Internal Forces and Moments

Introduction

To a very large extend this chapter is simply an extension of Section 6.3, The Method of Sections. The section on curved cables is new material. The section on pressures in fluids is another calculus review on centroids, and that application is covered again in a later course in fluid mechanics. The application the method of sections to beams to create shear and bending moment diagrams is critical to finding beam stresses and deflections. However, that topic is covered in detail again in a later course in the mechanics of solids. Furthermore, shear and moment diagrams are more easily automated by a calculus approach not covered in the Bedford Fowler text (i.e., the use of discontinuous distributions).

Therefore, this TK Solver review will mainly be restricted to the new topics of curved cables, and cables with multiple straight segments. The distributed loading on cables is defined either in terms of load per unit cable length, or in terms of per unit horizontal or vertical distance. If the distributed load per unit length is constant, then two special cable shapes occur: the Catenary for constant cable weight per unit length, and the parabolic cable for constant vertical load on the cable. Having the equation of the shape of a cable, you can find its slope (derivative). The slope information lets you split the axial cable tension into a changing vertical component that balances the supported weight, and a constant horizontal component.

Cable loaded with Its own weight (catenary)

The equation of the shape of a cable (relative to its horizontal tangent point) loaded with its own weight per unit length, w , and the resulting axial tension is

$$y(x) = \frac{T_0}{w}(\cosh\frac{wx}{T_0} - 1)$$

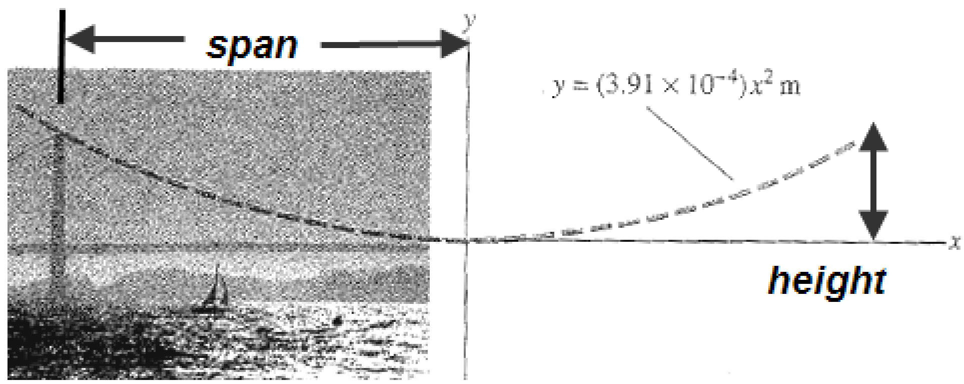
where T_0 is the tension at the horizontal tangent location. The point here is that, on the earth, cables are never straight. However, you are usually justified in assuming that a cable with a "short" span is straight, as you have done up to this point.

Cable loaded with a constant vertical load (parabolic cable)

When the vertical load, say *v*, is constant with respect to the horizontal position, *x*, the shape equation is much simpler: $y(x) = \frac{v}{2T_0}x^2$, but it is still not straight. When the height change to span width ratio is small, a catenary can be approximated as a parabolic cable with *v = w*.

Example 10.6 Golden Gate Bridge (a parabolic cable half-span approximation)

This example is simplified because symmetry allows you to locate the horizontal tangent point by inspection. Locate the coordinate system there. Then, the shape of the parabolic cable is given above. The shape equation is included in the TK model (see Figure 1) so that you could plot the shape if desired (as a list solve output). However, you usually evaluate the shape equation at the support location(s) as a way to determine the constant a, since the horizontal force is initially unknown. In addition, evaluating the slope at the support point(s) allows you to apply equilibrium to determine the required support tension. In the outputs of Figure 2 a constant vertical line load was assumed to illustrate the large support forces required.



Status	Rule
Comment	; Parabolic cable with constant vertical load per horizontal span
Satisfied	a = v / H ; ratio of unit vertical load to horizontal point tension
Satisfied	y = a * x^2 / 2 ; x & y from (theoretical) horizontal tangent point
Satisfied	T = H * sqrt (1 + a^2 * x^2) ; tension in the cable
Satisfied	T^2 = H^2 + (v * x)^2 ; tension in the cable
Satisfied	2 * s = x * T / H + log (a * x + T / H) / a ; length from horizontal point
Comment	; equilibrium (H to left at x=y=0)
Satisfied	-V + T_end * sin (ang_end) = 0 ; Σ F_y = 0
Satisfied	-H + T_end * cos (ang_end) = 0 ; Σ F_x = 0
Satisfied	V = v * span ; supported load
Comment	; related
Satisfied	height = a * span^2 / 2
Satisfied	slope = a * span
Satisfied	tan(ang_end) = slope
Satisfied	L = L1 + L2
Satisfied	2 * L1 = span * sqrt (1 + a^2 * span^2)
Satisfied	2 * L2 = ln (a * span + sqrt (1 + a^2 * span^2)) / a

Figure 1 Rules for a half span symmetric parabolic cable

Input	Name	Output	Unit	Comment
				Parabolic cable, constant vertical load per horizontal span
				Ex 10.6, 3rd Ed, Bedford Fowler
	a	.0007813	1/m	ratio vertical unit load to horizontal tension
320	x		m	horizontal position
	y	40	m	vertical position
	s	233.7071	m	length of cable from horizontal point
	T	1.3194E8	N	tension at x (and y)
	V	64000000	N	cable vertical load
100000	v		N/m	vertical load per horizontal length
640	span		m	horizontal dist from horiz tangent to cable end
160	height		m	vertical dist from horiz tangent to cable end
	L	665.7464	m	cable length from horiz tangent
	T_end	1.4311E8	N	end support tension
	H	1.28E8	N	horizontal tension at horizontal tangent
	ang_end	26.56505	degrees	end cable angle
	L1	357.7709	m	work term
	L2	307.9756	m	work term
	slope	.5		slope at cable end

Figure 2 Forces and length of half span cable

Example 10.7 Maximum tension in parabolic cable

This example is more common in that you know the total span distance and the cable end heights, but not the location of the horizontal slope point (Figure 3). This is basically two problems consisting of a left part and a right part sharing the same horizontal point tension. You either have to rearrange the shape equation to eliminate constant *a* (as done in the text), or you iteratively solve for the two span lengths that define the unknown horizontal point. The TK rules in Figure 4 let you solve each shape equation for the spans that match the heights.

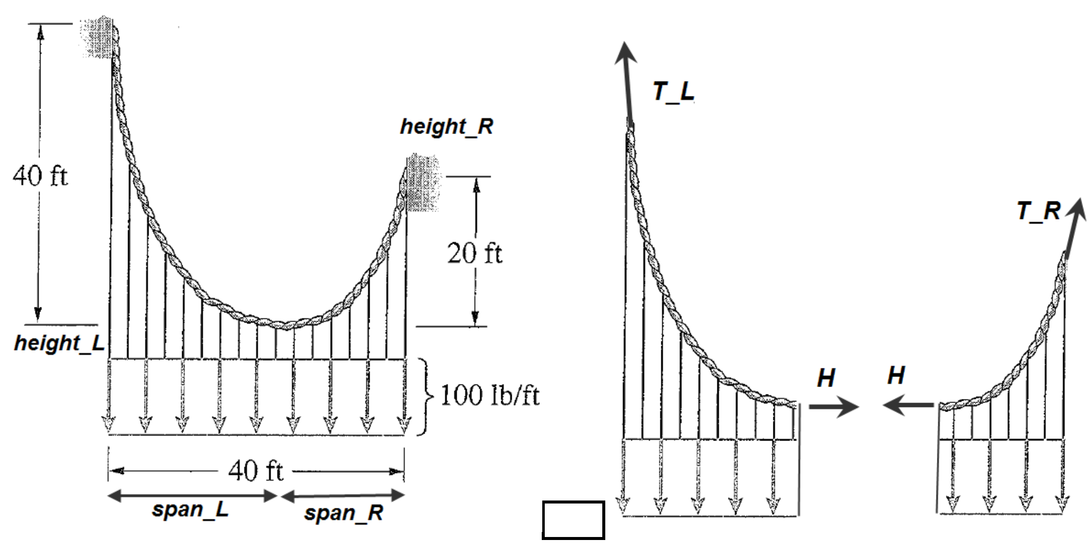


Figure 3 A typical parabolic cable geometry

Status	Rule
Comment	; Parabolic cable with constant vertical load per horizontal span
Satisfied	a = v / H ; ratio of unit vertical load to horizontal point tension
* Unsatisfi	y = a * x^2 / 2 ; x & y from (theoretical) horizontal tangent point
* Unsatisfi	T = H * sqrt (1 + a^2 * x^2) ; tension in the cable
* Unsatisfi	T^2 = H^2 + (v * x)^2 ; tension in the cable
* Unsatisfi	2 * s = x * T / H + log (a * x + T / H) / a ; length from horizontal point
Satisfied	V = v * span ; total supported load
Satisfied	V_R = v * span_R ; right supported load
Satisfied	V_L = v * span_L ; left supported load
Comment	; equilibrium of right cable segment (H to left at x=y=0)
Satisfied	–V_R + T_R * sin (ang_R) = 0 ; Σ F_y = 0
Satisfied	–H + T_R * cos (ang_R) = 0 ; Σ F_x = 0
Comment	; equilibrium of left cable segment (H to right at x=y=0)
Satisfied	–V_L + T_L * sin (ang_L) = 0 ; Σ F_y = 0
Satisfied	H – T_L * cos (ang_L) = 0 ; Σ F_x = 0
Comment	; related
Satisfied	L = L_L + L_R ; total length
Satisfied	L_R = L1 + L2 ; right length
Satisfied	2 * L1 = span_R * sqrt (1 + a^2 * span_R^2)
Satisfied	2 * L2 = ln (a * span_R + sqrt (1 + a^2 * span_R^2)) / a
Satisfied	L_L = L3 + L4 ; left length
Satisfied	2 * L3 = span_L * sqrt (1 + a^2 * span_L^2)
Satisfied	2 * L4 = ln (a * span_L + sqrt (1 + a^2 * span_L^2)) / a
Satisfied	span = span_L + span_R
Satisfied	height_L = a * span_L^2 / 2
Satisfied	height_R = a * span_R^2 / 2
Satisfied	height_L * span_R^2 = height_R * span_L^2
Satisfied	slope_R = a * span_R
Satisfied	tan(ang_R) = slope_R
Satisfied	slope_L = a * span_L
Satisfied	tan(ang_L) = slope_L

Figure 4 Rules for two parabolic spans with common minimum tension

The outputs in Figure 5 required an iterative solution. An initial guess (half the span) was provided for the left span size, and for the left support tension (half the total load). Once again, the x-variable was provided to allow a list solve for the shape (y-values) in case a plot of the shape was desired for checking or reporting purposes. The shape for the current example is presented in Figure 6.

Input	Name	Output	Unit	Comment
				Parabolic cable, constant vertical load per horizontal span
				Ex 10.6, 3rd Ed, Bedford Fowler
	a	.1457107	1/ft	ratio vertical unit load to horizontal tension
	x		m	horizontal position
	y		m	vertical position
	s		m	length of cable from horizontal point
	T		N	tension at x (and y)
	V	3999.982	lb	cable vertical load
	V_L	2343.135	lb	left cable vertical load
	V_R	1656.847	lb	right cable vertical load
100	v		lb/ft	vertical load per horizontal length
40	span		ft	horizontal cable span
	span_L	23.43146	ft	left horizontal span
	span_R	16.56854	ft	right horizontal span
	L	75.53319	ft	total cable length
40	height_L		ft	left vertical height
20	height_R		ft	right vertical height
	ang_L	73.67505	degrees	left end cable angle from horizontal
	ang_R	67.5	degrees	right end cable angle from horizontal
	H	686.2885	lb	tension at horizontal tangent (minimum)
	T_L	2441.572	lb	left end support tension
	T_R	1793.358	lb	right end support tension
	L_L	48.34392	ft	left side cable length
	L_R	27.18927	ft	right side cable length
	L1	21.64784	ft	work term
	L2	5.54143	ft	work term
	L3	41.68043	ft	work term
	L4	6.663486	ft	work term
	slope_L	3.414214		left end slope
	slope_R	2.414214		right end slope

Figure 5 Resulting forces, spans, and lengths for a parabolic cable

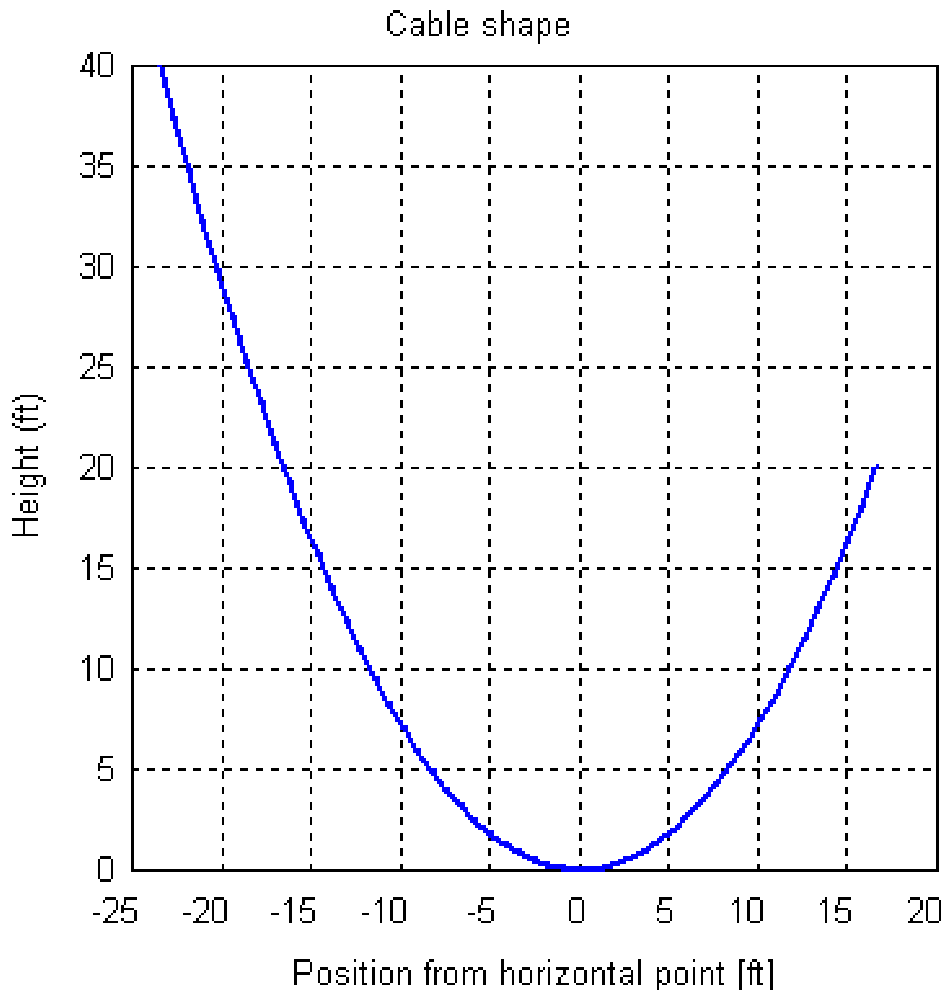
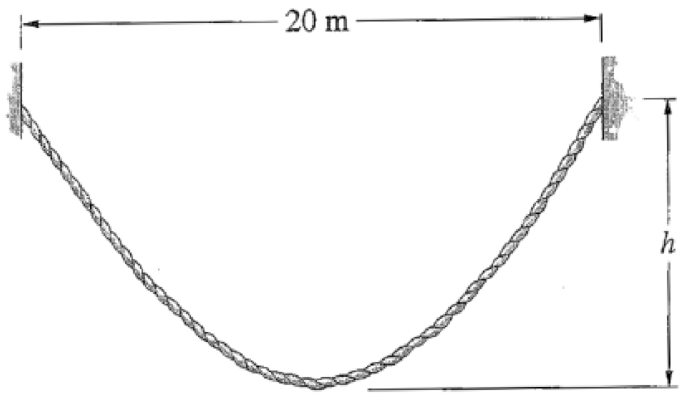


Figure 6 Shape of the parabolic cable

Example 10.8 Cable loaded by its own weight only

The rules for the catenary are quite similar to those for the parabolic cable. They are given in Figure 7. The outputs for the symmetrical span given in the text are in Figure 8. An additional set of outputs for an unequal height catenary are given in Figure 9 to illustrate the more common type of available data. There, the cable length, weight per unit length, and left side height are known. That problem was also list solved over the range of x values to produce the curve shape also in that figure.



Status	Rule
Comment	; Catenary cable with constant unit weight
Satisfied	a = w / H ; ratio of unit cable wt to horizontal point tension
* Unsatisfi	y = (cosh (a * x) - 1) / a ; x & y from horizontal tangent point
* Unsatisfi	T = H * cosh (a * x) ; tension in the cable
* Unsatisfi	T^2 = H^2 + (w * s)^2 ; tension in the cable
* Unsatisfi	a * s = sinh (a * x) ; length from horizontal pt
Satisfied	W = w * L ; total supported load
Satisfied	W_R = w * L_R ; right supported load
Satisfied	W_L = w * L_L ; left supported load
Comment	; equilibrium of right cable segment (H to left at x=y=0)
Satisfied	-W_R + T_R * sin (ang_R) = 0 ; Σ F_y = 0
Satisfied	-H + T_R * cos (ang_R) = 0 ; Σ F_x = 0
Comment	; equilibrium of left cable segment (H to right at x=y=0)
Satisfied	-W_L + T_L * sin (ang_L) = 0 ; Σ F_y = 0
Satisfied	H - T_L * cos (ang_L) = 0 ; Σ F_x = 0
Comment	; related
Satisfied	L = L_L + L_R ; total length
Satisfied	a * L_R = sinh (a * span_R) ; right length
Satisfied	a * L_L = sinh (a * span_L) ; left length
Satisfied	span = span_L + span_R
Satisfied	height_L = (cosh (a * span_L) - 1) / a
Satisfied	height_R = (cosh (a * span_R) - 1) / a
Satisfied	slope_R = sinh (a * span_R)
Satisfied	tan(ang_R) = slope_R
Satisfied	slope_L = sinh (a * span_L)
Satisfied	tan(ang_L) = slope_L

Figure 7 The cantenary cable shape and rules

Input	Name	Output	Unit	Comment
				Catenary cable, constant unit cable weight
				Ex 10.8, 3rd Ed, Bedford Fowler
	a	.1962	1/m	ratio vertical unit load to horizontal tension
	x		m	horizontal position
	y		m	vertical position
	s		m	length of cable from horizontal point
	T		N	tension at x (and y)
20	span		m	horizontal cable span
10	span_L		m	left horizontal span
	span_R	10	m	right horizontal span
	height_L	13.3897	m	left vertical height
	height_R	13.3897	m	right vertical height
9.81	w		N/m	cable weight per unit length
	W	348.6481	N	tota; cable weight
	W_L	174.3241	N	left side weight
	W_R	174.3241	N	right side weight
	ang_L	73.99594	degrees	left end cable angle from horizontal
	ang_R	73.99594	degrees	right end cable angle from horizontal
50	H		N	tension at horizontal tangent (minimum)
	T_L	181.3529	N	left end support tension
	T_R	181.3529	N	right end support tension
	L	35.54008	m	total cable length
	L_L	17.77004	m	left side cable length
	L_R	17.77004	m	right side cable length
	slope_L	3.486481		left end slope
	slope_R	3.486481		right end slope

Figure 8 Results for the example cantenary cable

Input	Name	Output	Unit	Comment
				Catenary cable, constant unit cable weight
				Ex 10.8, 3rd Ed, Bedford Fowler
	a	.0142929	1/ft	ratio vertical unit load to horizontal tension
	x		ft	horizontal position
	y		ft	vertical position
	s		ft	length of cable from horizontal point
	T		N	tension at x (and y)
100	span		ft	horizontal cable span
	span_L	60.67496	ft	left horizontal span
	span_R	39.32504	ft	right horizontal span
28	height_L		ft	left vertical height
	height_R	11.3457	ft	right vertical height
200	w		lb/ft	cable weight per unit length
	W	21999.9	lb	total cable weight
	W_L	13714.22	lb	left side weight
	W_R	8285.681	lb	right side weight
	ang_L	44.42367	degrees	left end cable angle from horizontal
	ang_R	30.63119	degrees	right end cable angle from horizontal
	H	13992.93	lb	tension at horizontal tangent (minimum)
	T_L	19592.91	lb	left end support tension
	T_R	16262.06	lb	right end support tension
110	L		ft	total cable length
	L_L	68.57141	ft	left side cable length
	L_R	41.42859	ft	right side cable length
	slope_L	.980082		left end slope
	slope_R	.5921333		right end slope

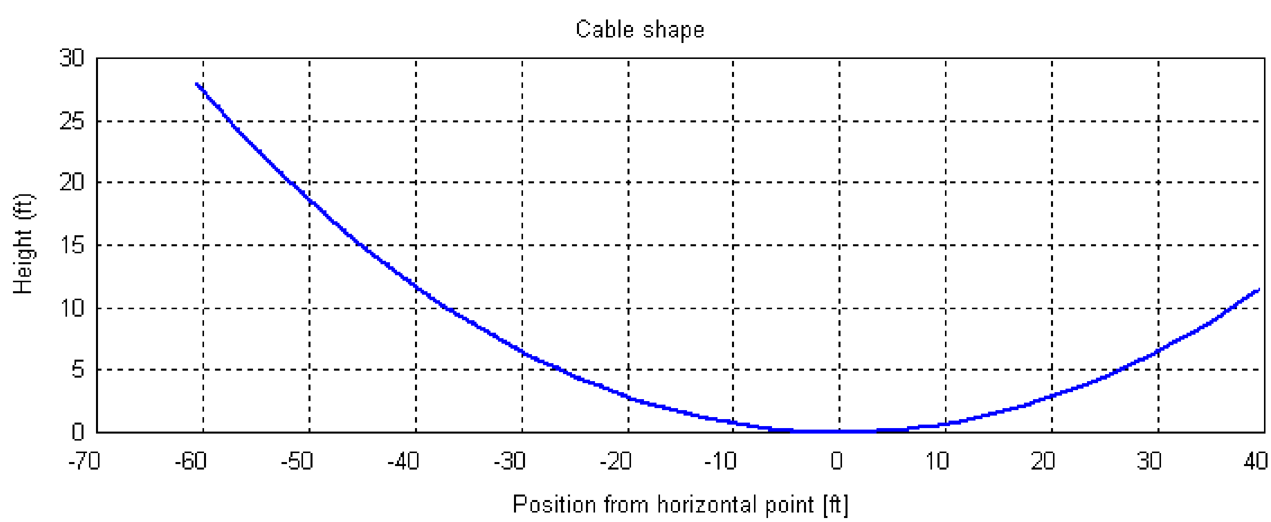


Figure 9 Results for a cantenary cable with two end heights

Example 10.9 Straight cable segments with two discrete loads

The text description of the analysis of a straight segment cable is correct, but incomplete. It describes, from internal equilibrium, how to get (n+1) equations with (n+2) unknowns. It is not necessary to know one of the heights (like h1) to find a solution. By adding the external equations of equilibrium, you can solve a wider range of problems. Since there are an infinite number of cable lengths that will support the same loads, in the same *horizontal* positions, it is necessary to either specify one of the heights or the total length of the cable.

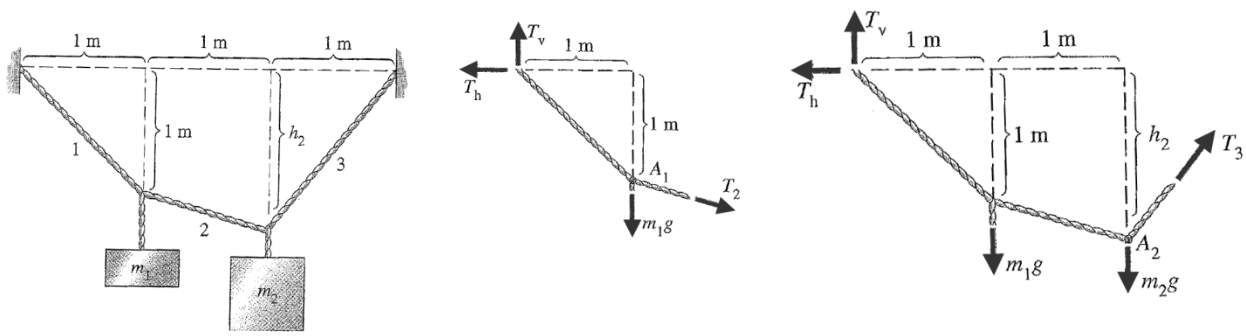


Figure 10 A straight cable segment with two point loads

Figure 10 shows the internal free body diagrams. The rules in Figure 11 add the external equilibrium equations as well. Also, the rules include the special nature of the *first* and *last* cable segments. The length components of the first and last cable form similar triangles to the force polygons for the first and last support tensions. If you did not see that relation, TK would yield the same solution. However, it is good to provide redundant rules for checking or to aid the iterative solver. The results for the given geometry in the text are listed in Figure 12. Note that the length of each segment is computed as well. When a height is specified the length is determined. When the cable length is given, all of the heights are determined. That result is given in Figure 13 where the length of the cable has been specified as 4,000 mm instead of the slightly longer one obtained in the first solution. Since the above set of rules are special (no loads between the first and last), they were expanded to solve a larger problem such as the one in problem P10.70.

Example P10.70 Cable with three point loads

The cable system of Figure 14 has at least on internal load in addition to the special first and last cable segments. The Figure 10 rules were extended for the input of a third weight, as given in Figure 15. In addition, rules (Figure 16) from the FBD of the middle weight were added so that all of the cable segment tension can be found. (Try adding those equations to the previous two-load cable example.) When the spans and first height are specified, you obtain the answers given in the text for the other two heights. The lengths of the cable segments were also output in Figure 17. A slightly shorter cable length of 1,900 mm was specified in Figure 18, and all three internal weight heights necessary to satisfy equilibrium were obtained.

<table>
<tr><th>Status</th><th>Rule</th></tr>
<tr><td>Comment</td><td>; Chap 10, Discrete Loaded Cable, with n=2 loads, r = n + 1</td></tr>
<tr><td>Comment</td><td>; T1_h b1 bn br, Σ b = span</td></tr>
<tr><td>Comment</td><td>; <— * ....</td></tr>
<tr><td>Comment</td><td>; ^ : h1 : hn : hr</td></tr>
<tr><td>Comment</td><td>; | T1_v : : * —> Tr_h</td></tr>
<tr><td>Comment</td><td>; | W1 | Wn ^ Tr_v</td></tr>
<tr><td>Comment</td><td>; v v |</td></tr>
<tr><td>Comment</td><td>; geometry (n=2 only)</td></tr>
<tr><td>Satisfied</td><td>span = b1 + bn + br</td></tr>
<tr><td>Satisfied</td><td>L1^2 = b1^2 + h1^2</td></tr>
<tr><td>Satisfied</td><td>Ln^2 = bn^2 + (hn - h1)^2</td></tr>
<tr><td>Satisfied</td><td>Lr^2 = br^2 + Δh^2</td></tr>
<tr><td>Satisfied</td><td>L = L1 + Ln + Lr</td></tr>
<tr><td>Satisfied</td><td>Δh = hn - hr</td></tr>
<tr><td>Satisfied</td><td>Weight = W1 + Wn</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Comment</td><td>; external force equilibrium</td></tr>
<tr><td>Satisfied</td><td>T1_v + Tr_v = Weight ; external reaction Σ F_y = 0</td></tr>
<tr><td>Satisfied</td><td>-T1_h + Tr_h = 0 ; external reaction Σ F_x = 0</td></tr>
<tr><td>Satisfied</td><td>T1^2 = T1_h^2 + T1_v^2 ; left resultant reaction</td></tr>
<tr><td>Satisfied</td><td>Tr^2 = Tr_h^2 + Tr_v^2 ; right resultant reaction</td></tr>
<tr><td>Comment</td><td>; external moment equilibrium</td></tr>
<tr><td>Satisfied</td><td>Tr_h * hr + Tr_v * span - W1 * b1 - Wn * (b1 + bn) = 0 ; ΣM_0</td></tr>
<tr><td>Satisfied</td><td>T1_h * hr - T1_v * span + W1 * (bn + br) + Wn * br = 0 ; ΣM_r</td></tr>
<tr><td>Comment</td><td>; internal section moment equilibrium</td></tr>
<tr><td>Satisfied</td><td>T1_h * h1 - T1_v * b1 = 0 ; Σ M_1 = 0</td></tr>
<tr><td>Satisfied</td><td>T1_h * hn - T1_v *(b1 + bn) + W1 * bn = 0 ; Σ M_n = 0</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Comment</td><td>; FBD of W1</td></tr>
<tr><td>Satisfied</td><td>T2_h = T1_h ; Σ F_x = 0</td></tr>
<tr><td>Satisfied</td><td>T1_v - W1 + T2_v = 0 ; Σ F_y = 0</td></tr>
<tr><td>Satisfied</td><td>T2^2 = T2_h^2 + T2_v^2</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Comment</td><td>; similar triangles (optional)</td></tr>
<tr><td>Satisfied</td><td>T1_h = T1 * b1 / L1</td></tr>
<tr><td>Satisfied</td><td>T1_v = T1 * h1 / L1</td></tr>
<tr><td>Satisfied</td><td>Tr_v = Tr * Δh / Lr</td></tr>
<tr><td>Satisfied</td><td>Tr_h = Tr * br / Lr</td></tr>
</table>

Figure 11 Special case rules for cable with two point loads

Input	Name	Output	Unit	Comment
				Bedford-Fowler 3rd Ed., Example 10.9
	T1	184.9791	N	left reaction force
	T1_h	130.8	N	left reaction force horizontal component (to left)
	T1_v	130.8	N	left reaction force vertical component (upwards)
1000	b1		mm	horizontal distance from left to first weight
1000	h1		mm	
98.1	W1		N	first weight
1000	bn		mm	horizontal distance from first to weight n=2
	hn	1250	mm	
196.2	Wn		N	last weight
1000	br		mm	horizontal distance from weight n to right support
0	hr		mm	
	span	3000	mm	horizontal span of total cable
	Weight	294.3	N	total weight supported by the cable
	Tr	209.3822	N	right reaction force
	Tr_h	130.8	N	right reaction force horizontal component (to right)
	Tr_v	163.5	N	right reaction force vertical component (upwards)
	T2	134.8256	N	second cable segment tension
	T2_h	130.8	N	second cable horizontal component (to right)
	T2_v	-32.7	N	second cable vertical component (upwards)
	L	4045.771	mm	total cable length
	L1	1414.214	mm	length of cable 1
	Ln	1030.776	mm	length of cable n
	Lr	1600.781	mm	length of last cable
	Δh	1250	mm	change in height of last cable

Figure 12 Support and cable tensions for two-point loaded cable with h1 given

Input	Name	Output	Unit	Comment
				Bedford-Fowler 3rd Ed., Example 10.9
	T1	187.5046	N	left reaction force
	T1_h	134.3478	N	left reaction force horizontal component (to left)
	T1_v	130.8	N	left reaction force vertical component (upwards)
1000	b1		mm	horizontal distance from left to first weight
	h1	973.5921	mm	
98.1	W1		N	first weight
1000	bn		mm	horizontal distance from first to weight n=2
	hn	1216.99	mm	
196.2	Wn		N	last weight
1000	br		mm	horizontal distance from weight n to right support
0	hr		mm	
	span	3000	mm	horizontal span of total cable
	Weight	294.3	N	total weight supported by the cable
	Tr	211.6166	N	right reaction force
	Tr_h	134.3478	N	right reaction force horizontal component (to right)
	Tr_v	163.5	N	right reaction force vertical component (upwards)
	T2	138.2702	N	second cable segment tension
	T2_h	134.3478	N	second cable horizontal component (to right)
	T2_v	-32.7	N	second cable vertical component (upwards)
4000	L		mm	total cable length
	L1	1395.665	mm	length of cable 1
	Ln	1029.195	mm	length of cable n
	Lr	1575.14	mm	length of last cable
	Δh	1216.99	mm	change in height of last cable

Figure 13 Support and cable tensions for two-point loaded cable with length given

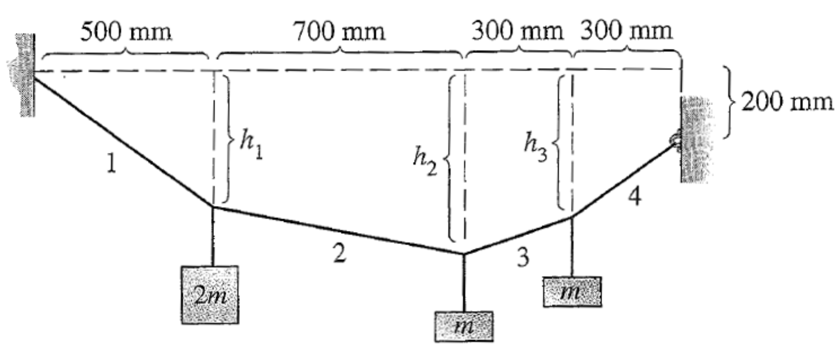


Figure 14 Cable with n=3 point loads

<table>
<tr><th>Status</th><th>Rule</th></tr>
<tr><td>Comment</td><td>; Chap 10, Discrete Loaded Cable, with n=3 loads, r = n + 1</td></tr>
<tr><td>Comment</td><td>; T1_h b1 b2 bn br, Σ b = span</td></tr>
<tr><td>Comment</td><td>; <— * ....</td></tr>
<tr><td>Comment</td><td>; ^ : h1 : h2 : hn : hr</td></tr>
<tr><td>Comment</td><td>; | T1_v : : : * —> Tr_h</td></tr>
<tr><td>Comment</td><td>; | W1 | W2 | Wn ^ Tr_v</td></tr>
<tr><td>Comment</td><td>; v v v |</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Comment</td><td>; geometry</td></tr>
<tr><td>Satisfied</td><td>span = b1 + b2 + bn + br</td></tr>
<tr><td>Satisfied</td><td>L1^2 = b1^2 + h1^2</td></tr>
<tr><td>Satisfied</td><td>L2^2 = b2^2 + (h2 – h1)^2</td></tr>
<tr><td>Satisfied</td><td>Ln^2 = bn^2 + (hn – h2)^2</td></tr>
<tr><td>Satisfied</td><td>Lr^2 = br^2 + Δh^2</td></tr>
<tr><td>Satisfied</td><td>L = L1 + L2 + Ln + Lr</td></tr>
<tr><td>Satisfied</td><td>Δh = hn – hr</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Satisfied</td><td>Weight = W1 + W2 + Wn</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Comment</td><td>; external force equilibrium</td></tr>
<tr><td>Satisfied</td><td>T1_v + Tr_v = Weight ; external reaction</td></tr>
<tr><td>Satisfied</td><td>–T1_h + Tr_h = 0 ; external reaction</td></tr>
<tr><td>Satisfied</td><td>T1^2 = T1_h^2 + T1_v^2 ; left resultant reaction</td></tr>
<tr><td>Satisfied</td><td>Tr^2 = Tr_h^2 + Tr_v^2 ; right resultant reaction</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Comment</td><td>; external moment equilibrium</td></tr>
<tr><td>Satisfied</td><td>Tr_h * hr + Tr_v * span – W1 * b1 – W2 * (b1 + b2) – Wn * (b1 + b2 + bn) = 0 ; ΣM_0</td></tr>
<tr><td>Satisfied</td><td>T1_h * hr – T1_v * span + W1 * (b2 + bn + br) + W2 * (bn + br) + Wn * br = 0; ΣM_r</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Comment</td><td>; internal section moment equilibrium</td></tr>
<tr><td>Satisfied</td><td>T1_h * h1 – T1_v * b1 = 0 ; Σ M_1 = 0</td></tr>
<tr><td>Satisfied</td><td>T1_h * h2 – T1_v *(b1 + b2) + W1 * b2 = 0 ; Σ M_2 = 0</td></tr>
<tr><td>Satisfied</td><td>T1_h * hn – T1_v *(b1 + b2+ bn) + W1 * (b2 + bn) + W2 * bn = 0 ; Σ M_3 = 0</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Comment</td><td>; similar triangles</td></tr>
<tr><td>Satisfied</td><td>T1_h = T1 * b1 / L1</td></tr>
<tr><td>Satisfied</td><td>T1_v = T1 * h1 / L1</td></tr>
<tr><td>Satisfied</td><td>Tr_v = Tr * Δh / Lr</td></tr>
<tr><td>Satisfied</td><td>Tr_h = Tr * br / Lr</td></tr>
</table>

Figure 15 Positions and support rules for cable with n=3 point loads

Status	Rule
Comment	; additional tension segments (FBD of each weight)
Satisfied	T1_v - W1 + T2_v = 0 ; W1 FBD
Satisfied	-T1_h + T2_h = 0
Satisfied	Δh2 = h2 - h1
Satisfied	T2_h = T2 * b2 / L2
Satisfied	T2_v = -T2 * Δh2 / L2
Satisfied	-T2_v - W2 + Tn_v = 0 ; W2 FBD, reverse T2
Satisfied	-T2_h + Tn_h = 0
Satisfied	Δh3 = hn - h2
Satisfied	Tn_h = Tn * bn / Ln
Satisfied	Tn_v = -Tn * Δh3 / Ln

Figure 16 Additional rules to recover remaining cable tensions

Input	Name	Output	Unit	Comment
				Bedford-Fowler 3rd Ed., Example 10.9
				Data from P10.70
	T1	1063.796	N	left reaction force
	T1_h	830.6855	N	left reaction force horizontal component (to left)
	T1_v	664.5484	N	left reaction force vertical component (upwards)
500	b1		mm	horizontal distance from left to first weight
400	h1		mm	
588.6	W1		N	first weight
700	b2		mm	horizontal distance from first to second weight
	h2	464	mm	
294.3	W2		N	second weight
300	bn		mm	horizontal distance from second to weight n
	hn	385.1429	mm	
294.3	Wn		N	third weight
300	br		mm	horizontal distance from weight n to right support
200	hr		mm	
	span	1800	mm	horizontal span of total cable
	Weight	1177.2	N	total weight supported by the cable
	Tr	976.1404	N	right reaction force
	Tr_h	830.6855	N	right reaction force horizontal component (to right)
	Tr_v	512.6516	N	right reaction force vertical component (upwards)
	L	2005.954	mm	total cable length
	L1	640.3124	mm	length of cable 1
	L2	702.9196	mm	length of cable 2
	Ln	310.191	mm	length of cable n
	Lr	352.5307	mm	length of last cable
	Δh	185.1429	mm	change in height of last cable
	Δh2	64	mm	change in height from first weight
	T2	834.1502	N	second cable force
	T2_h	830.6855	N	second cable force horizontal component (to right)
	T2_v	-75.94839	N	second cable force vertical component (upward)
	Δh3	-78.85714	mm	change in height from second weight
	Tn	858.9038	N	third (n) cable force
	Tn_h	830.6855	N	third cable force horizontal component (to right)
	Tn_v	218.3516	N	third cable force vertical component (upward)

Figure 17 Three load cable results with h1 given

Input	Name	Output	Unit	Comment
				Bedford-Fowler 3rd Ed., Example 10.9
				Data from P10.70, but all h unknown
	T1	1448.957	N	left reaction force
	T1_h	1261.708	N	left reaction force horizontal component (to left)
	T1_v	712.4398	N	left reaction force vertical component (upwards)
500	b1		mm	horizontal distance from left to first weight
	h1	282.3315	mm	
588.6	W1		N	first weight
700	b2		mm	horizontal distance from first to second weight
	h2	351.0383	mm	
294.3	W2		N	second weight
300	bn		mm	horizontal distance from second to weight n
	hn	310.5074	mm	
294.3	Wn		N	third weight
300	br		mm	horizontal distance from weight n to right support
200	hr		mm	
	span	1800	mm	horizontal span of total cable
	Weight	1177.2	N	total weight supported by the cable
	Tr	1344.585	N	right reaction force
	Tr_h	1261.708	N	right reaction force horizontal component (to right)
	Tr_v	464.7602	N	right reaction force vertical component (upwards)
1900	L		mm	total cable length
	L1	574.2047	mm	length of cable 1
	L2	703.3638	mm	length of cable 2
	Ln	302.7255	mm	length of cable n
	Lr	319.7059	mm	length of last cable
	Δh	110.5074	mm	change in height of last cable
	Δh2	68.70674	mm	change in height from first weight
	T2	1267.771	N	second cable force
	T2_h	1261.708	N	second cable force horizontal component (to right)
	T2_v	-123.8398	N	second cable force vertical component (upward)
	Δh3	-40.53084	mm	change in height from second weight
	Tn	1273.171	N	third (n) cable force
	Tn_h	1261.708	N	third cable force horizontal component (to right)
	Tn_v	170.4602	N	third cable force vertical component (upward)

Figure 18 Three load cable results with cable length given, heights unknown

