all our intended interpretations of piratesNextTo but is not true for some "nonsense" interpretations we want to exclude. (This will be a formula without an analog in the WaterWorld domain axioms[1].)

Solution:

(solution set will be posted later)

[1] http://cnx.rice.edu/modules/m10528/latest/