13 Concepts of Thermal Analysis

13.1 Introduction

There are three different types of **heat transfer**: conduction, convection, and radiation. A temperature difference must exist for heat transfer to occur. Heat is always transferred in the *direction* of decreasing temperature. Temperature is a scalar, but heat flux is a vector quantity.

Conduction takes place within the boundaries of a body by the diffusion of its internal energy. The temperature within the body, T, is given in units of degrees Celsius [C], Fahrenheit [F], Kelvin [K], or Rankin [R]. Its variation in space defines the temperature gradient vector, ∇T, with units of [K/m] say. The heat flux vector, $\boldsymbol{q}$, is define by Fourier's Conduction Law, as the thermal conductivity, k, times the negative of the temperature gradient, $\boldsymbol{q} = - k \nabla T$. Thermal conductivity has the units of [$W/m\text{-}K$] while the heat flux has units of [W/m^2]. The conductivity, k, is usually only known to two or three significant figures. For solids it ranges from about $417\ W/m\text{-}K$ for silver down to $0.76\ W/m\text{-}K$ for glass.

A perfect insulator material ($k \equiv 0$) will not conduct heat; therefore the heat flux vector must be parallel to the insulator surface. A plane of symmetry (where the geometry, k values, and heat sources are mirror images) acts as a perfect insulator. In finite element analysis, all surfaces default to perfect insulators unless you give a specified temperature, a known heat influx, a convection condition, or a radiation condition.

Convection occurs in a fluid by mixing. Here we will consider only *free convection* from the surface of a body to the surrounding fluid. *Forced convection*, which requires a coupled mass transfer, will not be considered. The magnitude of the heat flux normal to a solid surface by free convection is $q_n = h\, A_h\, (T_h - T_f)$ where h is the convection coefficient, A_h is the surface area contacting the fluid, T_h is the convecting surface temperature, and T_f is the surrounding fluid temperature, respectively. The units of h are [$W/m^2\text{-}K$]. Its value varies widely and is usually known only from one to four significant figures. Typical values for convection to air and water are *5-25* and $500\text{-}1000\ W/m^2\text{-}K$, respectively.

Radiation heat transfer occurs by electromagnetic radiation between the surfaces of a body and the surrounding medium. It is a highly nonlinear function of the absolute temperatures of the body and medium. The magnitude of the heat flux normal to a solid surface by radiation is $q_r = \varepsilon\, \sigma\, A_r\, (T_r^{\,4} - T_m^{\,4})$. Here T_r is the absolute temperature of the body surface, T_m is the absolute temperature of the surrounding medium, A_r is the body surface area subjected to radiation, $\sigma = 5.67 \times 10^{8}\ W/m^2\text{–}K^4$ is the Stefan-Boltzmann constant, and ε is a surface factor ($\varepsilon = 1$ for a perfect black body).

Transient, or unsteady, heat transfer in time also requires the material properties of specific heat at constant pressure, c_p in [$k\ J/kg\text{-}K$], and the mass density, ρ in [kg/m^3]. The specific heat is typically known to 2 or 3 significant figures, while the mass density is probably the most accurately known material property with 4 to 5 significant figures.

13.1.1 One-dimensional thermal-structural analogy

The one-dimensional governing differential equation for transient heat transfer through an area A, of conductivity k_x, density ρ, specific heat c_p with a volumetric of heat generation, Q, for the temperature T at time t is $\partial(k_x\, \partial T/\partial x)/\partial x + Q(x) = \rho\, c_p\, \partial T/\partial t$, for $0 \le x \le L$ and time $t \ge 0$. It requires initial conditions to describe the beginning state, and boundary conditions for later times. For a steady state condition ($\partial T/\partial t = 0$) the typical boundary conditions of one of the following:

1. T prescribed at 0 and L, or
2. T prescribed at one end and a heat source at the other, or

3. T prescribed at one end and a convection condition at the other, or
4. A convection condition at one end and a heat source at the other, or
5. A convection condition at both ends.

These thermal conditions, in 1D, are related to the displacements and stress in an axial bar as summarized in Table 13-1.

Table 13-1 Terms of the 1D thermal-structural analogy

Thermal Analysis Item, [units], symbol	**Structural Analysis Item, [units], symbol**
Unknown: Temperature [K], T	*Unknown:* Displacements [m], u
Gradient: Temperature Gradient [K/m], ∇T	*Gradient:* Strains [m/m], ε
Flux: Heat flux [W/m^2], q	*Flux:* Stresses [N/m^2], σ
Source: Heat Source for point, line, surface, volume [W], [W/m], [W/m^2], [W/m^3], Q	*Source:* Force for point, line, surface, volume [N], [N/m], [N/m^2], [N/m^3], Q
Restraint: Prescribed temperature [K], T	*Restraint:* Prescribed displacement [m], u
Reaction: Heat flow resultant [W], Q	*Reaction:* Force component [N], Q
Material Property: Thermal conductivity [W/m-K], k	*Material Property:* Elastic modulus [N/m^2], E
Material Law: Fourier's law	*Material Law:* Hooke's law

13.1.2 Three-dimensional formulation

In the 3D case the differential equation becomes the anisotropic Poisson Equation (see Chapter 16). That is, the above diffusion term (second derivatives in space) is expanded to include derivatives with respect to y and z, times their corresponding thermal conductivity values.

13.2 Thermal analysis input properties

The thermal material properties available in SW Simulation are listed in Table 13-3. Only the conductivities are theoretically needed for a steady state study, but SW Simulation always requests the mass density. Any transient (time dependent) thermal analysis involves the product of the mass density and specific heat, as seen in the above equation.

Table 13-2 Isotropic thermal properties

Symbol	Label	Item	Application
ρ	DENS	Mass density	Transient
c	C	Specific heat, at constant pressure	Transient
k	KX	Thermal conductivity	Steady state and transient

Table 13-3 Anisotropic thermal properties in principal material directions

Symbol	Label	Item
ρ	DENS	Mass density
c	C	Specific heat, at constant pressure
$\mathbf{k_x}$	KX	Thermal conductivity in material X direction
$\mathbf{k_y}$	KY	Thermal conductivity in material Y direction
$\mathbf{k_z}$	KZ	Thermal conductivity in material Z direction

13.3 Finite Element Thermal Analysis

13.3.1 Thermal rod element

From the above analogy the matrix equations of a single element (from sections 2.3 and 2.4) is

$$k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

where $k \equiv k_x A / L$ may be referred to as the thermal stiffness of the rod of length, L, area ,A, and thermal conductivity k_x. In this case, T corresponds to a nodal temperature, and F corresponds to the resultant nodal heat power from the various heat sources. The typical units of the above three matrices are W/C, C, and W.

13.3.2 Algebraic equations

The finite element method creates a set of algebraic equations by using an equivalent governing integral form that is integrated over a mesh that approximates the volume and surface of the body of interest. The mesh consists of elements connected to nodes. In a thermal analysis, there will be one simultaneous equation for each node. The unknown at each node is the temperature. Today, a typical thermal mesh involves 20,000 to 100,000 nodes and thus temperature equations. The restraints are specified temperatures (or a convection condition since it includes a specified fluid temperature). The reactions are is the resultant heat power necessary to maintain a specified temperature. All other conditions add load or source terms. The default surface condition is an insulated boundary, which results in a zero source (load) term.

The assembled matrix equations for thermal equilibrium have exactly the same partitioned form as the structural systems of section 2.5:

$$\begin{bmatrix} \boldsymbol{K_{uu}} & \boldsymbol{K_{ug}} \\ \boldsymbol{K_{gu}} & \boldsymbol{K_{gg}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{T_u} \\ \boldsymbol{T_g} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F_g} \\ \boldsymbol{F_u} \end{Bmatrix}$$

where now $\mathrm{T}_{\mathbf{g}}$ represents the given (restrained) nodal temperatures, $F_{\boldsymbol{g}}$ represents the known resultant nodal heat power at the node. This system of equations is solved just as described in section 2.5. The thermal restraints items for steady state analysis are given in Table 13-4.

Most programs offer only a temperature restraint. SW Simulation also offers the ability to define a non-ideal material interface, as illustrated in Figure 13-1. This is often needed in practice and is referred to as a contact resistance. It basically defines a temperature jump across an interface for a given heat flux through the interface. The necessary resistance input, R, depends on various factors. The R value is the same concept used is specifying home insulation. Table 13-5 gives typical R values, while Table 13-6 cites values of its reciprocal, the conductance.

The thermal load (source) items for steady state analysis are given in Table 13-7. Both convection and radiation require inputs of the estimated surface conditions. Typical convection coefficients are given in

Table 13-8. Note that there is a wide range in such data. Therefore, you will often find it necessary to run more that one study to determine the range of answers that can be developed in your thermal study. Having supplied all the restraints, loads, and material properties you can run a thermal analysis and continue on to post-processing and documenting the results.

Table 13-4 Restraints in steady state thermal analysis

Restraint Type	Geometric Entities	Required Input
Temperature	Vertexes, edges, faces and parts	Temperature value and units
Contact resistance	Two contacting faces	Total thermal resistance or unit thermal resistance. See discussion.

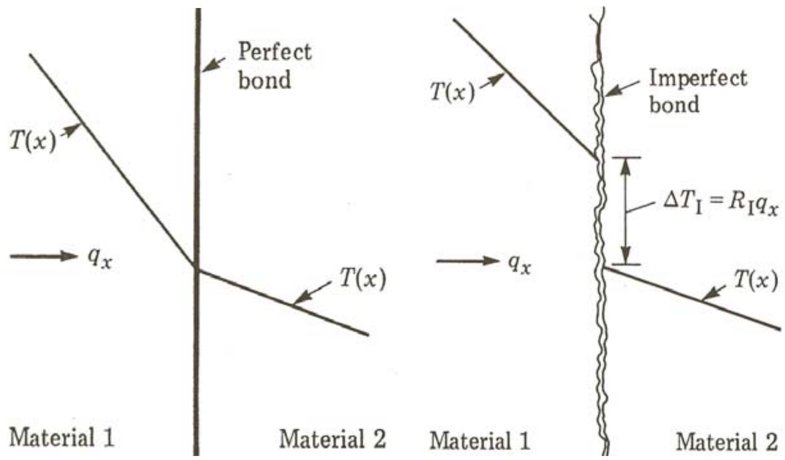


Figure 13-1 Ideal and thermal contact resistance interfaces

Table 13-5 Typical contact resistance values, *R x e4*, [m^2 K/W]

Contact Pressure	"Moderate"	100 kN/m^2	10,000 kN/m^2
Aluminum/aluminum/air	0.5	1.5-5.0	0.2-0.4
Copper/copper/air	0.1	1-10	0.1-0.5
Magnesium/magnesium/air		1.5-3.5	0.2-0.4
Stainless steel/stainless steel/ air	3	6-25	0.7-4.0

Table 13-6 Typical contact conductance values, *C*, [W/m^2 K]

Contacting Faces (pressure unknown)	Conductance
Aluminum / aluminum / air	2200 - 12000
Ceramic / ceramic / air	500 - 3000
Copper / copper / air	10,000 - 25,000
Iron / aluminum / air	45,000
Stainless steel / stainless steel / air	2000 - 3700
Stainless steel / stainless steel / vacuum	200 - 1100

Table 13-7 Loads for steady state thermal analysis

Load Type	Geometric Entities	Required Input
Convection	Faces	Film coefficient and bulk temperature in the desired units
Heat Flux	Faces	Heat flux (heat power/unit area) value in the desired units
Heat Power	Vertexes, edges, faces and parts	Total heat power value and units (rate of heat generation per unit volume times the part volume)
Insulated (Adiabatic)	Faces	None. This is the *default condition* for any face not subject to one of the three above conditions
Radiation	Faces	Surrounding temperature, emissivity values and units, and view factor for surface to ambient radiation

Table 13-8 Typical heat convection coefficient values, *h*, [W/m^2 K]

Fluid Medium	*h*
Air (natural convection)	5-25
Air / superheated steam (forced convection)	10-500
Oil (forced convection)	60-1800
Steam (condensing)	5000-120,000
Water (boiling)	2500-60,000
Water (forced convection)	300-6000

13.3.3 Post-processing

The temperature often depends only on geometry. The heat flux, and the thermal reaction, always depends on the material thermal conductivity. Therefore, it is always necessary to examine both the temperatures and heat flux to assure a correct solution. The heat flux is determined by the gradient (derivative) of the approximated temperatures. Therefore, it is less accurate than the temperatures. The user must make the mesh finer in regions where the heat flux vector is expected to rapidly change its value or direction. The heat flux should be plotted both as magnitude contours, and as vectors.

The temperatures should be plotted as discrete color bands or as contour lines. The temperature contours should be perpendicular to insulated boundaries. Near surfaces with specified temperatures, the contours should be nearly parallel to the surfaces. These “eyeball” checks are illustrated in Figure 13-2. The heat flux vectors should be parallel to insulated surfaces. They should be nearly perpendicular to surfaces with a specified constant temperature. Those flux checks are illustrated in Figure 13-3. The items available for output after a thermal analysis run are given in Table 13-9.

The exact temperature gradient is discontinuous at an interface between different materials because their thermal conductivities will be different. Pretty continuous color contours (the default) tend to prevent these important engineering checks. The temperature and temperature gradient vector can depend only on the geometry in some problems.

In SW Simulation it is possible to list, sum, average, and graph results along selected edges, lines, curves or surfaces. Thus, you should plan ahead and add "split lines" to the mesh where you expect to find such graphs informative. Written results should not be given with more significant figures than the material input data. For heat transfer problems that is typically three or four significant figures.

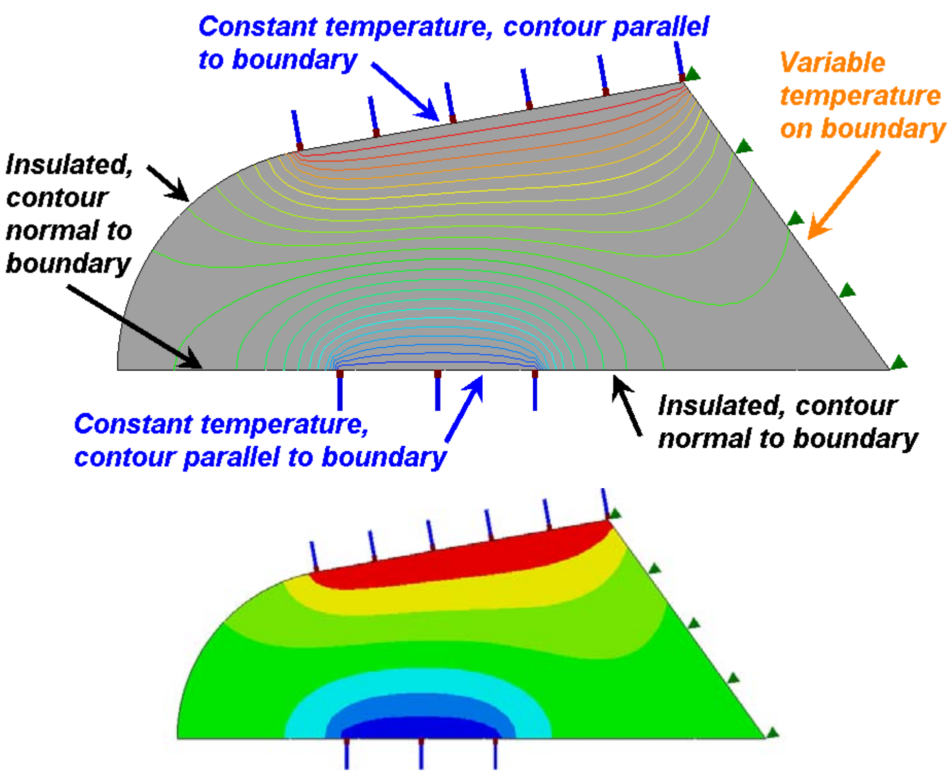


Figure 13-2 Guidelines for checking temperatures

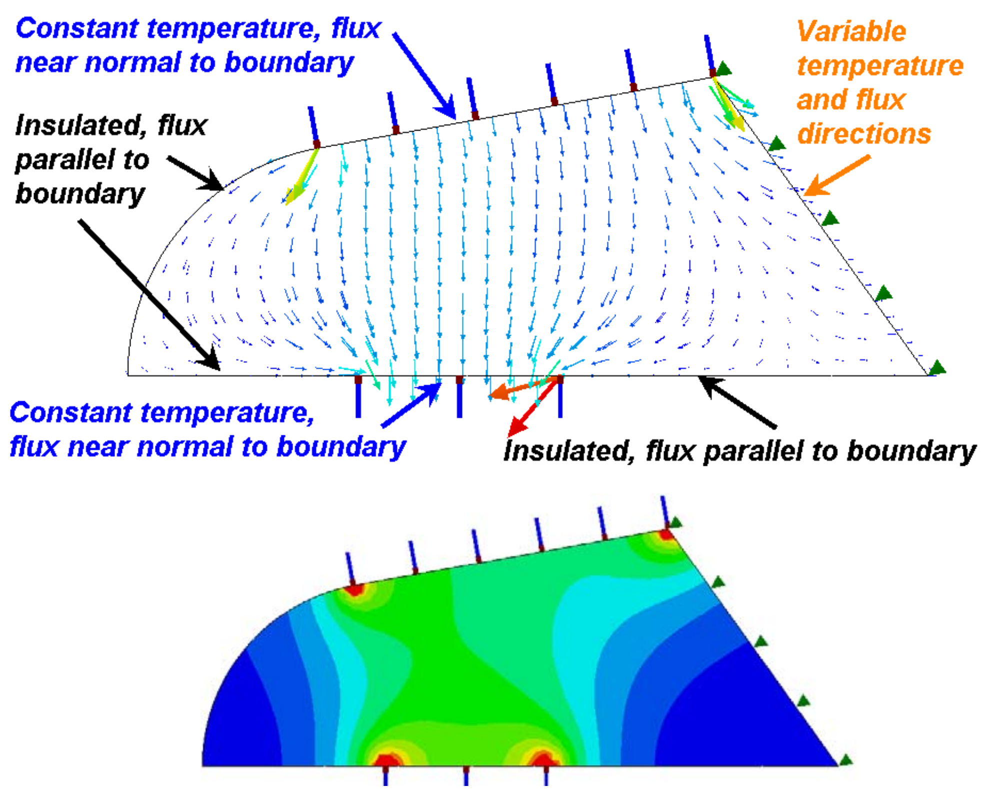


Figure 13-3 Graphical checks for heat flux vectors

Table 13-9 Thermal analysis output options

Symbol	Label	Item
T	TEMP	Temperature
$\partial T/\partial x$	GRADX	Temperature gradient in the selected reference X-direction
$\partial T/\partial y$	GRADY	Temperature gradient in the selected reference Y-direction
$\partial T/\partial z$	GRADZ	Temperature gradient in the selected reference Z-direction
$\|\nabla T\|$	GRADN	Resultant temperature gradient magnitude
q_x	HFLUXX	Heat flux in the X-direction of the selected reference geometry
q_y	HFLUXY	Heat flux in the X-direction of the selected reference geometry
q_z	HFLUXZ	Heat flux in the X-direction of the selected reference geometry
q	HFLUXN	Resultant heat flux magnitude

SW Simulation also offers p-adaptive elements (p is for polynomial). Keeping the mesh unchanged, it can automatically run a series of cases where it uses complete second, third, fourth, and finally fifth order polynomial interpolations. It allows the user to specify the allowable amount of error. That is, it can solve a given problem quite accurately. However, you still must define the geometry, materials, load and restraint locations, and load and restraint values as well as interpret the results properly. You still have the age old problem of garbage-in garbage-out, so avoid computer aided stupidity.

13.4 Classical 1D thermal solutions

There are a few well know thermal problems that have known simple solutions that give you some insight into the phenomenon and are easily verified with a SW Simulation analysis. A few of these will be presented in the following sections. The first of these is a planar wall with a temperature difference on each side. This is often approximated as a semi-infinite wall, which reduces the problem to a one-dimensional study. The solution [5] shows that the temperature through the wall is linear in space. Therefore, the heat flux, per unit area, will be constant. Any finite element model should give the exact result everywhere [2].

13.4.1 Heat transfer through a planar wall

The heat transfer through a wall will be illustrated by a SW Simulation model. It could be solved with a single layer of elements through the wall. Here it is assumed that the analytic solution is not known, so several thousand unknowns are used to clearly illustrate the response. The wall in this case is five inches thick and made of alloy steel. A unit cross-sectional area is used. The outer (left) side is kept at *100 F* while the inner side is at *0 F*. Those two restraints must be explicitly applied. The other four faces of the body are planes of symmetry and are automatically treated as insulated. The mesh is shown along with the resulting linear temperature drop distribution. The linear temperature change with position is clearly seen in Figure 13-4.

Note that at a position 40 % through the wall the temperature difference has dropped 40 % to *60 F*. This result will be compared to a cylindrical wall later. The heat flux should be constant. Constant values do not contour well so the contour bounds must be set to give a reasonable plot. The flux values at the inlet and outlet faces are selected and listed in tables shown in Figure 13-5. It shows that each square inch of the outer wall requires about *0.0134 BTU/s* of power to maintain the outer temperature. For a planar wall made up of constant thickness layers of different materials the heat flux must still remain constant, but the temperature difference will occur as linear changes from one interface to the next. The linear distribution of temperature is more easily seen with a graph along one edge of the mesh.

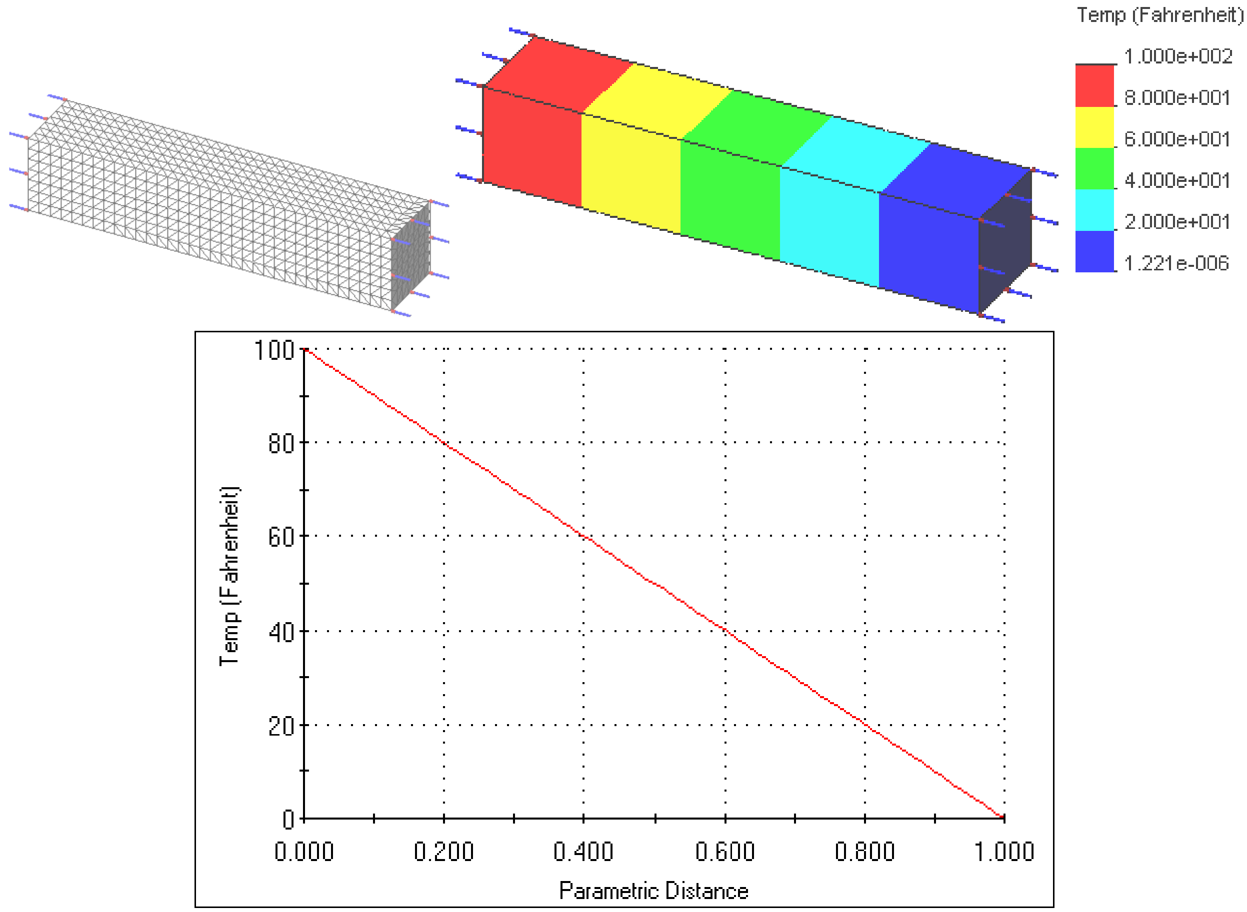
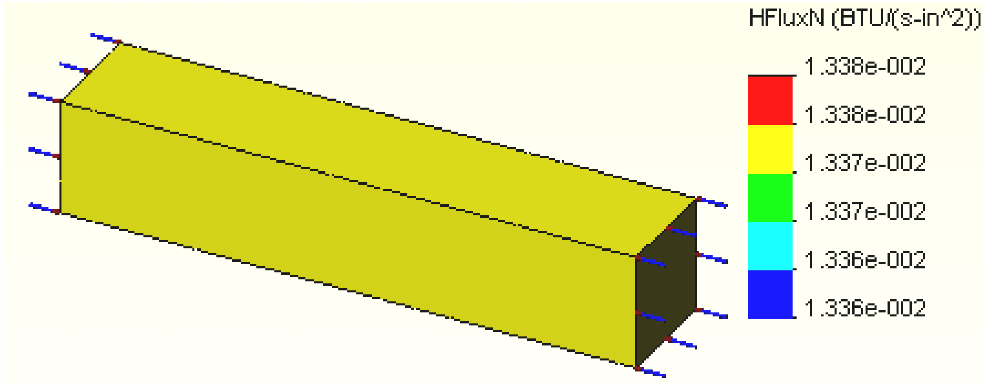


Figure 13-4 Temperatures of a homogeneous wall

13.4.2   Heat transfer through cylindrical walls or pipes

Another well known heat transfer problem with a simple analytic solution is that of radial conduction through an infinite pipe, or curved wall. In that case, the temperature difference varies in a logarithmic manner through the wall thickness. That means that the heat flux must also vary through the wall, since it passes through more material as the radius increases.

The example here [4], will be for an alloy steel pipe with an inner radius of *10 inches* and with a thickness of *5 inches*. Thus it is very similar to the previous example having inner and outer temperatures of *100 F* and *0 F*, respectively. In this case, each of those restraints is applied to cylindrical faces. The other four faces are insulated and do not require specific action. The geometry, a very fine mesh, the resulting temperature contours, and the radial variation of the temperature are given in Figure 13-6. The contour plot there might appear to again be linear, but the graph of the temperature along a radial edge is actually logarithmic. Compared to Figure 13-4, you see that at a distance of 40 % through the wall the temperature has dropped more than 40 % to about *56.4 F*. The non-constant nature of the corresponding heat flux is seen in the contour plot and in the radial edge heat flux graph of Figure 13-7.



Summary

	Value	Units
Total Heat	0.013375	BTU/s
Avg	0.013375	BTU/(s-in^2
Max	0.013375	BTU/(s-in^2
Min	0.013375	BTU/(s-in^2
RMS	0.013375	BTU/(s-in^2

Summary

	Value	Units
Total Heat	-0.013375	BTU/s
Avg	0.013375	BTU/(s-in^2
Max	0.013375	BTU/(s-in^2
Min	0.013375	BTU/(s-in^2
RMS	0.013375	BTU/(s-in^2

Figure 13-5 Constant heat flux through a wall

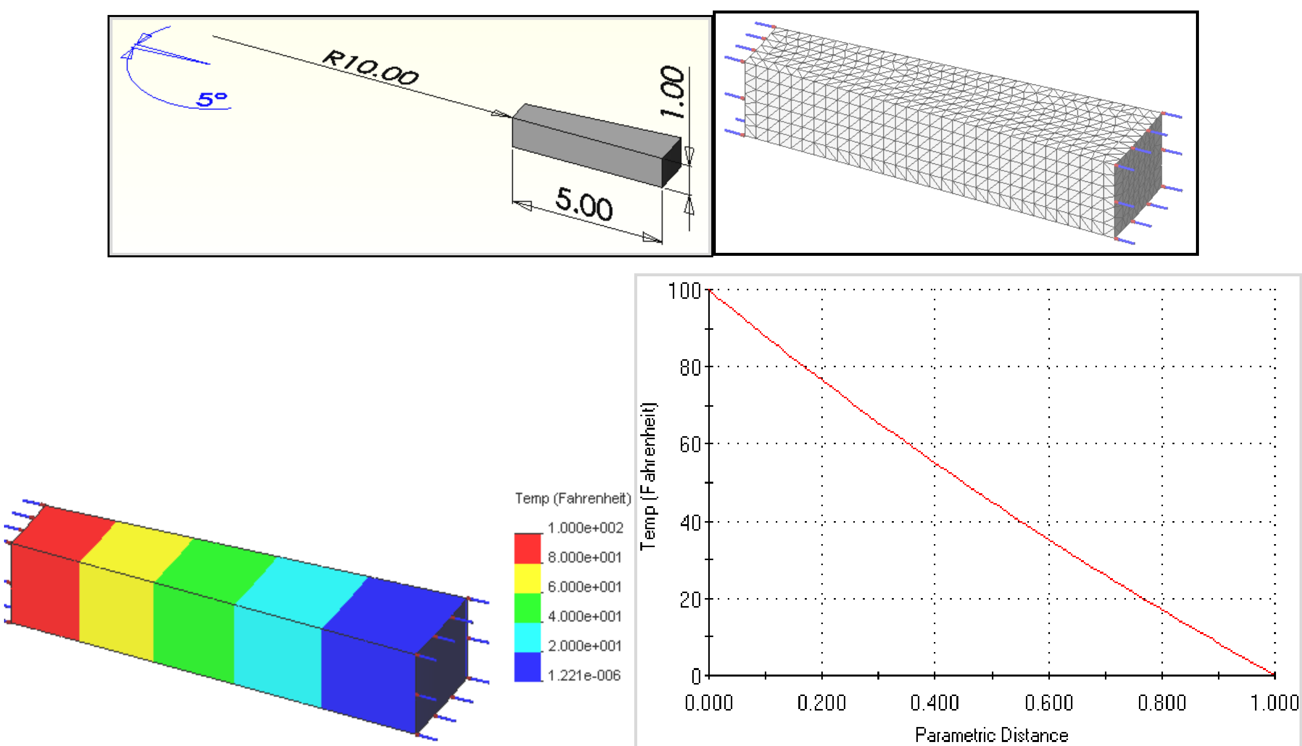


Figure 13-6 Radial temperature through a cylindrical wall

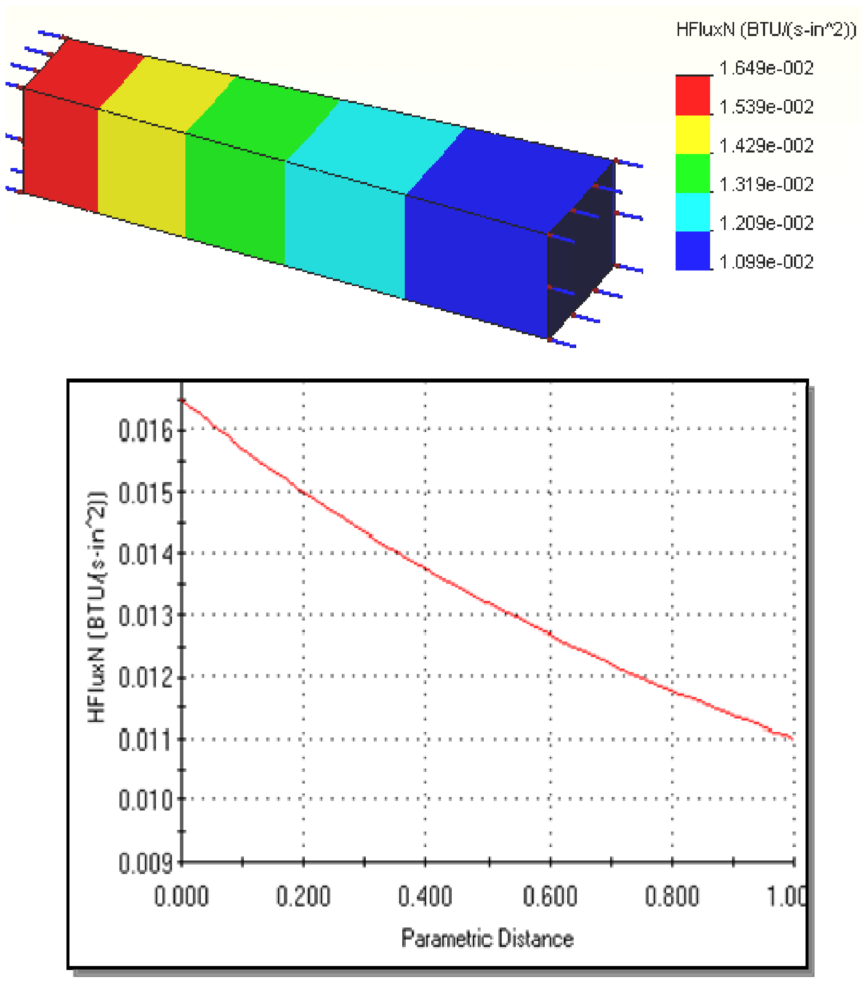


Figure 13-7 Contours and graph of radial heat flux in a cylindrical wall

13.4.3 Shell thermal radial model

The last radial heat transfer example could have also been solved by using the SW Simulation mid-surface shell element, which has one temperature unknown per mesh node. When the *5 degree* solid segment of the cylinder (top) is meshed as a mid-surface shell (in the circumferential direction) the mesh is placed in the middle of a plane of constant thickness. Here the mesh is generated in a constant axial (z) plane. Clearly, it has only a few percent as many equations as the solid mesh above. The two temperature restraints are applied to the two circular arc edges. The two straight edges and the shell face(s) are insulated. The temperature results agree very closely with the much more expensive solid computations. That is easily seen by examining the temperature results given in Figure 13-8. Likewise, the heat flux contours and radial graph values in Figure 13-9 are also in close agreement with the solid model (and the analytic solution).

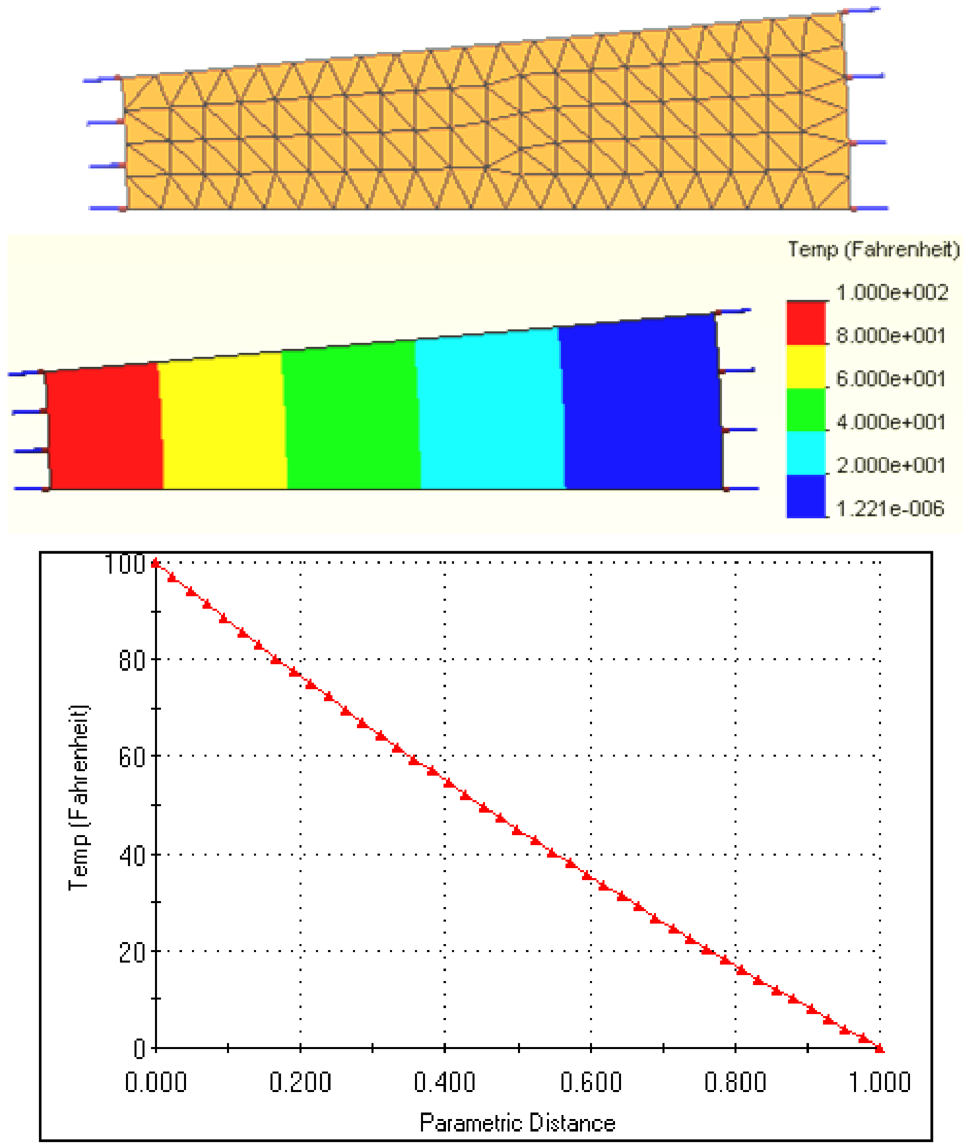


Figure 13-8 Pipe segment temperatures from mid-surface shell mesh

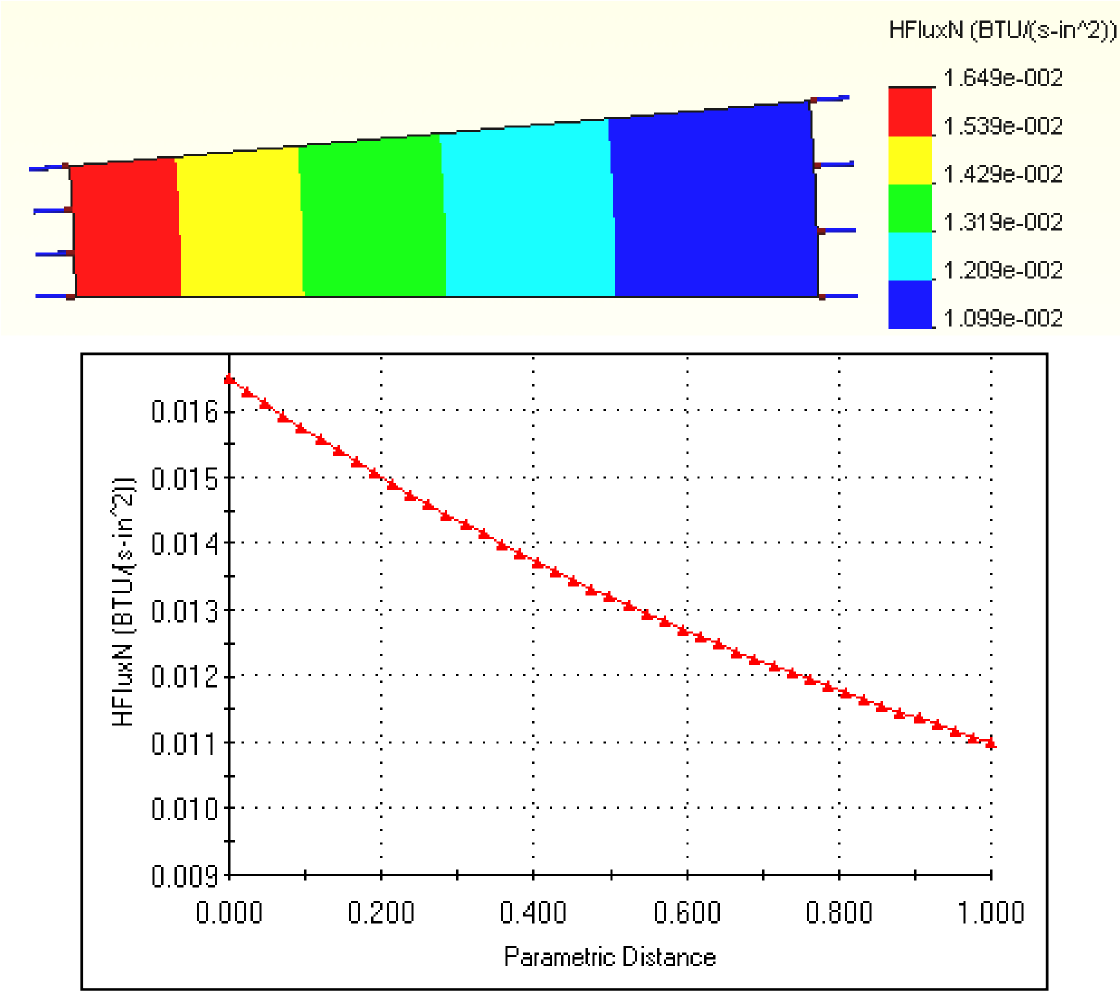


Figure 13-9 Mid-surface shell heat flux result for the pipe

13.5 Heat transfer with an orthotropic material

13.5.1 Introduction

It is becoming more common to encounter materials which have properties that are directionally dependent (anisotropic). A common case is that of orthotropic materials that have their properties completely defined in terms of three perpendicular directions. Those three principal material directions are usually defined by a user defined coordinate system or a user defined reference plane. SW Simulation employs the reference plane approach. The input reference system provides the data necessary to compute the direction cosines between the material directions and the global *x-y-z*-axes. That defines a coordinate transformation matrix, say $\boldsymbol{T}$, that converts the principal properties, say $\boldsymbol{K_{123}}$, to the corresponding global properties as $\boldsymbol{K_{xyz} = T^T K_{123} T}$. For the common isotropic case this reduces to $\boldsymbol{K_{xyz}} = k\,\boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix.

It can be confusing to input orthotropic properties into commercial software so it is wise to begin with a problem with a known solution. There are few such problems but [4] presents the exact solution for temperatures in an orthotropic rectangular block with a constant internal heat generation rate, *Q*. The block is *2 by 1 by 0.1 m* thick and its outer edge faces are held at a constant temperature of *0 C*. The thermal conductivity in the long direction is *2 W/m-K* while that short direction is 1.2337 W/m-K. It is assumed that no