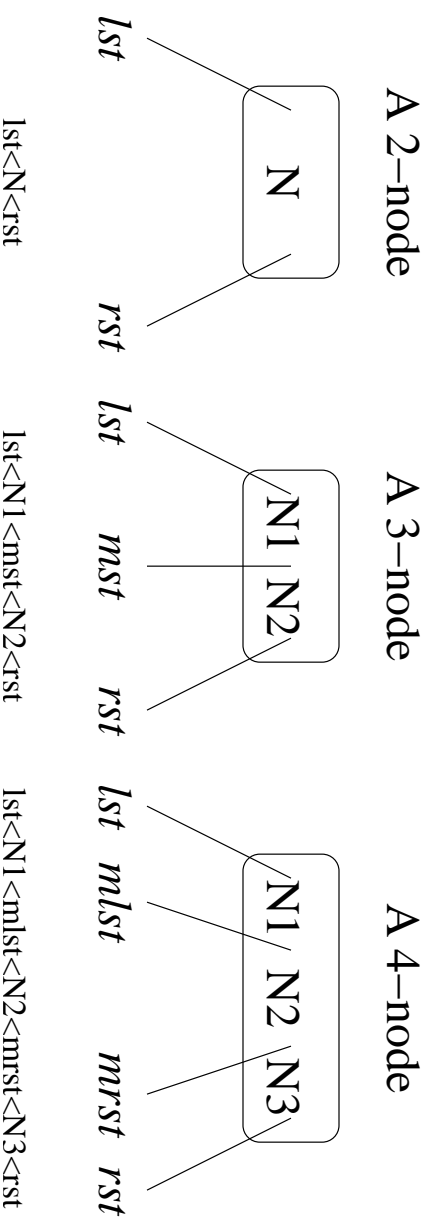


Overview

- Height Balanced Trees: 2-3-4 Trees

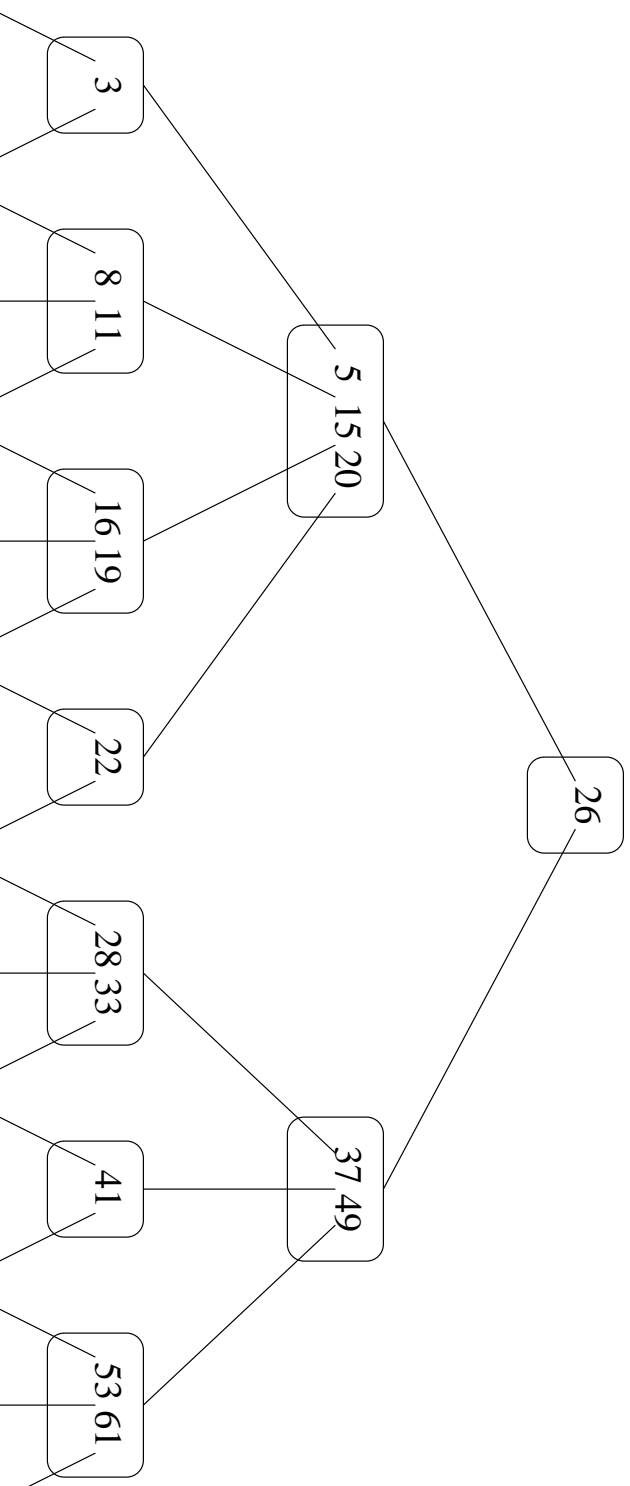
2-3-4 Trees

- A 2-3-4 Tree is suitable for use as an ordered container.
 - A 2-3-4 Tree can be empty (contains no data elements)
 - A non-empty 2-3-4 Tree can be in one of the three states:



2-3-4 Trees: An Example

- For simplicity, suppose that the keys are integers.

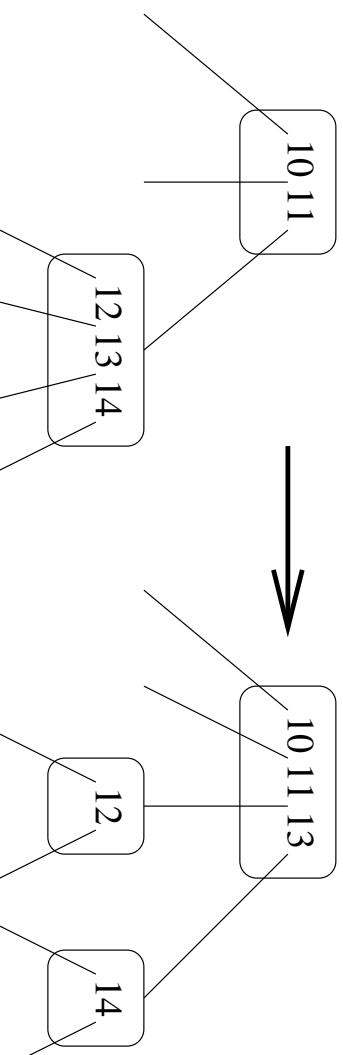
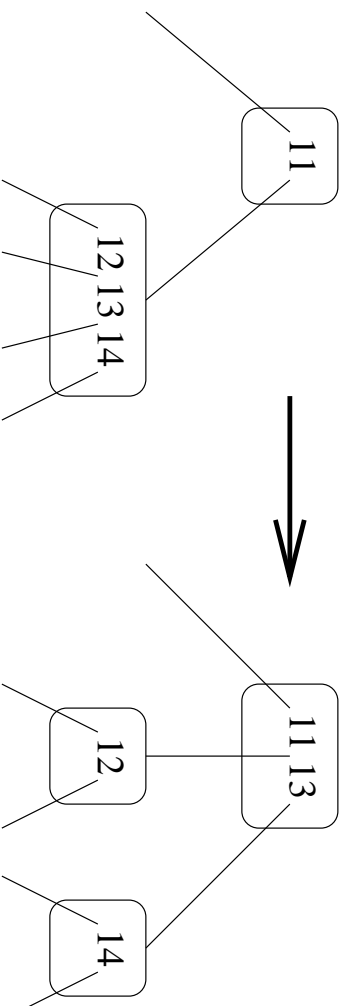


2-3-4 Trees: Insertion

- The algorithm for inserting an integer N into a 2-3-4 tree is:
 - Start at the root of the tree.
 - Follow the path down the tree in search of N .
 - * At each tree node along the path, if that tree node is a 4-state tree node, split it into two 2-state tree nodes and merge its middle data element with its parent tree node (if any).
 - * Finally, if the tree node is a leaf, insert N .
 - Note: The leaf may become a 4-state tree node. We don't split this tree node. We only split 4-state tree nodes encountered before inserting N .

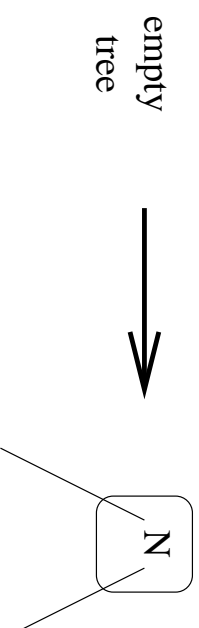
2-3-4 Trees: Insertion (cont'd)

- Suppose that I want to insert 15 into the following trees:



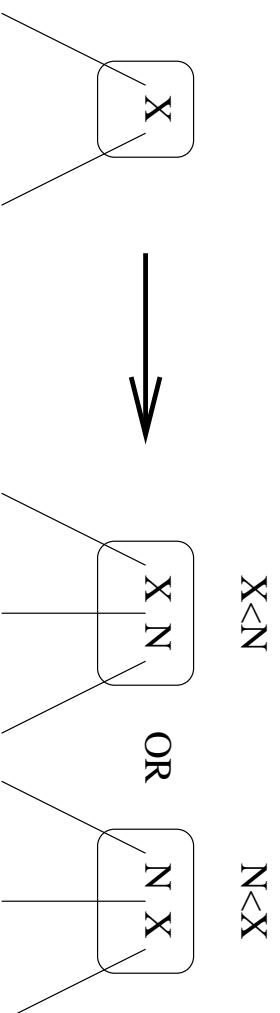
Insertion: Empty to 2-state

insert an integer N into an empty tree



Insertion: The 2-state Case

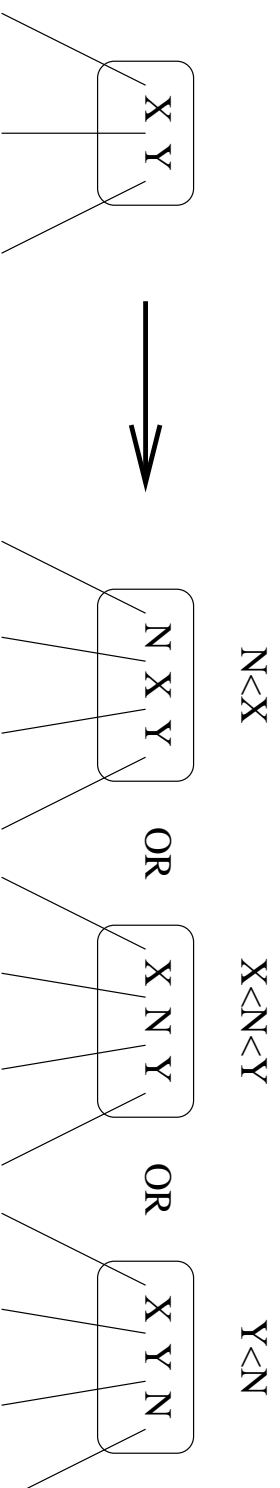
- If T is a leaf, then T changes to a 3-state tree containing N and X in the proper order, with all empty subtrees.



- If T is not a leaf, then N is inserted into *lst* when $N < X$, N is inserted into *rst* when $X < N$.

Insertion: The 3-state Case

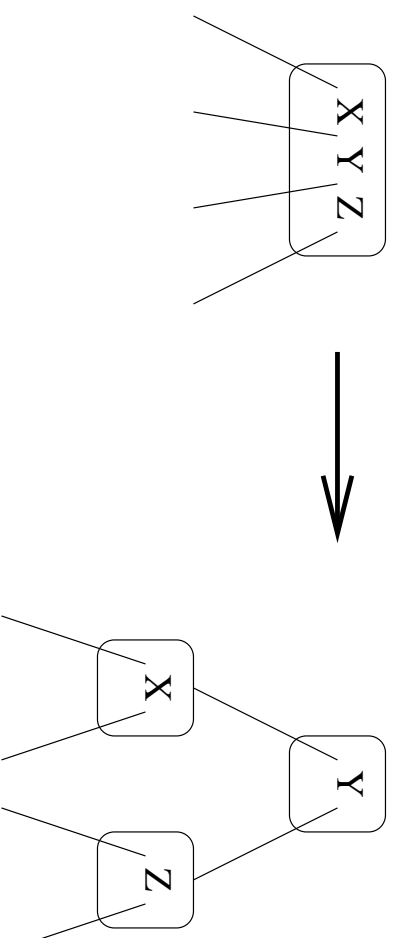
- If T is a leaf, then T changes to a 4-state tree containing N , X , and Y in the proper order, with all empty subtrees.



- If T is not a leaf, then N is inserted into *lst* when $N < X$, N is inserted into *mst* when $X < N < Y$, and N is inserted into *rst* when $Y < N$.

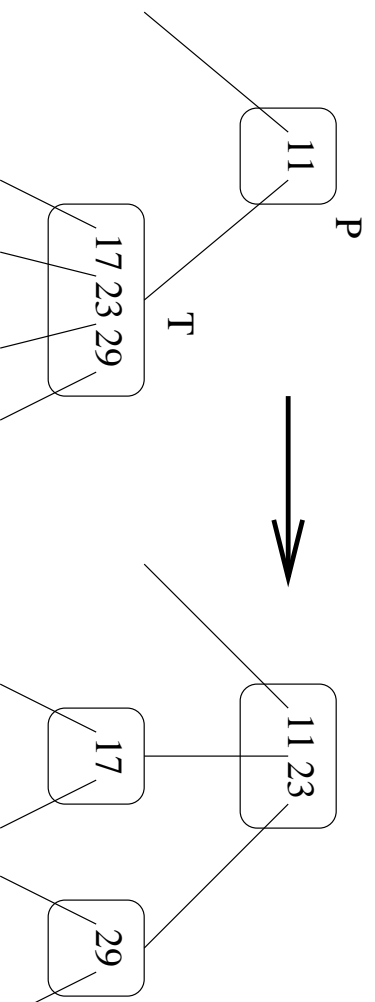
Insertion: The 4-state Case

- If T is a leaf, then T changes to a 4-state tree containing N , X , and Y in the proper order, with all empty subtrees.



Insertion: The 4-state Case (cont'd)

- If T has a parent tree, P, then T merges with its parent tree in the following way before N is inserted.
 - As we consistently split all 4-state trees on the way down the tree, the parent tree, P, can only be a 2-state tree or a 3-state tree.
 - If P is a 2-state tree, then there are two cases ... the following diagram illustrates one of the cases. The other case is simply the mirror image.



Insertion: The 4-state Case (cont'd)

- If P is a 3-state tree, then there are three cases ... the following diagram illustrates one of the cases.

