Ron's Rules for Mathematics

Rule #1: An Example is NOT a Proof

- Many Examples are still NOT a Proof.
- Many, Many Examples are still NOT a Proof.
- A student will get Zero Credit -- No Partial Credit -- if they provide an Example or Examples when we ask for a Proof.
- But . . . See Rule #2.

Rule #2: Examples are a Good Place to Start

- Examples are useful for many reasons:

  -- to help understand the meaning of a problem;

  -- to generate hypotheses;

  -- to help in the search for a general solution.

- Students *should* work out many examples, but they *should not* confuse an example or examples with a proof.

- See Rule #1.

Rule #3: Standard Procedures for Problem Solving

1. Read the question -- twice.
2. Determine the problem you need to solve.
3. List all the pertinent parameters.
4. Write LARGE.
5. Draw a diagram whenever possible.
    - Make the diagram LARGE.
    - Label ALL the variables in the diagram.
6. Do not get into a fight with Algebra.
    - Algebra is your friend, not your enemy.
7. After you solve the problem, verify your answer.

Rule #4: Use the Correct Style for Proofs by Induction

A. *Base Case:* First verify the base case, using the following template:

LHS(0)=something

RHS(0)=something

and say *base case holds*.

DO NOT WRITE:

LHS(0)=RHS(0)

BEFORE showing they are both equal to some common result.

B. *Inductive Hypothesis:* Clearly specify your induction hypothesis.
Omitting this step can cause serious confusion.

C. For the *inductive step*, NEVER start from the conclusion

LHS(n+1)=RHS(n+1)

and perform arithmetic operations on both sides to derive a well-known fact such as

0=0.

This style is VERY BAD and the semantics are incorrect.

q and $p \Rightarrow q$, do not imply p.

You MAY verify the given statement in this way.
You MAY NOT prove the result in this way.

D. *Induction:* I strongly recommend one of the following three correct approaches:

i. Start from LHS(n+1), show

LHS(n+1) = ...
= ....
= ....
= RHS(n+1)

ii. Start from RHS(n+1), show

RHS(n+1) = ...
= ....
= ....
= LHS(n+1)

iii. Start from LHS(n+1), show

LHS(n+1) = ...
= ....
= ....
= some intermediate result;

and do the same thing on RHS(n+1)

RHS(n+1) = ...
= ....
= ....
= same intermediate result;

E. *Conclusion:* After you finish the inductive step, you should state a conclusion, summarizing what your proof has shown.

Rule #5: The Same BUT Simpler

If you cannot solve a hard problem, try first to solve an analogous but simpler problem.

A. *Geometry*
   Replace a difficult problem in three dimensions by an analogous but simpler problem in one or two dimensions.

B. *Algebra*
   Replace a difficult problem in high degree polynomials by an analogous but simpler problem in polynomials of degree one or degree two.

C. *Counting, Combinatorics, Number Theory*
   Replace a difficult problem in large numbers by an analogous but simpler problem in very small numbers.

<u>Rule #6: Induction in the Handmaiden of Recursion</u>

- Use Induction to prove results about:

  -- recursive functions

  -- recursive programs

  -- iteration and iterative programs

- Use Induction when you see a <u>discrete pattern</u> in algebra, geometry, or calculus and *cannot see in any other way* why this pattern holds.

Rule #7: Avoid Proofs by Surprise

- A proper proof is like a good murder mystery: you must prepare your reader with all the necessary clues.
- Bringing in new suspects or fresh evidence in the final paragraph is forbidden.
- Proofs with a surprise ending are impossible for readers to follow because the readers will have no idea where they are headed.
- If you need to invoke a result from somewhere else to complete the proof, mention this result before you start your proof.