Mech 417/517 Homework Set 2
Problems are due in hardcopy form at the "In Box" in the wall beside the mailboxes across from ME108 by 5pm Jan. 22

2.4. Derive a one-dimensional linear interpolation formula for a function $u = u(x)$ that is valid in the range u_1 through u_2 as shown in Fig. 2-3.

Buchanan page 38

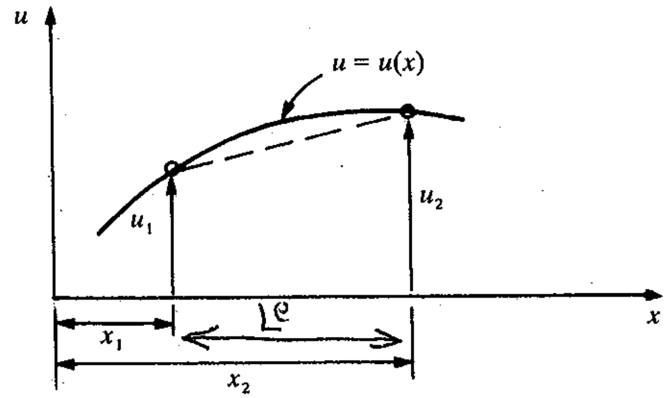

Fig. 2-3

The function $u(x)$ is shown in Fig. 2-3. A linear equation that would approximate $u(x)$ between u_1 and u_2 is assumed as

$$u = A + Bx = \underset{1\times 2}{\tilde{P}(x)}\ \underset{2\times 1}{\tilde{C}^e} \qquad (a)$$

where A and B are constants. Substituting the boundary conditions $u(x_1) = u_1$ and $u(x_2) = u_2$ gives two equations that can be solved for A and B:

polynomial

$$u(x_j) = \tilde{P}(x_j)\,\tilde{C} \longrightarrow \begin{aligned} u_1 &= A + Bx_1 \\ u_2 &= A + Bx_2 \end{aligned} \rightarrow \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} \rightarrow \underset{2\times 1}{\tilde{u}^e} = \underset{2\times 2}{\tilde{G}^e}\ \underset{2\times 1}{\tilde{C}^e}$$

Solving for A and B and substituting into Eq. (a) gives the interpolating polynomial:

shape function

$$A = \frac{u_1 x_2 - u_2 x_1}{x_2 - x_1} \qquad B = \frac{u_2 - u_1}{x_2 - x_1}$$

$$u(x) = \underset{1\times 2}{\tilde{N}(x)}\ \underset{2\times 1}{\tilde{u}^e} \longrightarrow u = u_1 \frac{x_2 - x}{x_2 - x_1} + u_2 \frac{x - x_1}{x_2 - x_1} = u_1 N_1(x) + u_2 N_2(x) \qquad (b)$$

1. For $x \in L^e$, sketch the value of $N_1(x)$ and $N_2(x)$.
2. Integrate $\int_{L^e} N_j(x)\,dx$, $j = 1, 2$, the area under the above curve,
3. For $x \in L^e$, sketh the products N_1^2 and $N_1(x) N_2(x)$.
4. What are the areas under the two above curves?
5. Evaluate the gradient, du/dx, for $x \in L^e$ interms of the nodal values, $\tilde{u}^e$.

6. The force equilibrium differential equation for an elastic axially loaded bar is

$$d\{ E(x)\, A(x)\, [du(x)/dx - \alpha(x)\, \Delta T(x)] \} / dx + X(x)\, A(x) = 0, \qquad (1)$$

plus two boundary conditions. Here, E and α are the material elastic modulus and coefficient of thermal expansion, A is the cross-sectional area, X the body force per unit volume, ΔT is the temperature change, and u(x) is the axial displacement. The Galerkin concept for creating an integral form is to require that the solution, u(x), be orthogonal to any error in the differential equation. That is, it sets the integral, over the domain, of u(x) times the differential equation to zero:

$$\int_0^L u(x)\, (d\{ EA\, [du(x)/dx - \alpha\Delta T\} / dx + XA)\, dx \equiv 0, \qquad (2)$$

where L is the length of the bar. Note that no boundary conditions have been specified yet.

a) Write this as the sum of three integrals set equal to zero.
b) Evaluate the first integral through integration by parts. Describe a physical meaning of the parts of the two new boundary terms that appear.

Mech 517

7. List four even order differential equations, with boundary conditions, that you have used in physical applications