

①

$$w_1 = 1$$

$$x_1 = -0.577350269189625764509149$$

$$w_2 = 1$$

$$x_2 = 0.577350269189625764509149$$

$$a) \int_{-1}^1 da = w_1 f(x_1) + w_2 f(x_2)$$

$$= 1(1) + 1(1)$$

$$= \boxed{2}$$

$$b) \int_{-1}^1 a da = w_1 f(x_1) + w_2 f(x_2)$$

$$= -0.577 + 0.577$$

$$= \boxed{0}$$

②

$$w_1 = 0.5$$

$$x_1 = 0.21132486540518711774543$$

$$w_2 = 0.5$$

$$x_2 = 0.78867513459481288225457$$

$$a) \int_0^1 dr = w_1 f(x_1) + w_2 f(x_2)$$

$$= 0.5(1) + 0.5(1)$$

$$= \boxed{1}$$

$$b) \int_0^1 r dr = w_1 f(x_1) + w_2 f(x_2)$$

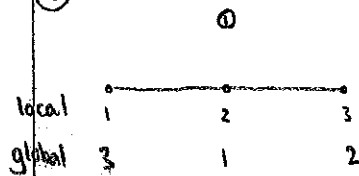
$$= 0.5(0.2113249) + 0.5(0.7886751)$$

$$= \boxed{0.5}$$

③ $\sum w_i$ must equal 1 X

the length of the parametric space. Here 2 and 1, respectively.

(4)



$$\begin{aligned} u_1^e &= u_3 \\ u_2^e &= u_1 \\ u_3^e &= u_2 \end{aligned}$$

one negative
one positive

$$\frac{EA}{3L} \begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix} \begin{bmatrix} u_1^e \\ u_2^e \\ u_3^e \end{bmatrix} = \begin{bmatrix} -EA \frac{d^2 f}{dx^2}(a) \\ 0 \\ -EA \frac{d^2 f}{dx^2}(L) \end{bmatrix} + \frac{LAF}{6} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

element level

$$\frac{EA}{3L} \begin{bmatrix} 16 & -8 & -8 \\ -8 & 7 & 1 \\ -8 & 1 & 7 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -EA \frac{d^2 f}{dx^2}(L) \\ -EA \frac{d^2 f}{dx^2}(0) \end{bmatrix} + \frac{LAF}{6} \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

system level

(5)

$$\begin{aligned} & \int_{-1}^1 N^T(a) A F da & x &= \frac{L}{2}(a+1) & \checkmark \\ & & dx &= \frac{L}{2} da & \checkmark \\ & = \frac{LAF}{2} \int_{-1}^1 N^T(a) da & & & \checkmark \\ & = \frac{LAF}{2} \int_{-1}^1 \begin{bmatrix} \frac{a^2-a}{2} \\ 1-a^2 \\ \frac{a^2+a}{2} \end{bmatrix} da \end{aligned}$$

$$\frac{1}{2} \int_{-1}^1 a^2 - a da = \frac{1}{2} \left[\frac{1}{3} a^3 - \frac{1}{2} a^2 \right]_{-1}^1 = \frac{1}{2} \left[\frac{1}{3}(1+1) - \frac{1}{2}(1-1) \right] = \frac{1}{3}$$

$$\int_{-1}^1 1 - a^2 da = \left[a - \frac{1}{3} a^3 \right]_{-1}^1 = \left[(1+1) - \frac{1}{3}(1+1) \right] = \frac{4}{3}$$

$$\frac{1}{2} \int_{-1}^1 a^2 + a da = \frac{1}{2} \left[\frac{1}{3} a^3 + \frac{1}{2} a^2 \right]_{-1}^1 = \frac{1}{2} \left[\frac{1}{3}(1+1) + \frac{1}{2}(1-1) \right]$$

$$\Rightarrow \int_{-1}^1 N^T(a) A F dx = \frac{LAF}{6} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \quad \checkmark$$