<u>Comp 360 -- Final Exam</u>

The following are the rules for this examination:

- a four hour time limit to finish the entire exam;

- you need not do the entire exam in one session, but once you read a problem you must finish the problem without taking time out (except for obvious emergencies);

- please write all your answers in **INK**, not in pencil! Pencil is difficult to read. If you want to change an answer, just cross out your old answer and write your new answer. Do not try to erase.

- **<u>closed</u> textbook, <u>closed</u> notes;**

- all work must be entirely your own;

- you may not confer on any problems with any person except the professor;

- the exam must be handed in no later than noon in my office (DH-3116) on Wednesday, December 16.

There are 5 problems on the exam. Each problem is worth 40 points. For full credit you must do **ALL** 5 problems.

Good luck.

1. (40 points)

Let N be a unit vector, and let u,v be two unit vectors in the plane perpendicular to N with $N \parallel u \times v$ and $\angle(u,v) = \phi/2$. Recall that by definition

$$q(N,\phi) = \cos(\phi/2)\,O + \sin(\phi/2)N$$

and that for any quaternion q

$$S_q(w) = q\,w\,q^*$$

$$T_q(w) = q\,w\,q$$

where the multiplication here is quaternion multiplication and the product of two quaternions is given by the formula:

$$(aO + u)(bO + v) = (a\,b - u \cdot v)O + (b\,u + a\,v + u \times v).$$

Show that:

a. $q(N,\phi) = -v\,u$

b. $S_{q(N,\phi)}(w) = (vu)\,w\,(uv)$

c. $S_{q(N,\phi)} = T_v \circ T_u$

d. Explain why it follows from part c that every rotation in 3-dimensions is the composite of two reflections.

2. (40 points)

Let $D(s) = (x(s), y(s), z(s))$ be a parametrized curve lying in a plane in 3-space, and let $Q = (q_1, q_2, q_3)$ be a fixed point in 3-space not in the plane of $D(s)$. The parametric surface:

$$C(s,t) = (1-t)D(s) + tQ,$$

is called the *generalized cone* over the curve $D(s)$.

a. Compute the normal $N(s,t)$ at an arbitrary point on the cone $C(s,t)$.

b. Suppose that you already have an algorithm to compute the intersection points of any line in the plane of $D(s)$ with the curve $D(s)$. Based on this algorithm, develop a procedure to find the intersection points of an arbitrary line in 3-space with the surface $C(s,t)$.

c. Based on parts a and b, explain how to ray trace the surface $C(s,t)$.

3. (40 points)

Consider the surface defined by the implicit equation

$$xyz - 1 = 0 .$$

a. Find the normal vector to this surface at the point $P = (2, 1, 0.5)$.

b. Find the implicit equation of this surface after rotating the surface by $\pi / 6$ radians around the z-axis and then translating the surface by the vector $v = (1,3,2)$.

c. Explain how you would ray trace the rotated surface in part b.

4. (40 points)

a. Suppose that the color of the light source is yellow and the color of the surface is cyan. What is the color perceived by the viewer for:

   i. diffuse reflection

   ii. specular reflection (homogenous material)

   iii. specular reflection (inhomogeneous material)

b. Show how to compute t^2 incrementally using only two additions and no multiplications per pixel.

c. Let $R(s) = R + s\,u$ and $P(t) = P + t\,v$ be two intersecting lines in 3-space. Let $v_\perp$ be the component of v perpendicular to u. Show that the lines $R(s)$ and $P(t)$ intersect at the parameter value

$$t = \frac{(R - P) \bullet v_\perp}{v \bullet v_\perp}.$$

d. The radiosity equations in matrix form are:

$$\underbrace{\begin{pmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1N} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_N F_{N1} & -\rho_N F_{N2} & \cdots & 1-\rho_N F_{NN} \end{pmatrix}}_{M} \underbrace{\begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix}}_{B} = \underbrace{\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix}}_{E}$$

Prove that the Radiosity Equations are diagonally dominant.

5. (40 points)

a. Let $B_k^n(t)$ denote the Bernstein basis functions of degree n over the interval [0,1]. Using the dual functional property of the blossom, prove the following identities:

i. $\sum_{k=0}^{n} B_k^n(t) \equiv 1$

ii. $\sum_{k=0}^{n} \binom{k}{j} B_k^n(t) \equiv \binom{n}{j} t^j \qquad j = 0, \ldots, n$

iii. $\sum_{k=0}^{n} (-1)^k B_k^n(t) \equiv (1-2t)^n$

b. Based on recursive subdivision, develop an algorithm to intersect::

i. a Bezier curve with a Bezier surface.

ii. two Bezier patches.