Example 6-24

A curved beam of square cross-section is loaded in pure bending. The allowable tension or compression stress is 20 ksi. Determine a) the maximum moment for 20 ksi compression at the outer radius, b) the maximum moment for 20 ksi tension at the inner radius, c) the stress on the opposite side of the beam for the maximum moment, d) the maximum moment from straight beam theory. The beam has a base width of 2 inches and its inner and outer radii are 9 and 11 inches, respectively.

Solution: a) The base width and inner and outer radii were input, and the stress at the outer fiber was input as – 20 ksi. The M_z moment was set as a "Guess" variable. The solution of M_z = 28.5e3 in-lb is seen in Figure 1. Also note that the inner fiber stress for that moment is 22.9 ksi > 20 ksi so you know this option is not workable.

Input	Name	Output	Unit	Comment
				Stress in rectangular curved beams, J.E. Akin 2007
				Ref: Mech. of Elastic Structures, J.T. Oden 1981
				==>> data for Hibbeler Ex6-24 a
2	base		in	base width of rectangle
	depth	2	in	radial depth of recrangle
9	r_inner		in	inner radius of the section
11	r_outer		in	outer radius of the section
	R	10	in	radius to centroid (y = 0 = z)
	k	.1	1/in	curvature at centroid, 1 / R. Zero for straight beam.
	A	4	in^2	cross-sectional area
	J_z	1.34139092	in^4	>0, reduces to section moment of inertia I_z for k = 0
	J_y	1.33333333	in^4	>0, reduces to section moment of inertia I_y for k = 0
	I_z	1.33333333	in^4	section moment of inertia
	I_y	1.33333333	in^4	section moment of inertia
0	N_s		lb	axial force normal to section
	M_z	28460.7337	in-lb	moment about z-axis, changes radius R
0	M_y		in-lb	moment about y-axis,
	M_total	28460.7337	in-lb	resultant moment
	M_angle	0	deg	resultant moment plane w.r.t. y-axis
	NA_angle	0	deg	neutral axis angle from z
0	z		in	distance from centroid, normal to curved plane
	y	-1	in	distance from centroid, in curved plane
11	r		in	radius to point y. Move to input for plot, fill its list,
				do a list serve, then plot
-20000	σ_s		psi	normal stress at r,z and y,z in y-s plane
	σ_s_inner	22863.2926	psi	normal stress at r_inner,z
	σ_s_outer	-20000	psi	normal stress at r_outer,z

Figure 1

b) Next, the inner fiber stress is set to +20 ksi and TK uses a “Guess” solve to obtain M_z = 24.9e3 in-lb, as shown in Figure 2. You also note that the outer fiber stress is -17.5 ksi, which is allowed. Thus, you obtain solution c) at the same time. The straight beam result is illustrated along with this curved beam result in Figure 3, and you can see there is very little difference here.

Input	Name	Output	Unit	Comment
				Stress in rectangular curved beams, J.E. Akin 2007
				Ref: Mech. of Elastic Structures, J.T. Oden 1981
				==>> data for Hibbeler Ex6-24 b & c
2	base		in	base width of rectangle
	depth	2	in	radial depth of recrangle
9	r_inner		in	inner radius of the section
11	r_outer		in	outer radius of the section
	R	10	in	radius to centroid (y = 0 = z)
	k	.1	1/in	curvature at centroid, 1 / R. Zero for straight beam.
	A	4	in^2	cross-sectional area
	J_z	1.34139092	in^4	>0, reduces to section moment of inertia I_z for k = 0
	J_y	1.33333333	in^4	>0, reduces to section moment of inertia I_y for k = 0
	I_z	1.33333333	in^4	section moment of inertia
	I_y	1.33333333	in^4	section moment of inertia
0	N_s		lb	axial force normal to section
	M_z	24896.4436	in-lb	moment about z-axis, changes radius R
0	M_y		in-lb	moment about y-axis,
	M_total	24896.4436	in-lb	resultant moment
	M_angle	0	deg	resultant moment plane w.r.t. y-axis
	NA_angle	0	deg	neutral axis angle from z
0	z		in	distance from centroid, normal to curved plane
	y	1	in	distance from centroid, in curved plane
9	r		in	radius to point y. Move to input for plot, fill its list,
				do a list serve, then plot
20000	σ_s		psi	normal stress at r,z and y,z in y-s plane
	σ_s_inner	20000	psi	normal stress at r_inner,z
	σ_s_outer	-17495.2929	psi	normal stress at r_outer,z

Figure 2

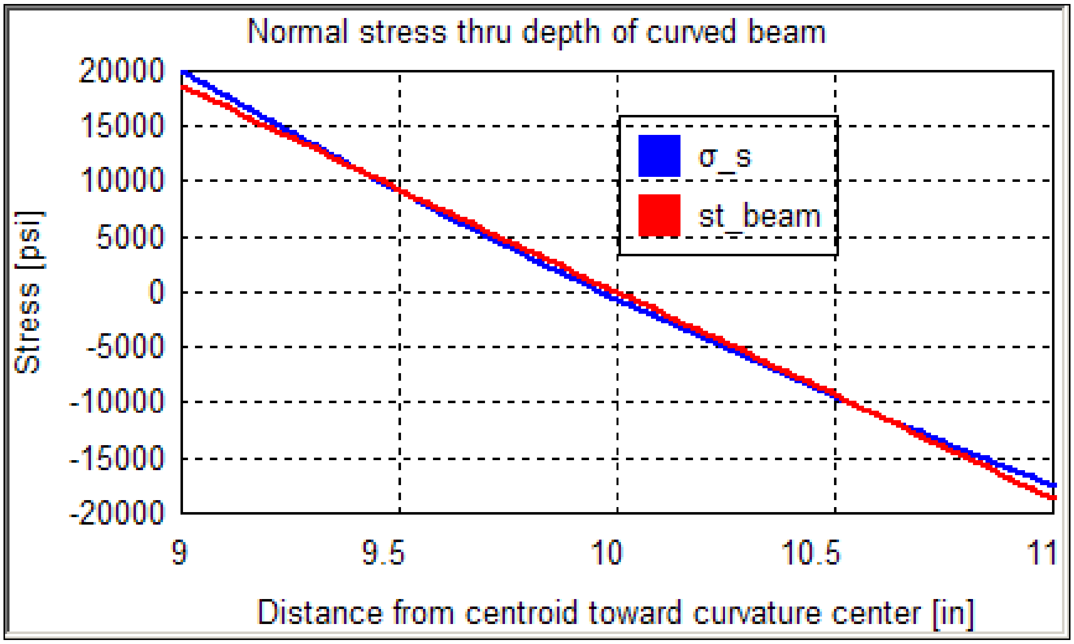

Figure 3

Since the straight beam equations are included for plotting, the answer to part d) is at hand. Just set the inner finer stress to +20 ksi and the moment solution of M_z = 26.7e3 in-lb is obtained, as seen in Figure 4. The corresponding rules are given in Figure 5.

Input	Name	Output	Unit	Comment
				Stress in rectangular curved beams, J.E. Akin 2007
				Ref: Mech. of Elastic Structures, J.T. Oden 1981
				==>> data for Hibbeler Ex6-24 d
2	base		in	base width of rectangle
	depth	2	in	radial depth of recrangle
9	r_inner		in	inner radius of the section
11	r_outer		in	outer radius of the section
	R	10	in	radius to centroid (y = 0 = z)
	k	.1	1/in	curvature at centroid, 1 / R. Zero for straight beam.
	A	4	in^2	cross-sectional area
	J_z	1.34139092	in^4	>0, reduces to section moment of inertia I_z for k = 0
	J_y	1.33333333	in^4	>0, reduces to section moment of inertia I_y for k = 0
	I_z	1.33333333	in^4	section moment of inertia
	I_y	1.33333333	in^4	section moment of inertia
0	N_s		lb	axial force normal to section
	M_z	26666.6667	in-lb	moment about z-axis, changes radius R
20000	st_in		psi	straight beam normal stress at inner radius
	st_out	-20000	psi	straight beam normal stress at outer radius
	st_beam	20000	psi	normal stress in straight beam

Figure 4

Rule
; Normal stress in curved beams, J.E. Akin 2007
; Ref: Mech. of Elastic Structures, J.T. Oden 1981
s_term = (N_s - M_z * k) / A ; work term
y_term = (M_z * J_y - M_y * J_yz) / (J_y * J_z - J_yz^2) ; work term
z_term = (M_y * J_z - M_z * J_yz) / (J_y * J_z - J_yz^2) ; work term
denom = 1 - y * k ; work term
if given('R) then k = 1 / R ; for curved beam, give k=0 for straight beam
if NOT (given('J_y)) then J_y = J_z ; default to circular section
σ_s = s_term + (y_term * y + z_term *z) / denom ; normal stress
σ_s_inner = s_term + (y_term * y_inner+ z_term *z) / (1-y_inner*k) ; normal stress
σ_s_outer = s_term + (y_term * y_outer+ z_term *z) / (1-y_outer*k) ; normal stress
; NEUTRAL AXIS ITEMS equation and angle w.r.t. z : y = m *z + e
if NOT (given ('k)) then e = -s_term / (y_term - s_term) else e = 0
m = -z_term / (y_term - s_term)
NA_angle = atand (m)
R_na = R - e
M_total = sqrt (M_z^2 + M_y^2) ; Resultant moment value
M_angle = atan2d (M_y, M_z) ; Resultant moment plane w.r.t. y-axis
; GENERAL SECTION GEOMETRY
J_z * k^3 = A_m - k * A ; Brickford. Geometric identity
J_z * k^2 = Z * A ; ; Seely-Smith. Tabulated geometric factor
First_s = A + J_z * k^2 ; Geometric identity = integral 1 / (1-y / R) dA
First_y = J_z * k ; Geometric identity = integral y / (1-y / R) dA
First_z = J_yz * k ; Geometric identity = integral z / (1-y / R) dA
r = R - y ; Radius from center of R to y
r_min = R - y_inner ; plot range
r_max = R - y_outer ; y_max < 0, plot range
A_m = First_s * k ; Geometric identity = integral 1 / r dA
; STRAIGHT BEAM COMPARISONS
st_beam = N_s / A + M_z * y / I_z ; straight beam normal stress
st_in = N_s / A + M_z * y_inner/ I_z ; straight beam normal stress at inner radius
st_out = N_s / A + M_z * y_outer/ I_z ; straight beam normal stress at outer radius
[illegible] PROPERTIES

Figure 5

