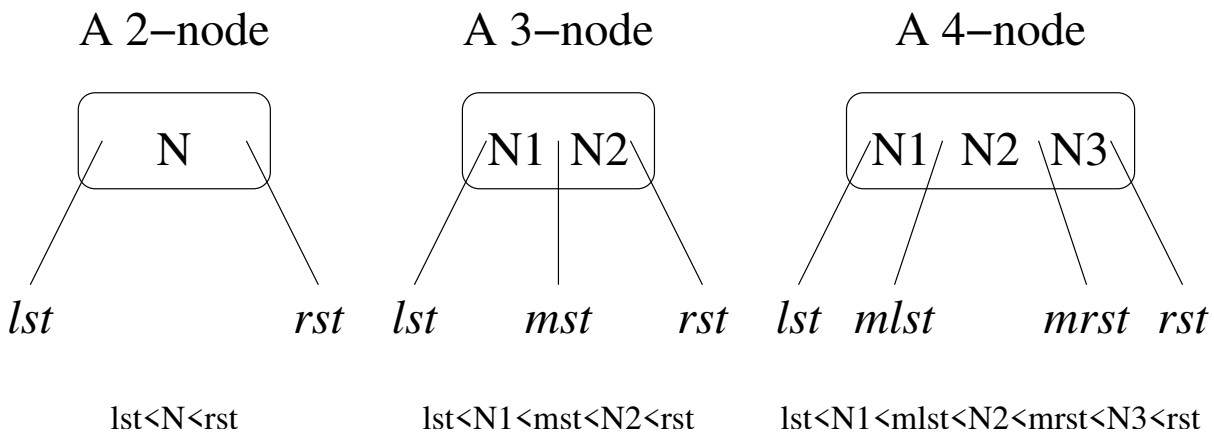
Overview

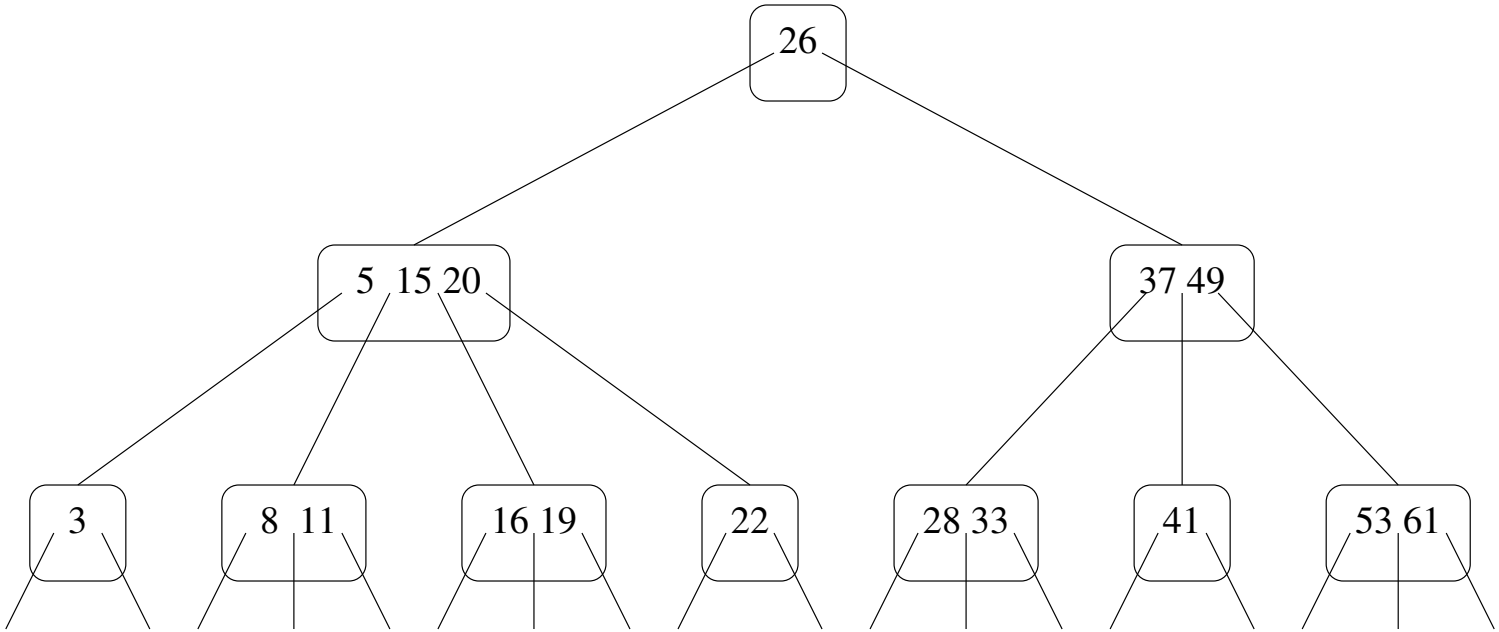
- Height Balanced Trees: 2-3-4 Trees

2-3-4 Trees

- A 2-3-4 Tree is suitable for use as an ordered container.
  - A 2-3-4 Tree can be empty (contains no data elements)
  - A non-empty 2-3-4 Tree can be in one of the three states:

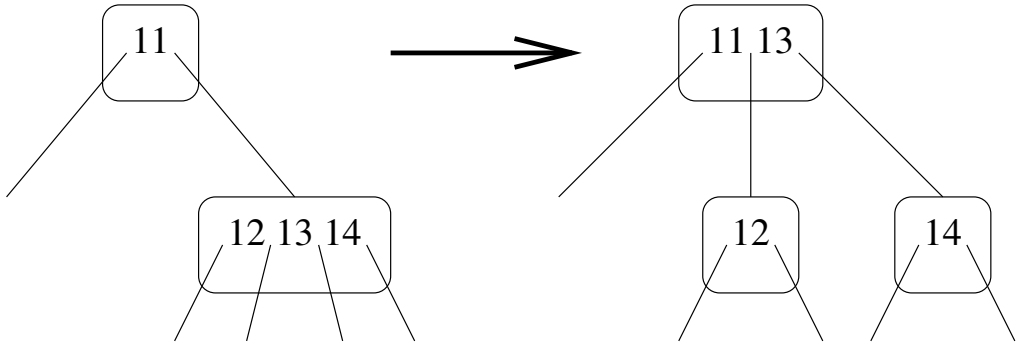


2-3-4 Trees: An Example

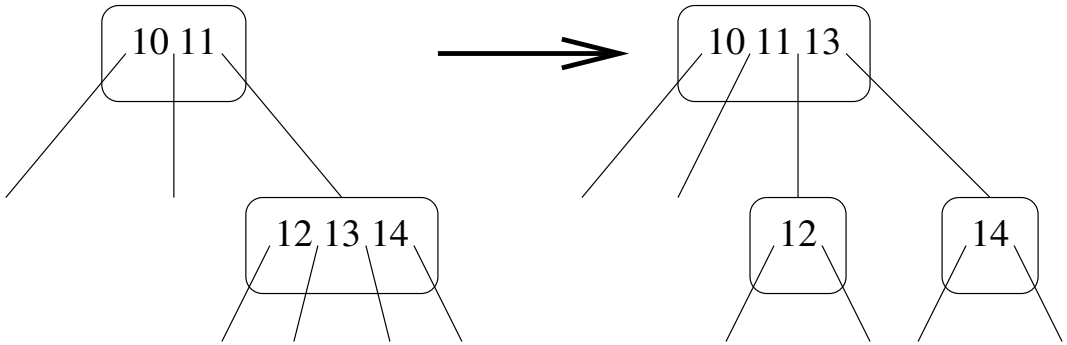


2-3-4 Trees: Insertion

- For simplicity, suppose that the data elements are integers.
- The algorithm for inserting an integer N into a 2-3-4 tree is:
  - Go down the search path for N in the tree, and insert it into the appropriate leaf;
  - along the search path, *before inserting N*, if a 4-state tree is encountered, split it into two 2-state trees and merge its middle data element with its parent tree (if any).
    - Suppose that I want to insert 15 into the following trees:

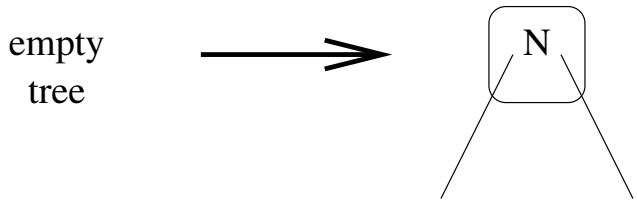


2-3-4 Trees: Insertion (cont'd)



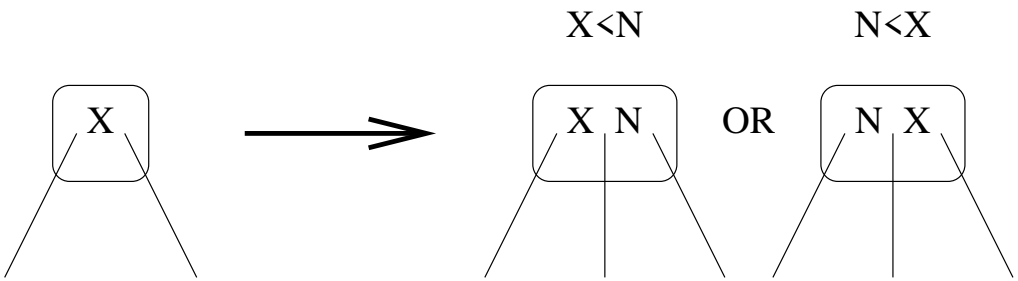
Insertion: Empty to 2-state

insert an integer N into an empty tree



Insertion: The 2-state Case

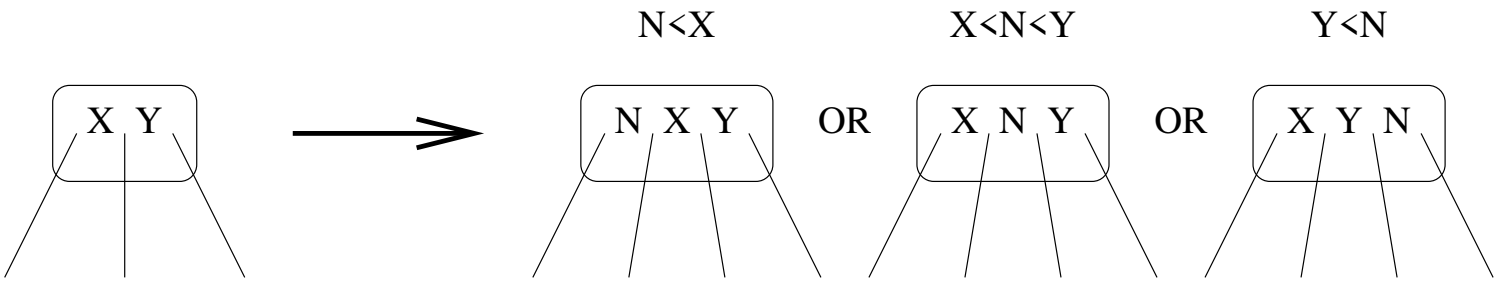
- If T is a leaf, then T changes to a 3-state tree containing N and X in the proper order, with all empty subtrees.



- If T is not a leaf, then N is inserted into *lst* when N<X, N is inserted into *rst* when X<N.

Insertion: The 3-state Case

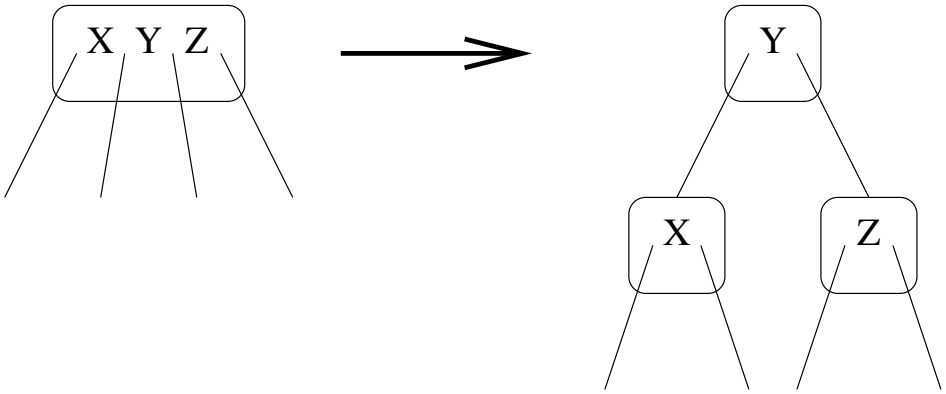
- If T is a leaf, then T changes to a 4-state tree containing N, X, and Y in the proper order, with all empty subtrees.



- If T is not a leaf, then N is inserted into *lst* when N<X, N is inserted into *mst* when X<N<Y, and N is inserted into *rst* when Y<N.

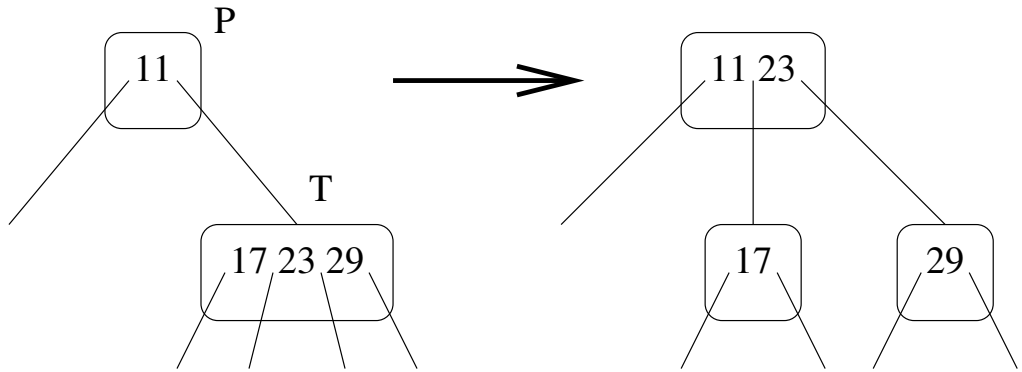
Insertion: The 4-state Case

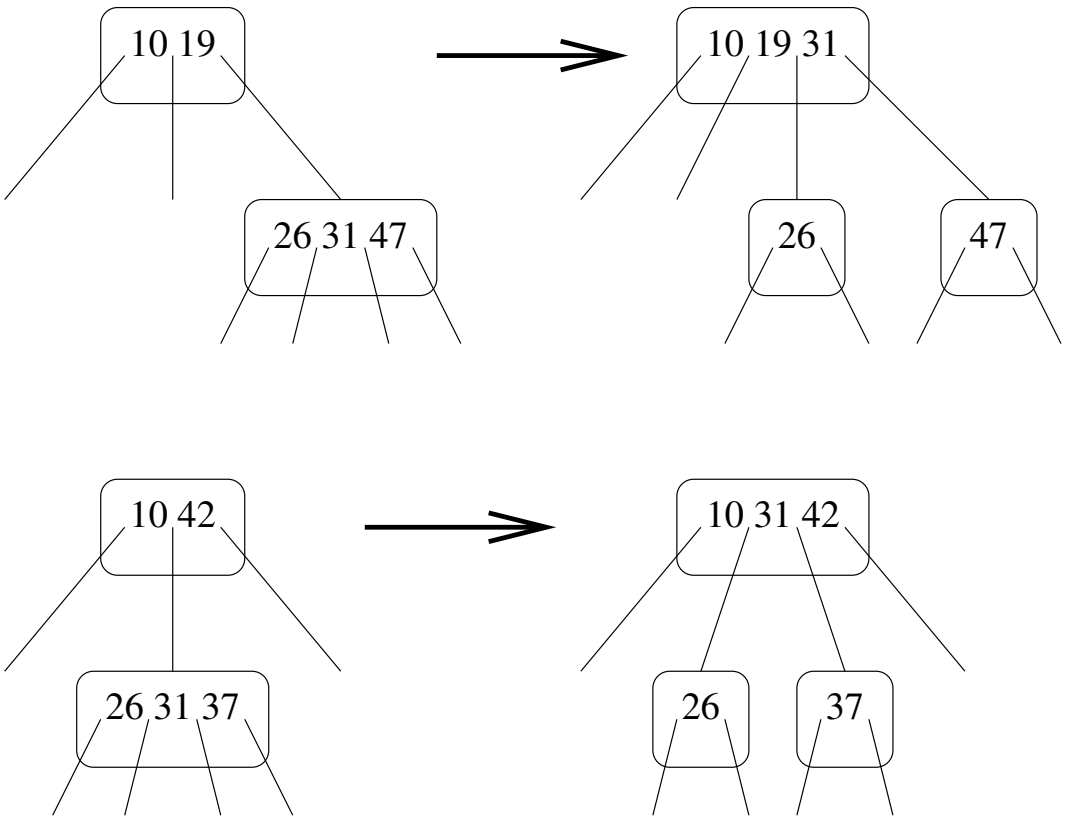
- If T is a leaf, then T changes to a 4-state tree containing N, X, and Y in the proper order, with all empty subtrees.



Insertion: The 4-state Case (cont'd)

- If T has a parent tree, P, then T merges with its parent tree in the following way before N is inserted.
  - As we consistently split all 4-state trees on the way down the tree, the parent tree, P, can only be a 2-state tree or a 3-state tree.
  - If P is a 2-state tree, then there are two cases ... the following diagram illustrates one of the cases. The other case is simply the mirror image.



Insertion: The 4-state Case (cont'd)

- If P is a 3-state tree, then there are three cases ... the following diagram illustrates one of the cases.