Stress Analysis Summary (from Chapter 3)

Finite element static stress analysis satisfies the differential equation of equilibrium. However, it does so by a mathematically equivalent method by minimizing the total potential energy of the system (a scalar quantity): $\Pi = U - W \rightarrow minimum$. Here, *U* is the strain energy (potential energy) stored in the structure and *W* is the external mechanical work done by forces acting on the structure. Detailed finite element analysis theory is covered in Mech 417.

These quantities are defined in terms of the displacement vector, $\vec{\delta} = u\vec{\imath} + v\vec{\jmath} + w\vec{k}$, at points within the solid. The displacement components (u, v, w) are the most accurate result from a stress study.

Force vectors, $\vec{F} = F_x\vec{\imath} + F_y\vec{\jmath} + F_z\vec{k}$, can be associated with each displacement vector.

From the Strain-Displacement Relations, the gradient components of the displacement components are used to create the six components of the strain tensor, $\varepsilon_{xx}, \varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yy}, \varepsilon_{yz}, \varepsilon_{zz}$.

A material law (Hooke's law) gives a constitutive tensor to define stresses in terms of strain. The material law uses the modulus of elasticity, E, and Poisson's ratio, ν. It defines the stresses as $\boldsymbol{\sigma} = \boldsymbol{E}\ \boldsymbol{\varepsilon}$. The corresponding six stress components are $\sigma_{xx}, \sigma_{xy}, \sigma_{xz}, \sigma_{yy}, \sigma_{yz}, \sigma_{zz}$. The stresses are the least accurate quantity in a stress analysis. The stresses or the strains are used to define a material failure criterion. For a 1D problem there is only a normal strain $\varepsilon_{xx} = \frac{\partial u}{\partial x}$ and normal stress $\sigma_{xx} = E\varepsilon_{xx}$.

Enough displacements must be prescribed to prevent rigid body motion of the system (three translations and three rotations). Otherwise, the solver will fail or give infinite displacements.

Solid bodies are often approximated by reduced geometrical models.

Solid s (Mech 516)→Surface model → Shell (Ceve 516) or plate or plane stress (Mech 311) or plane strain
Solids (Mech 516) → Line model → Beam (Mech 311) or bar or truss (Mech 211)
Beams, plates, and shells also introduce infinitesimal rotations as unknowns in the analysis.

Data reliability:

Geometry: the solid model is generally the most accurate data, but they are often changed to generate the mesh.
Material: measured properties are fairly accurate for standard materials, but only to a few (<=4) significant figures. Custom materials may have a wide range of properties.
Loads: are less accurate since they require assumptions about their magnitudes and where they are applied (e.g., at a point or over a small area)
Fixtures: displacement restraints are the least accurate data. They drastically affect the results. Thus, several reasonable assumptions should be investigated for each design. The most common error is to assume that air is supporting a solid at some surface regions.

Result reliability:

Displacements are most accurate. Stresses and failure criterion are least accurate.