<u>Comp 360 -- Final Exam (200 Points)</u>

The following are the rules for this examination:

- a <u>four</u> hour time limit to finish the entire exam;

- you need not do the entire exam in one session, but once you read a problem you must finish the problem without taking time out (except for obvious emergencies);

- please either **TYPE** you answers or write all your answers in **DARK INK**. Do **NOT** write in pencil! Pencil is difficult to read. If you want to change an answer, just cross out your old answer and write your new answer. Do not try to erase.

- **<u>closed</u> textbook, <u>closed</u> notes;**

- all work must be entirely your own;

- you may not confer on any problems with any person except the professor;

- the exam must be handed in no later than noon in my office (DH-3116) on Wednesday, December 14.

There are 5 problems on the exam. Each problem is worth 40 points. For full credit you must do **ALL** 5 problems.

Good luck.

1. (40 points)

Let N be a unit vector, and let u,v be two unit vectors in the plane perpendicular to N with $N \parallel u \times v$ and $\angle(u,v) = \phi/2$. Recall that by definition

$$q(N,\phi) = \cos(\phi/2)\,O + \sin(\phi/2)N$$

and that for any quaternion q

$$S_q(w) = q\,w\,q^*$$

$$T_q(w) = q\,w\,q$$

where the multiplication here is quaternion multiplication and the product of two quaternions is given by the formula:

$$(aO + u)(bO + v) = (a\,b - u \cdot v)O + (b\,u + a\,v + u \times v).$$

Show that:

a. $q(N,\phi) = -v\,u$

b. $S_{q(N,\phi)}(w) = (vu)\,w\,(uv)$

c. $S_{q(N,\phi)} = T_v \circ T_u$

d. Explain why it follows from part *c* that every rotation in 3-dimensions is the composite of two reflections.

2. (40 points)

Let $D(s) = \left(x(s), y(s), z(s)\right)$ be a parametrized curve lying in a plane in 3-space, and let $v = (v_1, v_2, v_3)$ be a fixed vector in 3-space not in the plane of $D(s)$. The parametric surface:

$$C(s,t) = (1 - t)\,D(s) + t\,v\,,$$

is called the *generalized cylinder* over the curve $D(s)$.

a. Compute the normal $N(s,t)$ at an arbitrary point on the cylinder $C(s,t)$.

b. Suppose that you already have an algorithm to compute the intersection points of any line in the plane of $D(s)$ with the curve $D(s)$. Based on this algorithm, develop a procedure to find the intersection points of an arbitrary line in 3-space with the surface $C(s,t)$.

c. Based on parts a and b, explain how to ray trace the surface $C(s,t)$.

3. (40 points)

a. Develop an algorithm to intersect a straight line with a right circular cone bounded by:

   i. Two disks.

   ii. One disk and the cone vertex.

b. Explain in detail how to ray trace an elliptical cone. In particular, explain how you would:

   i. Model the elliptical cone.

   ii. Compute the surface normals to the elliptical cone.

   iii. Find ray–surface intersections for the elliptical cone.

Treat both the bounded and the unbounded elliptical cone.

4. (40 points)

a. Suppose that the color of the light source is yellow and the color of the surface is cyan. What is the color perceived by the viewer for:

i. diffuse reflection

ii. specular reflection (homogenous material)

iii. specular reflection (inhomogeneous material)

b. Show how to compute t^2 incrementally using only two additions and no multiplications per pixel.

c. Let $R(s) = R + s\,u$ and $P(t) = P + t\,v$ be two intersecting lines in 3-space. Let $v_\perp$ be the component of v perpendicular to u. Show that the lines $R(s)$ and $P(t)$ intersect at the parameter value

$$t = \frac{(R - P) \bullet v_\perp}{v \bullet v_\perp}.$$

d. The radiosity equations in matrix form are:

$$\underbrace{\begin{pmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1N} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_N F_{N1} & -\rho_N F_{N2} & \cdots & 1-\rho_N F_{NN} \end{pmatrix}}_{M} \underbrace{\begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix}}_{B} = \underbrace{\begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix}}_{E}$$

Prove that the Radiosity Equations are diagonally dominant.

5. (40 points)

a. Consider a Bezier curve with control points $P_0,\ldots,P_n$. Prove that:

i. The line segment P_0P_1 is tangent to the curve at P_0.

ii. The line segment $P_{n-1}P_n$ is tangent to the curve at P_n.

b. Given point and derivative data $(R_0, v_0),\ldots,(R_n, v_n)$, explain how to place Bezier control points to generate a smooth piecewise cubic curve to interpolate this data.