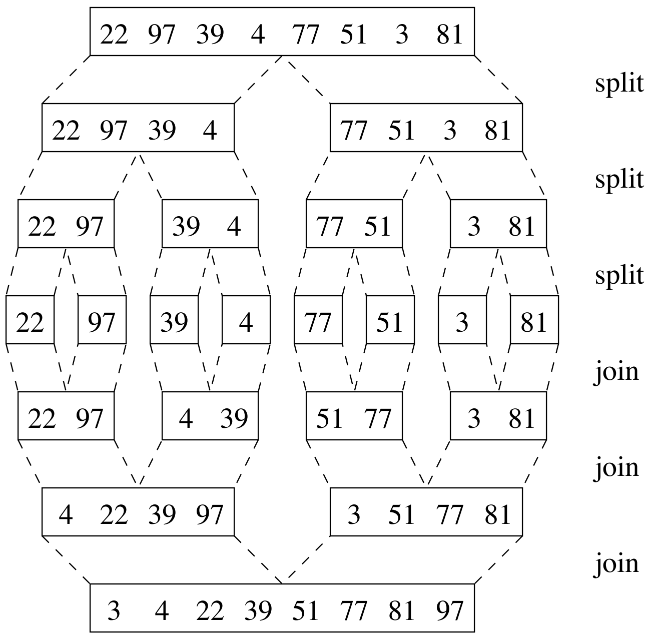
Merge Sort

- Merge Sort is an *easy-split, hard-join* method.



- `split()` is trivial.

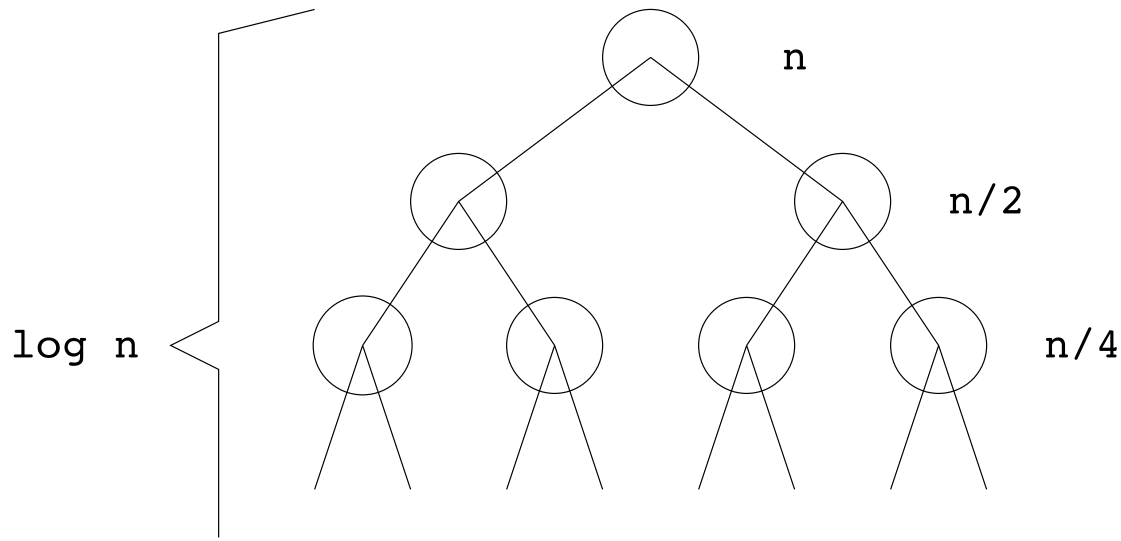
```
public int split(int[] A, int lo, int hi)
{
  return (lo + hi + 1)/2;
}
```

- `join()` merges two smaller, sorted arrays.
  - Specifically, it merges `A[lo:s-1]` and `A[s:hi]` into `_tempA[lo:hi]`, then copies `_tempA` back to `A`.

```
public void join(int[] A, int lo, int s, int hi)
{
  merge(A, lo, s, hi);
  for (int i = lo; i <= hi; i++) {
    A[i] = _tempA[i];
  }
}
```

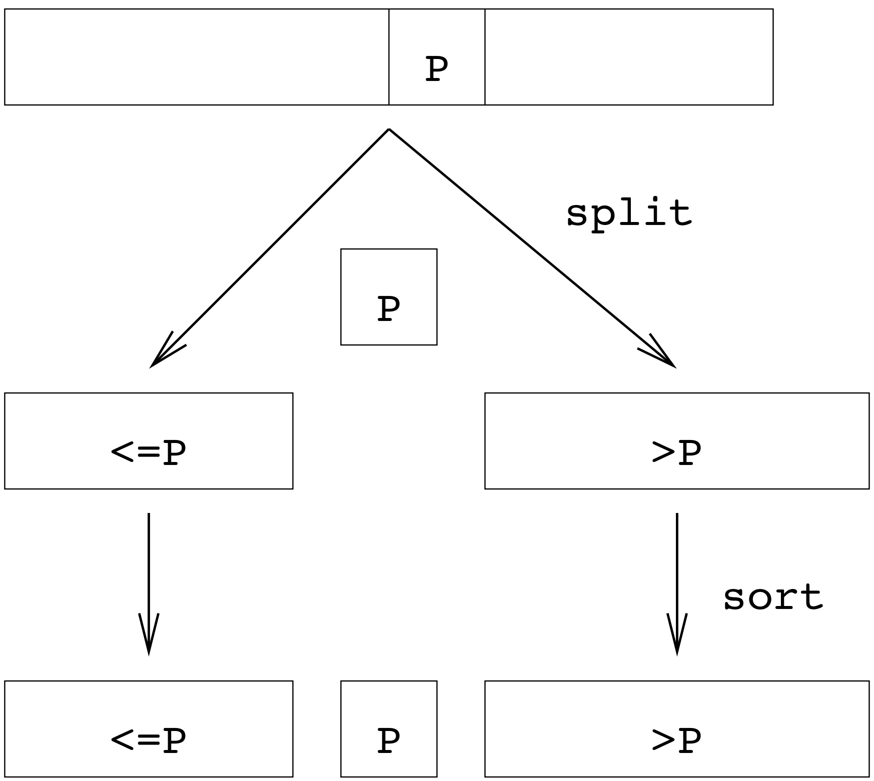
```
private void merge(int[] A, int lx, int mx, int rx)
{
  int i = lx;
  int j = mx;

  for (int k = lx; k <= rx; k++) {
    if ((i < mx) && (j <= rx)) {
      if (A[i] < A [j])
        _tempA[k] = A[i++];
      else
        _tempA[k] = A[j++];
    }
    else if (i < mx) {
      _tempA[k] = A[i++];
    }
    else if (j <= rx) {
      _tempA[k] = A[j++];
    }
  }
}
```

- Merge Sort takes $O(n\ log\ n)$ steps.
  - Because each `split()` divides the array into two (almost) equal-sized parts, each element is `join()`'ed $log\ n$ times.



Quick Sort

- Quick Sort is a *hard-split, easy-join* method.
- The following diagram illustrate one step.



Quick Sort (cont.)

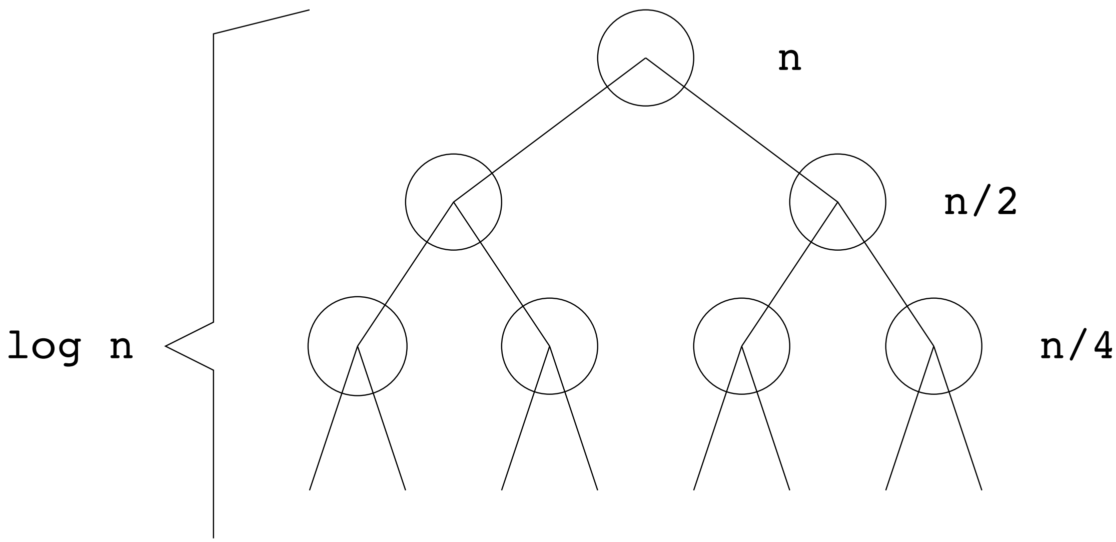
```
public int split(int[] A, int lo, int hi)
{
  int key = A[lo];
  int lx = lo;                // left index.
  int rx = hi;                // right index.

  // Invariant 1: key <= A[rx+1:hi].
  // Invariant 2: A[lo:lx-1] <= key.
  // Invariant 3: there exists ix in [lo:rx]
  //              such that A[ix] <= key.
  // Invariant 4: there exists jx in [lx:hi]
  //              such that key <= A[jx].

  while (lx <= rx) {
    while (key < A[rx]) {  // will terminate due to invariant 3.
      rx--;                // Invariant 1 is maintained.
    }                      // A[rx] <= key <= A[rx+1:hi];
```

```
                                // also ``Invariant 0'', lx <= rx.

    while (A[lx] < key) {       // will terminate due to invariant 4.
      lx++;                     // Invariant 2 is maintained.
    }                           // A[lo:lx-1] <= key <= A[lx]

    if (lx <= rx) {             // swap A[lx] with A[rx]:
      int temp = A[lx];
      A[lx] = A[rx];            // invariant 3 is maintained.
      A[rx] = temp;             // invariant 4 is maintained.
      rx--;                     // invariant 1 is maintained.
      lx++;                     // invariant 2 is maintained.
    }
  }

  // rx < lx, A[lo:lx-1] <= key <= A[rx+1:hi], and key = A[lx].

  return lx;
}
```

```
public void join(int[] A, int lo, int s, int hi)
{
  // nothing to do!
}
```

Quick Sort (cont.)

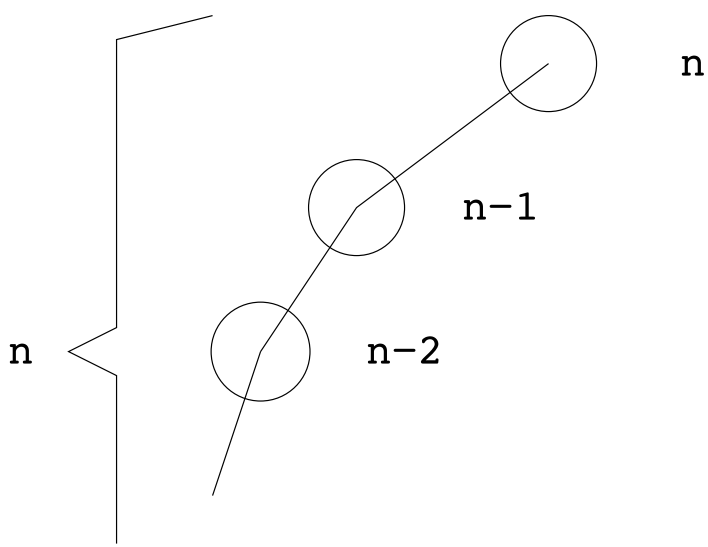
- If the pivot chosen by `split()` divides the array into two (almost) equal-sized parts, each element is `split()` $log\ n$ times.



- Thus, in this case, Quick Sort takes $O(n\ log\ n)$ steps.

Quick Sort (cont.)

- On the other hand, an unfortunate choice of the pivot could divide the array into two parts, one that contains no elements and another that contains $n-1$ elements.



- In this case, Quick Sort takes $O(n^2)$ steps.

Quick Sort (cont.)

- Various strategies are used to choose the pivot. (None is perfect.)
  - Pick the first element (worst-case scenario is a nearly-sorted or nearly-inverse-sorted array).
  - Take the median of the first, last, and middle elements. This is often used in practice, since it behaves well on the nearly-sorted case, which can be quite common in some applications.

Summary

Sort	Best-Case Cost	Worst-Case Cost
Selection	$O(n^2)$	$O(n^2)$
Insertion	$O(n)$	$O(n^2)$
Merge	$O(n\ log\ n)$	$O(n\ log\ n)$
Quick	$O(n\ log\ n)$	$O(n^2)$

where n is the size of the container