Objects in Equilibrium

Introduction

You have seen that in two dimensions you get three static equilibrium equations per FBD , while in three dimensions you get six static equilibrium equations per FBD. In two dimensions, the requirements that the resultant force vanish are $\sum F_x = 0$ and $\sum F_y = 0$. The requirement that the resultant couple (normal to the plane) vanish is $\sum M_A = 0$, where A is an arbitrary point. These are not the only independent set of three equilibrium equations. You could write one force equilibrium and two moment equations. Or, you could write three moment equilibrium equations if, and only if, the three points do not line on a straight line.

The moment equilibrium equations offer the user the choice of which point(s) or line(s) to select. Generally, it is best to select moment calculation points that have the most unknown force components. There is no unique way to solve most problems. Just make sure that you have at least as many equations of equilibrium as unknown forces or moments. The following examples reinforce these concepts.

If the number of external unknowns acting on a system exceeds the number of independent equations of equilibrium (3 or 6), then it is defined to be statically indeterminate. Often, such systems have connecting parts whose individual FBDs will be statically determinate. Then, you can typically work outwards to determine the external reactions as well.

Example 5.2

This two-dimensional system, shown in Figure 1, has three unknowns: force components A_x, A_y and z-moment component M_A. The geometry of the perpendicular lever arms is clear, so it will be easy to write moment equations. Here, the use of force equilibrium yields one equation with one unknown in both the x- and y-directions. Thus, it would be wise to solve them first and use a single moment equation*, taken at any point*, to determine the reaction moment. Selecting point A is the logical choice.

If you do not see the above easy approach, you could solve the problem the long way by writing any three moment equations. While not a recommended approach, this is illustrated in the TK rules of Figure 2 that uses moments at points B, D, and E. That approach requires to give a guess value to any one of the unknowns, as shown in Figure 3. Then the correct outputs are obtained. If you included the three simplest rules as redundant equations, you obtain the same output values, but without having to give any starting guess values. That happens because the two force equilibrium equations were solved first (each having one equation and one unknown). Then, TK case selects the moment about point A as having one equation and one unknown. Finally the original three moment equations were processed as redundant validation results. The three simplest rules that were appended to the Figure 2 rules are presented in Figure 4.

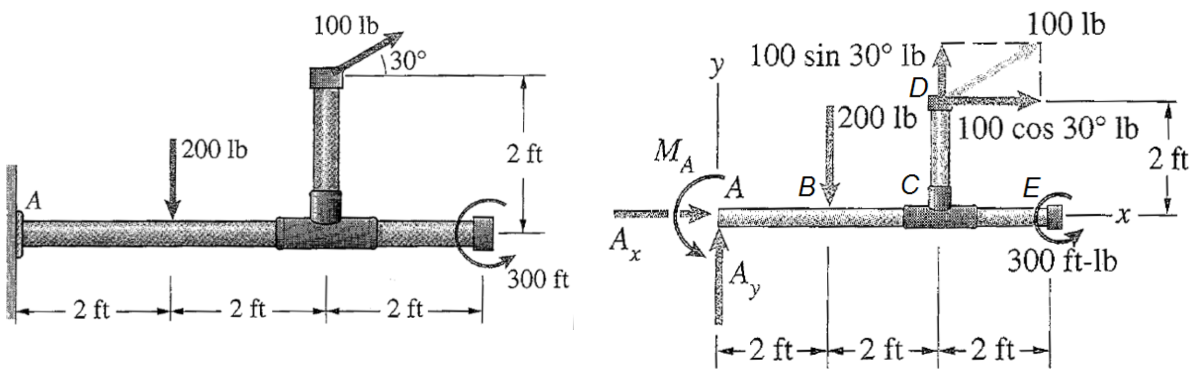

Figure 1 A statically determinate pipe system in equilibrium

Status	Rule
Comment	;Bedford-Fowler Ex 5.2 (the long way)
Comment	; using absolute values of forces and couple
Comment	;Z-moment equilibrium at three points
Satisfied	M_A - Ay * d - Dx * v + Dy * d + M_E = 0 ; Pt B
Satisfied	M_A + Ax * v - Ay * 2 * d + By * d + M_E = 0 ; Pt D
Satisfied	M_A - Ay * 3 * d + By * 2 * d - Dx * v - Dy * d + M_E = 0 ; Pt E
Satisfied	Dx = D * cos (angle)
Satisfied	Dy = D * sin (angle)

Figure 2 Obtaining equilibrium via moment rules only

Status	Input	Name	Output	Unit	Comment
					Bedford-Fowler Ex 5.2 (the long way)
					using absolute values of forces and couple
	30	angle		degrees	force angle at point D
	2	d		ft	horizontal spans
	2	v		ft	vertical height
	200	By		lb	downward force at B
	100	D		lb	magnitude of force at D
		Dx	86.60254	lb	component of D
		Dy	50	lb	component of D
	300	M_E		ft-lb	free end couple
		Ax		lb	wall horizontal force
		Ay		lb	wall vertical force
Guess	100	M_A		ft-lb	wall moment

Input	Name	Output	Unit	Comment
	Ax	-86.60254	lb	wall horizontal force
	Ay	150	lb	wall vertical force
	M_A	73.20487	ft-lb	wall moment

Figure 3 Variables output for the moments only approach

Status	Rule
Comment	; Check forces
Satisfied	Ax + Dx = 0
Satisfied	Ay - By + Dy = 0
Comment	; Check moments
Satisfied	M_A - By * d - Dx * v + Dy * 2 * d + M_E = 0 ; Pt A

Figure 4 Appending the simplest rules avoids needing a starting guess

Example 5.4

This simple system, in Figure 5, has a frictionless vertical slider support on the left. That requires the reaction force A to be horizontal. The top right pin support supplies both horizontal and vertical force components. Thus, those are the three unknowns for this system. If you have enough experience to see that this is a concurrent force system (as indicated by the dashed lines) then you could solve this problem by inspection. The only unknown vertical force is B_y. It has to be equal and opposite to the weight $B_y = W$. Then, from the concurrent force geometry, you can get B_x (acting along WB) from similar triangles. That is, $2\ m : 3\ m = B_x : B_y$ so $B_x = \frac{2}{3}B_y = \frac{2}{3}W$. Horizontal equilibrium requires A to oppose B_x so $A = -B_x = -\frac{2}{3}W$. However, if you do not see that from experience you pick any three equilibrium equations that you can from the FBD. The rules and results, using the absolute values of the forces and the directions assumed in the FBD, are given in Figures 6 and 7, respectively.

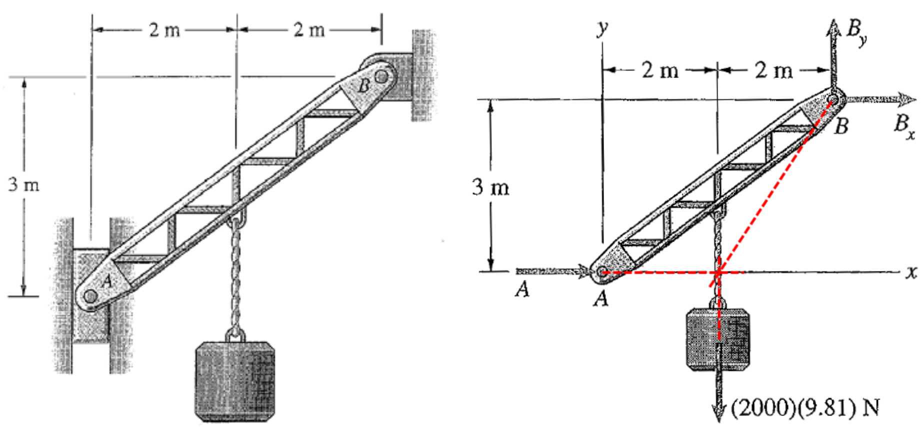

Figure 5 Slider-truss system and its FBD

Status	Rule
Comment	; Bedford-Fowler Ex 5.4, 3-rd Ed
Comment	; Arbitrary equilibrium rules
Comment	; Moment about point B
Satisfied	A * v_AB + W * h_WB = 0
Comment	; Moment where A line intersects W
Satisfied	-Bx * v_AB + By * h_WB = 0
Comment	; Horizontal equilibrium
Satisfied	A + Bx = 0
Comment	; Gravity
Satisfied	W = m * g

Figure 6 Three arbitrary equilibrium rules for the system

Input	Name	Output	Unit	Comment
				Bedford-Fowler Ex 5.4, 3-rd Ed
				Gravity
	W	19620	N	Magnitude of weight
2000	m		kg	Mass
9.81	g		m/sec^2	Gravitational constant
				Geometry
2	h_WB		m	Horizontal distance from W to B
3	v_AB		m	Vertical distance from A to B
				Forces
	A	-13080	N	Horizontal slider force
	Bx	13080	N	Horizontal pin force
	By	19620	N	Vertical pin force

Figure 7 Outputs for the three unknown forces

Example 5.8

The example system and FBD shown in Figure 8 has a ball joint and two cable supports. A ball joint transmits only a force with an unknown direction. Therefore, its unknown components are A_x, A_y and A_z. The cable force directions are known, so only the two magnitudes, T_{BD} and T_{BC} are unknown. That gives a total of five unknowns. The weight is acting at the midpoint of line AB.

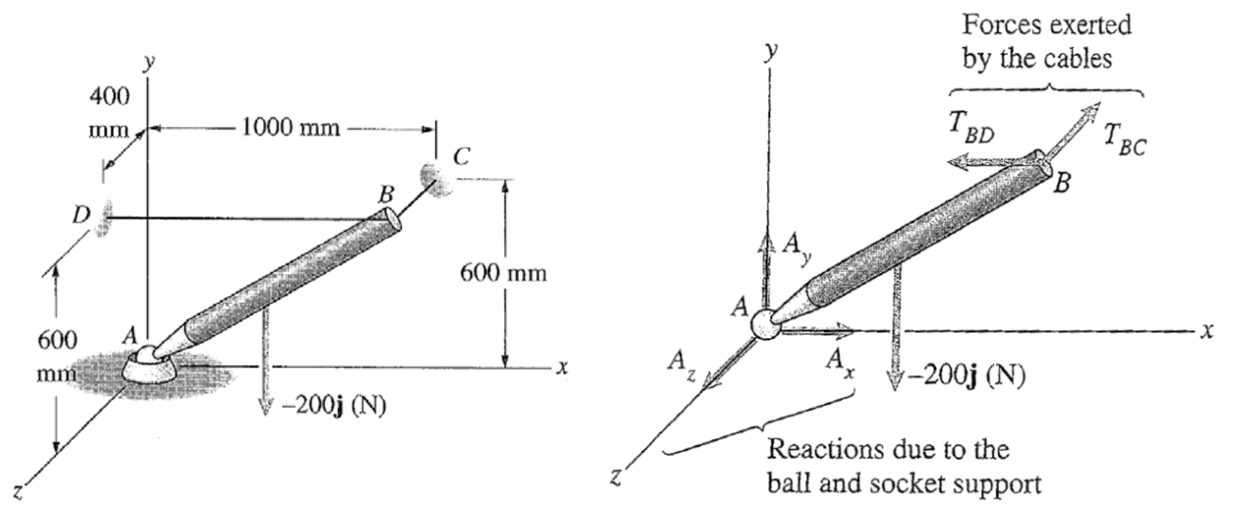

Figure 8 Cylinder ball-cable support system and FBD

This is a useful example of the fact that any force parallel to a line, or axis, does not have a moment about that line. Force T_{BD} is parallel to the x-axis and has no moment component about the x-axis. Force T_{BC} is parallel to the z-axis and has no moment component about the z-axis. Finally, the weight W is parallel to the y-axis and has no moment component about the y-axis. That general rule can let you solve problems faster by omitting calculations that you know will turn out to be zero.

From the FBD you should see that A_x, and T_{BD} will be equal and opposite. The same is true for $A_y$and W, as well as for A_z and T_{BC}. Taking moments about point A eliminates three of the unknowns, and lets you find both tensions. It also gives an optional redundant y-moment component equation. The six rules and their output are given in Figure 9a, and 9b, respectively.

Status	Rule
Comment	; Bedford-Fowler Ex 5.8, 3-rd Ed
Comment	; Moment about point A (weight at midpoint)
Satisfied	r_ABy * (-T_BC + 0 + 0) - r_ABz * (0 + 0 - W/2) = 0 ; Σ M_Ax
Satisfied	r_ABz * (0 - T_BD + 0) - r_ABx * (-T_BC + 0 + 0) = 0 ; Σ M_Ay
Satisfied	r_ABx * (0 + 0 - W / 2) - r_ABy * (0 - T_BD + 0) = 0 ; Σ M_Az
Comment	; Force equilibrium
Satisfied	Ax - T_BD = 0 ; Σ Fx
Satisfied	Ay - W = 0 ; Σ Fy
Satisfied	Az - T_BC = 0 ; Σ Fz

Figure 9a Ball-cylinder-cable system rules

Input	Name	Output	Unit	Comment
				Bedford-Fowler Ex 5.8, 3-rd Ed
				Positions
1	r_ABx		m	x-distance to cables
.6	r_ABy		m	y-distance to cables
.4	r_ABz		m	z-distance to cables
200	W		N	Weight (midway on r_AB)
				Supports
	T_BC	66.66667	N	Cable BC
	T_BD	166.6667	N	Cable BD
	Ax	166.6667	N	Ball joint component
	Ay	200	N	Ball joint component
	Az	66.66667	N	Ball joint component

Figure 9b Ball-cylinder-cable system outputs

Example 5.10

The plate is loaded with a known corner force and supported by two hinges and a cable. The hinge at B also prevents sliding along the z-axis. From the FBD in Figure 10, there are six unknowns. Only the cable force *T* does not intersect the z-axis. An experienced person would note that equilibrium about line BA would eliminate the forces at A and B and therefore give one equation for the one remaining unknown, *T*. If you do not see that, then a standard approach of writing force equilibrium gives three equations. Moment equilibrium about any point (B is the best choice) will give three more equations. You then have the six equations you need for all unknowns. For redundant checking equations, you could take moments about additional points. The arbitrary choice of equilibrium rules are given in Figure 11. The moments about the three lines are essentially the same as the taking the moment about point B, that was used in the text. The text summed t the cable moment, hinge A moment, and corner load moment at the point. That gave the resultant moment components about the x- y- and z- axes. Of course, the three lines picked in Figure 11 are the same axes. So, the text moment equilibrium and those in Figure 11 are the same thing, just written in a different order.

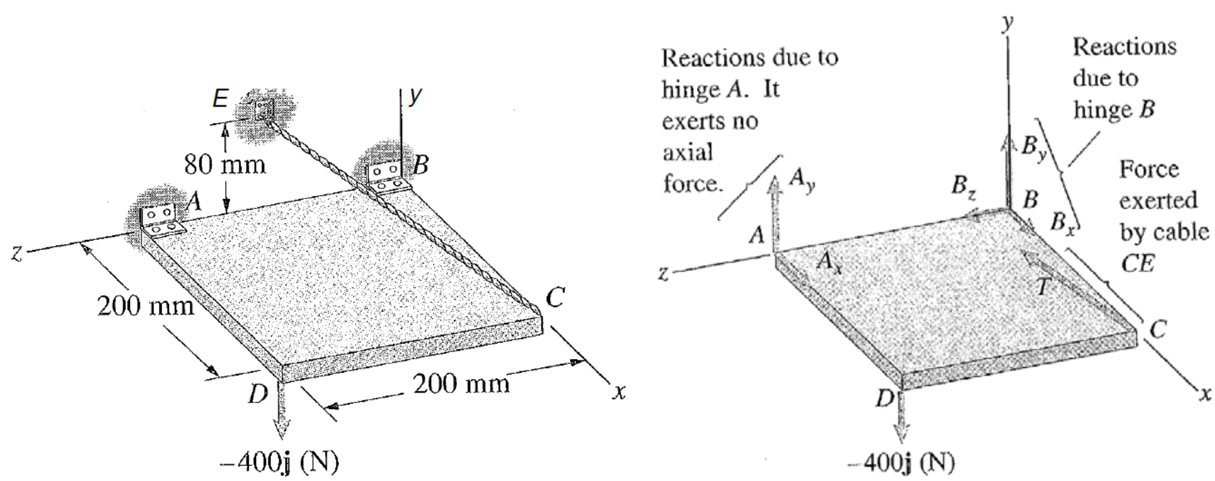

Figure 10 Hinged plate supported with a cable.

Status	Rule
Comment	; Bedford-Fowler Ex 5.10 3rd Ed
Comment	; Moments about three lines
Satisfied	0 - L_BA * (Ay + Dy) = 0 ; Moment about line BC (Ay & Dy only)
Satisfied	L_BC * (Dy + T * e_CEy) - 0 = 0 ; Moment about line BA (Dy & Ty only)
Satisfied	L_BA * Ax - L_BC * T * e_CEz = 0 ; Moment about y-axis (Ax & Tz only)
Comment	; Force equilibrium
Satisfied	Ax + Bx + T * e_CEx = 0 ; Σ F_x = 0
Satisfied	Ay + By + T * e_CEy + Dy = 0 ; Σ F_y = 0
Satisfied	Bz + T * e_CEz = 0 ; Σ F_z = 0
Comment	; Geometry
Satisfied	L_CE^2 = r_CEx^2 + r_CEy^2 + r_CEz^2
Satisfied	e_CEx = r_CEx / L_CE
Satisfied	e_CEy = r_CEy / L_CE
Satisfied	e_CEz = r_CEz / L_CE

Figure 11 Equilibrium and analytic geometry for the hinged plate

The results obtained in Figure 12 agree exactly with those of the text. What would you expect to happen to the cable force if you moved the cable attachment point E up or down (in the y-direction)? Lowering it half way is found to increases the cable tension by almost a factor of two, as shown in Figure 13. Lowering it to line BA would cause the tension to become infinite, and the cable clearly would fail.

Input	Name	Output	Unit	Comment
				Bedford-Fowler Ex 5.10 3rd Ed
.2	L_BA		m	Geometry data
.2	L_BC		m	Geometry data
	L_CE	.2374868	m	Length of cable
-.2	r_CEx		m	Geometry data
.08	r_CEy		m	Geometry data
.1	r_CEz		m	Geometry data
				Forces
	Ax	500	N	Hinge force
	Ay	400	N	Hinge force
	Bx	500	N	Force in restrained hinge
	By	-400	N	Force in restrained hinge
	Bz	-500	N	Force in restrained hinge
-400	Dy		N	Plate vertical corner load
	T	1187.434	N	Cable force
	e_CEx	-.8421519		Direction cosine of cable
	e_CEy	.3368608		Direction cosine of cable
	e_CEz	.421076		Direction cosine of cable

Figure 12 Support forces for the hinged plate

Input	Name	Output	Unit	Comment
-.2	r_CEx		m	Geometry data
.04	r_CEy		m	Geometry data
.1	r_CEz		m	Geometry data
				Forces
	Ax	1000	N	Hinge force
	Ay	400	N	Hinge force
	Bx	1000	N	Force in restrained hinge
	By	-400	N	Force in restrained hinge
	Bz	-1000	N	Force in restrained hinge
-400	Dy		N	Plate vertical corner load
	T	2271.563	N	Cable force

Figure 13 Lowering point E greatly increases the cable tension