Mech 417

HW #4 1-D parametric numerical integration

Let $\Box$ denote a non-dimensional parametric space, Then

$$\int_{\Box} f\, d\Box = \sum_{i=1}^{n_q} w_i f_i$$

where w_i is tabulated in $\Box$ at points where f is evaluated to form f_i. If f is a polynomial it can be exactly integrated in this way using Gaussian quadrature data. See Tables 4.1 (p. 121) & 4.2 (p. 121) of suggested text.

1. Using Table 4.1 integrate using $n_q = 2$

a) $\int_{-1}^{1} da$, (i.e. $f=1$).

b) $\int_{-1}^{+1} a\, da$ (the first moment)

2. Use Table 4.2 with $n_q = 2$ to compute the measure

a) $\int_0^1 dr$

and the first moment b) $\int_0^1 r\, dr$.

3. What do the part a) solutions tell you about the required value of $\sum_i w_i$?

4. The quadratic bar example used the same node numbering order in both parametric and physical space:

local node	1	2	3
physical node	1	2	3.

Physical node numbers can be assigned in any order. Then the gather and scatter operations are no longer identities. Let the physical numbering change to:

local node	1	2	3
physical node	3	1	2.

Write the matrix equations for the new physical order of unknowns, without EBC. Then apply the one essential boundary condition. Do not solve. (In other words re-do the parametric to physical equation scatter.)

5 17. Develop the constant source vector $\int \underline{N}^T A f \, dx$ for the quadratic interpolation done in the parametric space $-1 \le a \le 1$ with $N_1(a) = (a^2 - a)/2$, $N_2(a) = 1 - a^2$ and $N_3(a) = (a^2 + a)/2$.