Form-finding, soap film and membrane analogy in Engineering

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Outline

- Membrane analogy in torsion
- Form finding in civil engineering

Membrane analogy in torsion

- Prandtl in 1903 identified this analogy for the stress distribution of non-circular shafts under torsion
- A membrane is the 2D equivalent of a string under tension
- Example of membrane: a soap film
- Assume the cross-section of the shaft to be covered by a soap film (membrane) having the same shape of the cross-section, with pressure under it that keeps it in tension

Membrane analogy in torsion (cont'd)

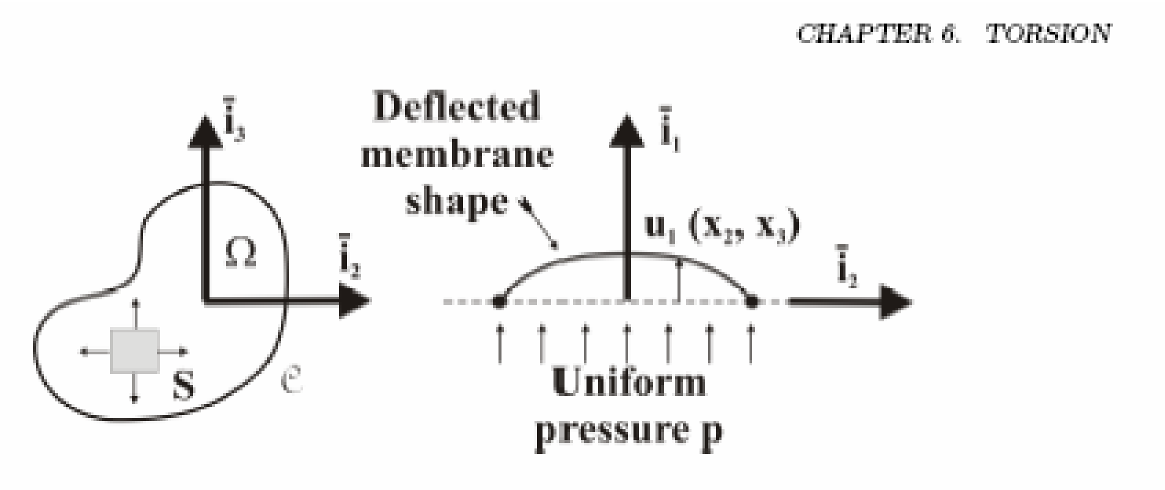
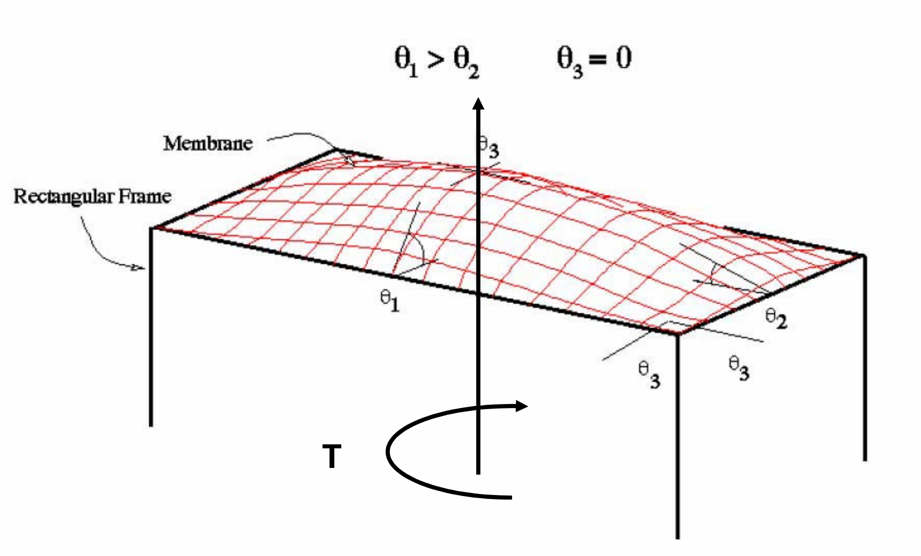


Figure 6.33: The thin membrane attached to the contour C.

Bauchau and Craig notes, August 2006

Membrane analogy in torsion (cont'd)

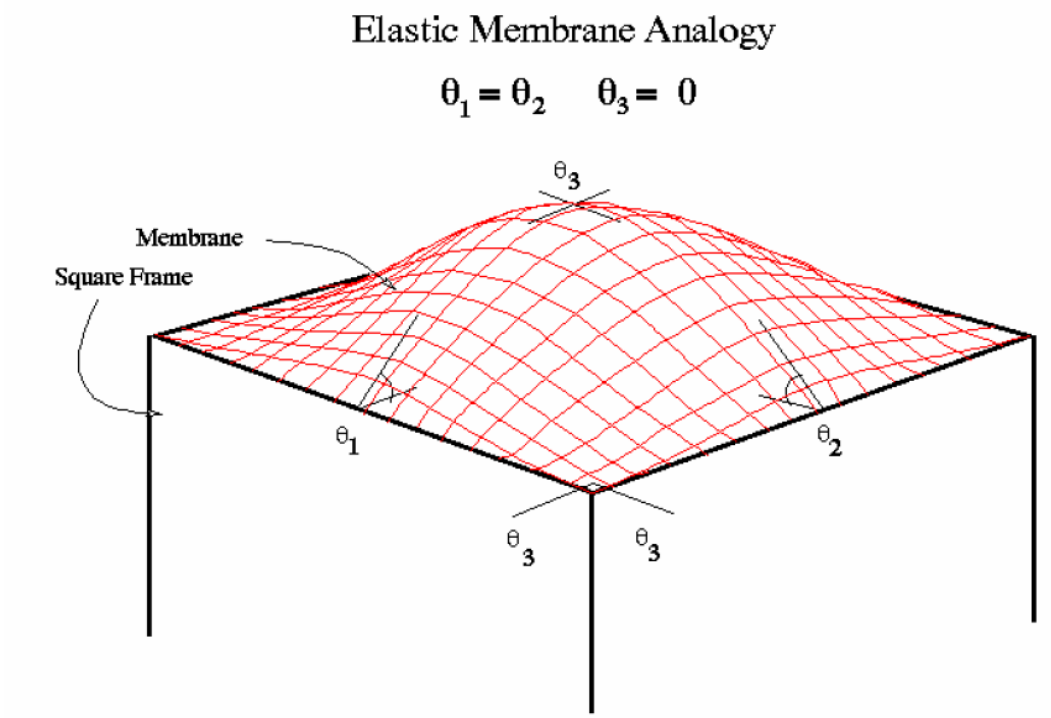


http://www.ae.msstate.edu/%7Emasoud/Teaching/SA2/A6.5_more2.html

Membrane analogy in torsion (cont'd)

	Membrane Problem	Torsion Problem
Main variables	displacement normal to membrane, $u1$ pressure under membrane, p tension along membrane, assumed constant, S	stress function, Φ shear modulus, G twist rate, assumed constant, κ1
Governing PDE	$\frac{\partial^2 u_1}{\partial x_2^2} + \frac{\partial^2 u_1}{\partial x_3^2} = -\frac{p}{S} \quad \text{on } \mathcal{A}.$ $\frac{\mathrm{d}u_1}{\mathrm{d}s} = 0 \quad \text{along } \mathcal{C}.$	$\frac{\partial^2 \Phi}{\partial x_2^2} + \frac{\partial^2 \Phi}{\partial x_3^2} = -2G\kappa_1. \quad \text{on } \mathcal{A}$ $\frac{\mathrm{d}\Phi}{\mathrm{d}s} = 0 \quad \text{along } \mathcal{C}.$
Critical items	Maximum slope, $\frac{\partial u_1}{\partial n}$	Maximum shear stress, $\tau_s = -\frac{\partial \Phi}{\partial n}$
Volume	volume under membrane, $Vol = \int_A u_1 \, dA$	resisting moment $M_1 = 2\int_A \Phi \, dA$

Membrane analogy in torsion (cont'd)



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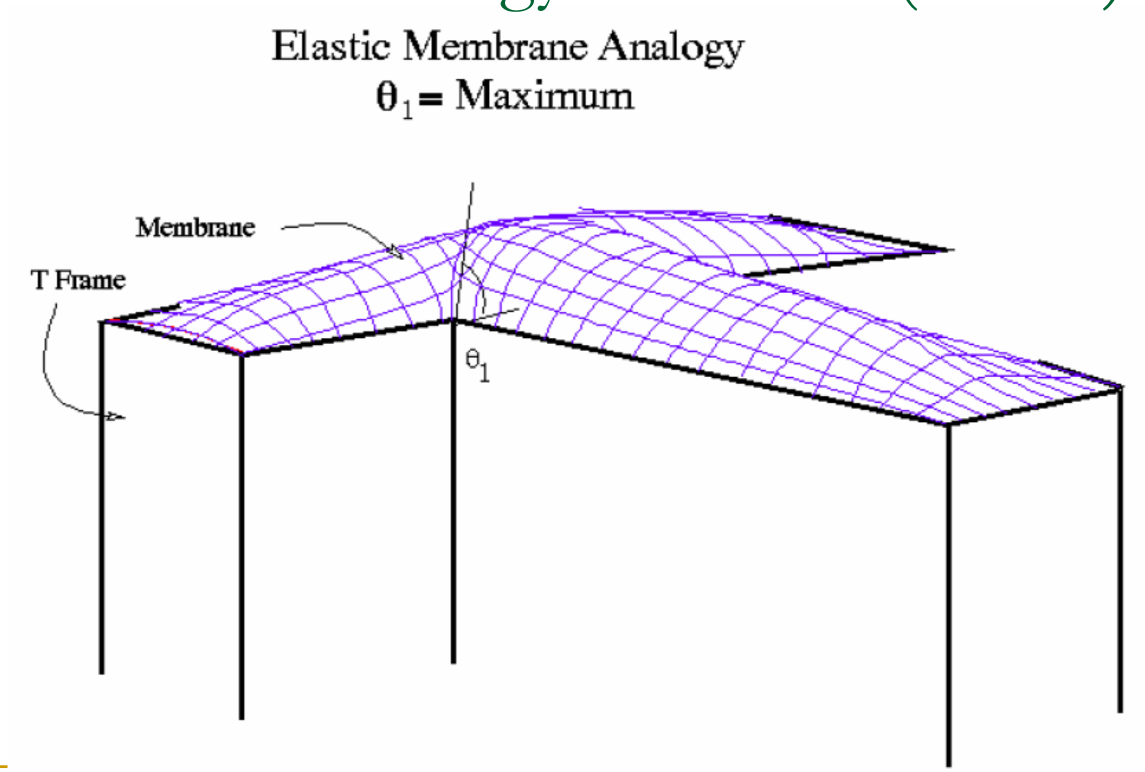
Membrane analogy in torsion (cont'd)



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