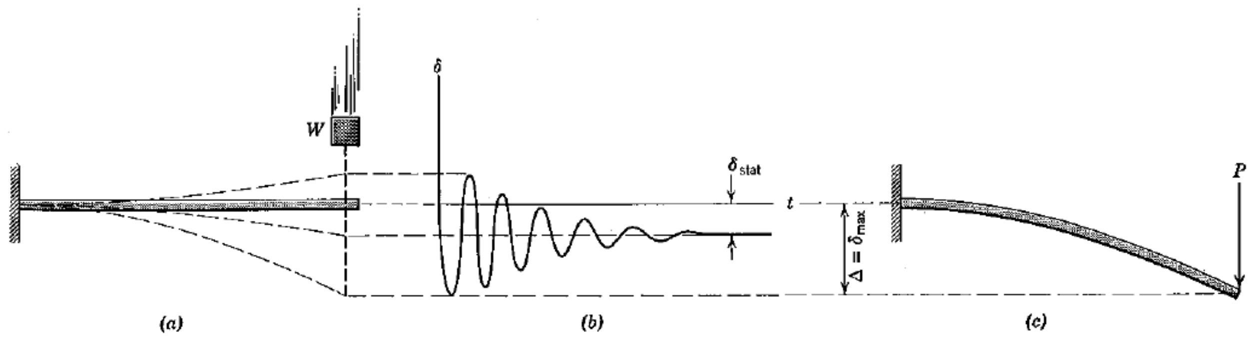
Impact Load Factors

Often a designer has a mass, with a known velocity, hitting an object and thereby causing a suddenly applied impact load. In a static stress analysis the static force (weight) of the mass must be increased by an "Impact Factor" so as to obtain a good approximation of the maximum deflection and stress. For hard elastic bodies the impact factor is greater than or equal to two.

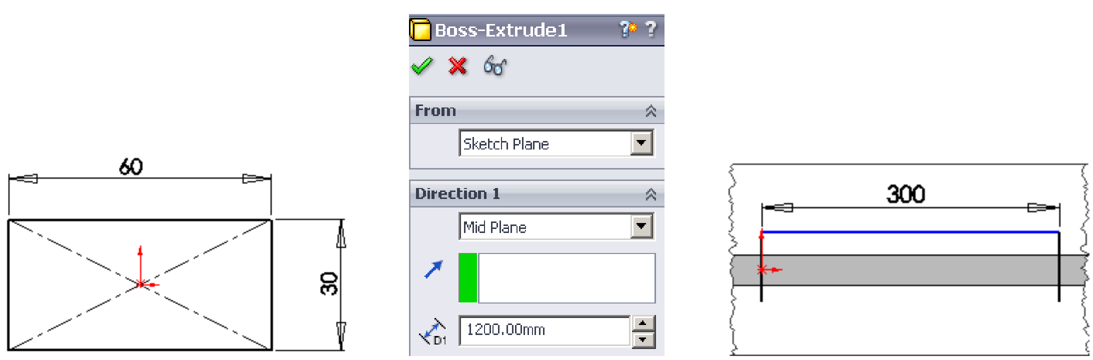


Static force P equivalent to a dropped weight W at the end of a cantilever beam

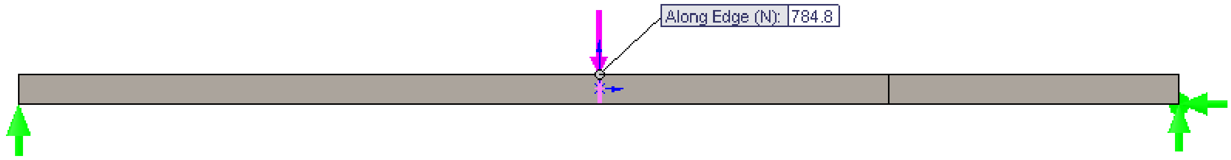
In the study of the mechanics of solids (Mech 311), an energy balance approximation is used to estimate the required static load. See the help file on impact forces for details. In the case of a weight dropped from a height of *h* the impact factor is $n = 1 + \sqrt{1 + \frac{2\,h}{\delta_{static}}}$, where δ_{static} is the static deflection of an object at the point where the weight is placed. There are handbook equations for the static deflections of bars and beams. In general the static deflection is $\delta_{static} = W/k$, where *k* is the stiffness of the member in the direction of the weight. For any object, a finite element static stress analysis can be probed to obtain the static deflection, in the direction of the force, at the point of loading.

Here, both a finite element simulation and a TK Solver worksheet will both be used to estimate the impact factor for a weight dropped on the middle of a simply supported beam of rectangular cross-section. Those examples are followed with a mechanics of materials solution for comparison. The mass of 80 kg (W = 784.8 N) is dropped 10 mm onto the center of a beam of length 1.2 m. The beam is steel (E = 200 GPA) with a cross-section base of 60 mm and a height of 30 mm.

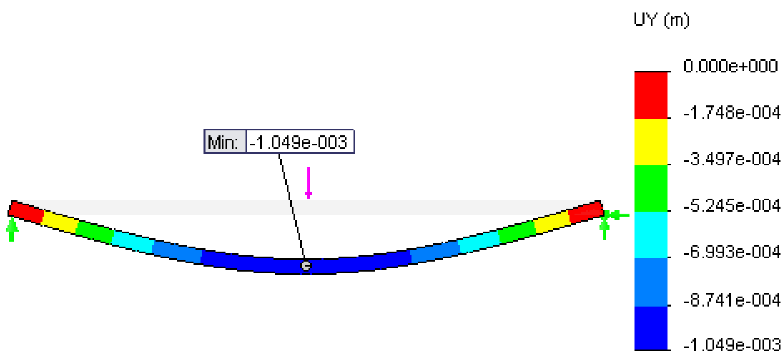
The SolidWorks model begins with the cross-section, extrudes it to the beam length, and adds split lines to the solid to allow loading at the middle and/or quarter points.



The member is pinned at the right and supported with rollers on the left. The static weight is applied at the center point.



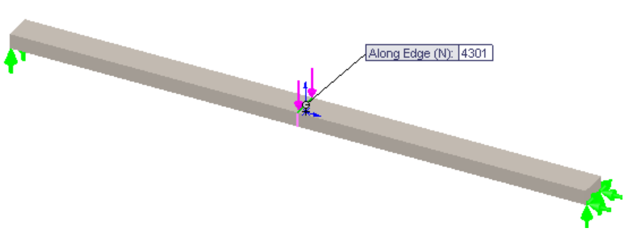
After assigning the material properties, the static deflection at the point of loading is obtained:

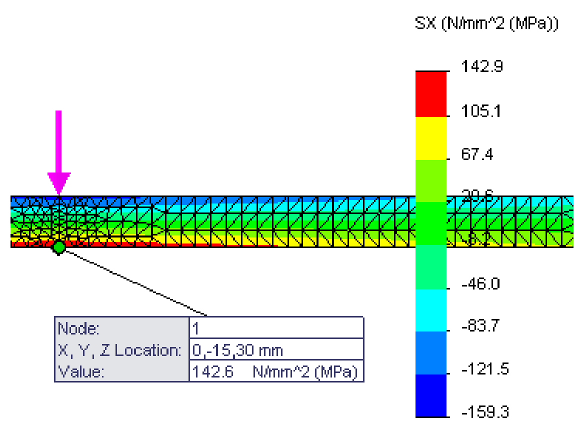


Its value, 0.001049 mm, is substituted into the above equation to yield

$$n = 1 + \sqrt{1 + \frac{0.02}{0.001049}} = 5.48,$$

so the actual static force need s to be *P = n W = 4,301 N*. Applying that load gives a maximum horizontal stress of about *143 MPa.*





The TK Solver worksheet begins by writing a set of rules (from mechanics of solids and dynamics):

Rules

Status	Rule
Comment	; Static load approximation of impact load
Comment	; J.E. Akin Oct. 2011
Satisfied	W = m * g ; gravity
Satisfied	sqrt (2 * g * h) = V ; impact speed
* Unsatisfied	F_static = W * Δ_max / Δ_s ; equivalent static load
* Unsatisfied	W / k = Δ_s ; static deflection of massless member
* Unsatisfied	Δ_max = Δ_s + sqrt (Δ_s^2 + 2*h*Δ_s) ; dynamic deflection
* Unsatisfied	Δ_max = Δ_s * Factor ; scale factor
* Unsatisfied	Factor = 1 + sqrt (1 + 2 * h / Δ_s) ; alternate impact factor definition
Comment	; Bar & cantilever Fixed -------------------L------------------- W
Satisfied	k_bar = EA / L ; stiffness at end bar of length L
Satisfied	k_cant = 3*EI / L^3 ; stiffness at end of cantilever
Comment	; Mid-span Pin ------- L/2------ W -------- L/2 ------- Pin
Satisfied	k_mid = 48 * EI / L^3 ; stiffness at middle of simple beam
Comment	; Overhang Pin --------- b --------- Pin ----- a -------- W, L = a + b
* Unsatisfied	k_over = 3 * EI / (a^2 * L) ; end stiffness of overhanging simple beam
Satisfied	E * A = EA ; axial property
Satisfied	E * I = EI ; bending property
* Unsatisfied	L = a + b ; overhang (a) and support span of beam
Satisfied	A = B * H ; rectangle area, base * height
Satisfied	I = B * H^3 / 12 ; rectangle moment of inertia
Comment	; NOTE Δ_s can be computed in a FEA study by applying the load at a point,
Comment	; and solving for the displacement at that point, in the load direction,

The initial input variables (below) were incomplete since neither the static deflection nor member stiffness was known. Thus, several rules were not satisfied.

Variables

Status	Input	Name	Output	Unit	Comment
					Find equivalent static force for an impact
					Data for sample 12.5, Pytel "Mech of Materials"
	80	m		kg	Mass of impacting object
		W	784.8	N	Weight of impacting object
	.01	h		m	Height of vertical drop
		V	.4429447	m/sec	Speed at impact
		k		N/m	Body stiffness at impact point
		Δ_s		m	Static deflection due to weight
		Δ_max		m	Maximum dynamic deflection
		F_static		N	Equivalent static force
		Factor			Amplification to get static load
	200000	E		MPa	Modulus of elasticity
	1.2	L		m	Length of bar or beam
		A	1800	mm^2	Area of bar
		EA	3.6E8	N	Axial property (optional)
		k_bar	3E8	N/m	Stiffness of axial bar
		EI	27000	N_m^2	Bending property (optional)
		I	1.35E-7	m^4	Moment of inertial of beam
		k_cant	46875	N/m	Stiffness at end of a cantilever beam
		k_mid	750000	N/m	Stiffness at middle of simple beam
		k_over		N/m	Stiffness at overhang end of simple beam
	9.81	g		m/sec^2	Gravitational constant
		a		m	Overhang of a simple beam, L=a+b
		b		m	Support span of overhanging simple beam
	.06	B		m	Rectangle base length
	.03	H		m	Rectangle height

There were enough inputs provided to calculate the stiffness of a bar and two types of beams. Since the weight is dropped on the center of a simple beam, the value of that stiffness (*7.5e5 N/m*) was copied from the output column, of k_mid, and pasted into the input column of k. Then, the next solve gives the desired results.

Variables

Status	Input	Name	Output	Unit	Comment
					Find equivalent static force for an impact
					Data for sample 12.5, Pytel "Mech of Materials"
	80	m		kg	Mass of impacting object
		W	784.8	N	Weight of impacting object
	.01	h		m	Height of vertical drop
		V	.4429447	m/sec	Speed at impact
	750000	k		N/m	Body stiffness at impact point
		Δ_s	.0010464	m	Static deflection due to weight
		Δ_max	.0057393	m	Maximum dynamic deflection
		F_static	4304.446	N	Equivalent static force
		Factor	5.484769		Amplification to get static load

Now the static deflection (*1.05e-3 m*) matches the finite element study well, as does the impact factor of about 5.5. Had the weight been held at the beam surface without toughing it (*h = 0*) and then released that calculation gives the impact factor equal to two.

Variables

Status	Input	Name	Output	Unit	Comment
	80	m		kg	Mass of impacting object
		W	784.8	N	Weight of impacting object
	0	h		m	Height of vertical drop
		V	0	m/sec	Speed at impact
	750000	k		N/m	Body stiffness at impact point
		Δ_s	.0010464	m	Static deflection due to weight
		Δ_max	.0020928	m	Maximum dynamic deflection
		F_static	1569.6	N	Equivalent static force
		Factor	2		Amplification to get static load

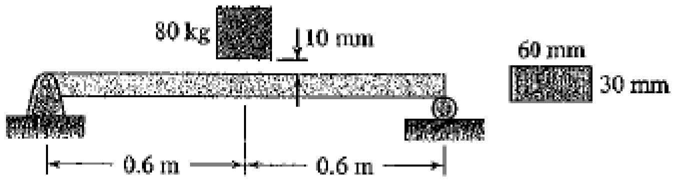
Had the weight been dropped on the end of a cantilever beam of the same size (*k = 46.9e3 N/m*) the input of that stiffness value gives an impact factor of about 36.

Variables

Status	Input	Name	Output	Unit	Comment
	80	m		kg	Mass of impacting object
		W	784.8	N	Weight of impacting object
	10	h		m	Height of vertical drop
		V	14.00714	m/sec	Speed at impact
	46875	k		N/m	Body stiffness at impact point
		Δ_s	.0167424	m	Static deflection due to weight
		Δ_max	.5956451	m	Maximum dynamic deflection
		F_static	27920.86	N	Equivalent static force
		Factor	35.57704		Amplification to get static load

Sample Problem 12.5 Pytel, Kiusalaas "Mech of Materials"

The 80-kg block hits the simply supported beam at its midspan after a drop of 10 mm as shown in Fig. (a). Determine (1) the impact factor; and (2) the maximum dynamic bending stress in the beam. Use $E = 200$ GPa for the beam. Assume that the block and the beam stay in contact after the collision.



(a)

Solution

Part 1

The moment of inertia of the cross section of the beam about the neutral axis is

$$I = \frac{bh^3}{12} = \frac{60(30)^3}{12} = 135.0 \times 10^3 \text{ mm}^4 = 135.0 \times 10^{-9} \text{ m}^4$$

According to Table 6.3 on page 233, the static midspan deflection of the beam under the weight of the 80-kg mass is

$$\delta_{st} = \frac{(mg)L^3}{48EI} = \frac{(80 \times 9.81)(1.2)^3}{48(200 \times 10^9)(135.0 \times 10^{-9})} = 1.0464 \times 10^{-3} \text{ m}$$

From Eq. (12.12b), the impact factor is

$$n = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{2(0.010)}{1.0464 \times 10^{-3}}} = 5.485 \qquad \textit{Answer}$$

Part 2

The maximum dynamic load P_{max} at the midspan of the beam is obtained by multiplying the static load by the impact factor:

$$P_{max} = n(mg) = 5.485(80 \times 9.81) = 4305 \text{ N}$$

The maximum bending moment caused by this load occurs at the midspan, as shown in Fig. (b). Its value is

$$M_{max} = \frac{4305}{2}(0.6) = 1291.5 \text{ N} \cdot \text{m}$$

which results in the maximum dynamic bending stress

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{1291.5(0.015)}{135.0 \times 10^{-9}} = 143.5 \times 10^6 \text{ Pa} = 143.5 \text{ MPa} \qquad \textit{Answer}$$