Heat Transfer Summary (from Chap. 13)

Mathematical terms:
scalar – a quantity with no subscripts, i.e., no directional dependence, has magnitude only
vector – a quantity with one subscript, has a magnitude and direction
tensor – a quantity with subscripts (usually 2 or 4) that obeys the tensor transformation law

There are three types of heat transfer: conduction (easy), convection (less easy) and radiation (difficult). The first two are linear problems, but radiation is non-linear. Heat transfer involves the scalar temperature, *T*, the heat flux vector, $\vec{q}$, and the scalar heat flow.

The governing equation for the temperature in 1D is $\frac{\partial}{\partial x}\left(k_{xx}\frac{\partial T}{\partial x}\right) + Q(x) = \rho c_p \frac{\partial T}{\partial t}$. Here k_{xx} is the material property thermal conductivity in the x-direction, $Q(x)$ is an internal rate of heat generation per unit length (such as from electrical resistance or chemical reaction), *t* denotes time, and ρc_p are the material properties of mass density and specific heat . For pure steady state conduction this reduces to $\frac{d}{dx}\left(k_{xx}\frac{dT}{dx}\right) = 0$. For a constant property material the solution between two points of known temperature is linear, i.e., a straight line independent of k_{xx} . (Verify $T(x) = (T_L - T_0)\frac{x}{L} + T_0$.)

Most materials are isotropic (have the same properties in all directions), then $k_{xx} = k_{yy} = k$. Otherwise, they are anisotropic and you must supply directionally dependent conductivities.

The heat flux vector per unit area, for and isotropic material, is defined by the Fourier law: $\vec{q} = -k\,\vec{\nabla}T$. Here $\vec{\nabla}$ is the gradient operator ($\frac{d}{dx}$ in 1D). A typical set of units for $\vec{q}$ is W/m^2.

Convection specifies the magnitude of the heat flux, per unit area, normal to a surface surrounded by a fluid at a temperature of T_m as $q = h(T - T\_m)$. Here *h* is a surface property known as the convection coefficient and T_m is the convection temperature. Except for a few common special cases, *h* can be difficult to determine and requires a formal course in heat transfer (like Mech 481).

Radiation specifies the magnitude of the heat flux, per unit area, normal to a surface surrounded by a medium at an absolute temperature of T_m as $q = \varepsilon\sigma(T^4 - T\_m^4)$. This includes the surface emissivity, ε, and the Stefan-Boltzmann coefficient, σ. Clearly this is non-linear since it involves the unknown surface temperature, *T*, raised to the fourth power.

Heat flow, normal to a surface, is a scalar obtained by integrating the normal heat flux over that surface. Say, $H \equiv \int_A \vec{q}\cdot\vec{n}\,dA = -\int_A k\vec{\nabla}T\cdot\vec{n}\,dA$. For a surface with a known temperature it is the amount of heat that must be supplied or removed to enforce such a known temperature boundary condition.