

1/15/09

Mech 417

HW 1

$$(a) \int_0^L x^2 dx = \left. \frac{x^3}{3} \right|_0^L = \frac{L^3}{3} \quad \checkmark$$

$$(b) x = Lr \Rightarrow \int_0^L x^2 dx = \int_0^1 x^2(r) \left( \frac{dx}{dr} \right) dr = \int_0^1 (L^2 r^2)(L) dr \quad \frac{20}{20}$$

$$= \int_0^1 L^3 r^2 dr = \left. \frac{L^3 r^3}{3} \right|_0^1 = \frac{L^3}{3} \quad \checkmark$$

$$(c) x = \frac{L(1+\alpha)}{2} \Rightarrow \int_0^L x^2 dx = \int_{-1}^1 x^2(\alpha) \left( \frac{dx}{d\alpha} \right) d\alpha$$

$$= \int_{-1}^1 \left( \frac{L+L\alpha}{2} \right)^2 \left( \frac{L}{2} \right) d\alpha = \int_{-1}^1 \left( \frac{L^2}{4} \right) (1+2\alpha+\alpha^2) \left( \frac{L}{2} \right) d\alpha$$

$$= \frac{L^3}{8} \int_{-1}^1 (1+2\alpha+\alpha^2) d\alpha = \frac{L^3}{8} \left[ \alpha + \alpha^2 + \frac{\alpha^3}{3} \right]_{-1}^1$$

$$= \frac{L^3}{8} \left[ \left( 1+1+\frac{1}{3} \right) - \left( -1+1-\frac{1}{3} \right) \right] = \frac{L^3}{8} \left( \frac{8}{3} \right) = \frac{L^3}{3} \quad \checkmark$$

$$(d) \int_0^L dx = x \Big|_0^L = L$$

$$\int_0^1 dr = r \Big|_0^1 = 1 \quad \checkmark$$

$$\int_{-1}^1 d\alpha = \alpha \Big|_{-1}^1 = 2$$