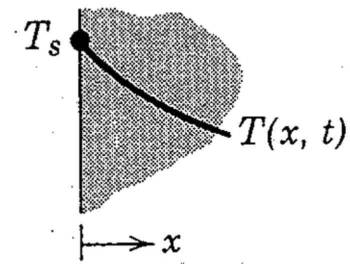
TABLE 11.1 ● One-dimensional initial-boundary-value problems (diffusion phenomena).

Application	Unknown	Physical/material properties			Interior load	Governing equation
		μ	α	β		
General mathematical formulation	U	μ	α	β	f	$\mu \frac{\partial U}{\partial t} - \frac{\partial}{\partial x}\left(\alpha \frac{\partial U}{\partial x}\right) + \beta U = f$
Heat conduction: diffusion of heat in a solid	T (temperature)	ϱc ϱ=mass density c=specific heat	k (thermal conductivity)	$\frac{hl}{A}$ (convection loss coefficient)	$Q + \frac{hlT_\infty}{A}$ Q=heat source $\frac{hlT_\infty}{A}$ = part of ambient convection	$\varrho c \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{hl}{A} T = Q + \frac{hlT_\infty}{A}$
Transient seepage: flow of fluid through a porous medium	H (hydraulic head)	S (storage coefficient)	T (hydraulic transmissivity)		Q (inflow from external source)	$S \frac{\partial H}{\partial t} - \frac{\partial}{\partial x}\left(T \frac{\partial H}{\partial x}\right) = Q$
Soil consolidation: diffusion of fluid through pores of an elastic medium	P (pore pressure of fluid)	C (compressibility)	$\frac{\varkappa}{\varrho}$ $\varkappa$=permeability ϱ=density			$C \frac{\partial P}{\partial t} - \frac{\partial}{\partial x}\left(\frac{\varkappa}{\varrho} \frac{\partial P}{\partial x}\right) = 0$
Atomic and nuclear physics: diffusion of electrons in a gas, or neutrons in matter	ϱ (spatial density of particles)	m (mass of particles)	a^2 (diffusion constant — proportional to p_a, the average momentum, and λ_a, the mean free path)	μ (mean rate of absorption of particles)	r (number of particles created per second per volume)	$m \frac{\partial \varrho}{\partial t} - \frac{\partial}{\partial x}\left(a^2 \frac{\partial \varrho}{\partial x}\right) + \mu \varrho = r$

Table 2.1 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

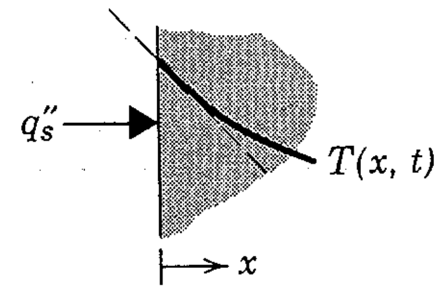
$$T(0,t) = T_s \quad (2.20)$$



2. Constant surface heat flux

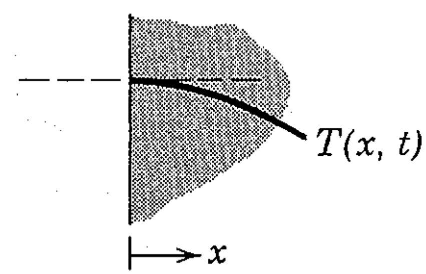
(a) Finite heat flux

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = q''_s \quad (2.21)$$



(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \quad (2.22)$$



3. Convection surface condition

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0,t)] \quad (2.23)$$