Overview

- Height Balanced Trees: 2-3-4 Trees
A 4-node

A 3-node

A 2-node

A non-empty 2-3-4 tree can be in one of the three states:

- A 2-3-4 tree can be empty (contains no data elements)
- A 2-3-4 tree is suitable for use as an ordered container.
For simplicity, suppose that the keys are integers.

2-3-4 Trees: An Example
Inserting $N$.

This tree node. We only split 4-state tree nodes encountered before.

Note: The leaf may become a 4-state tree node. We don't split.

Finally, if the tree node is a leaf, insert $N$.

Element with its parent tree node (if any).

Node, split it into two 2-state tree nodes and merge its middle data.

At each tree node along the path, if that tree node is a 4-state tree

Follow the path down the tree in search of $N$.

Start at the root of the tree.

The algorithm for inserting an integer $N$ into a 2-3-4 tree is:

2-3-4 Trees: Insertion
• Suppose that I want to insert 15 into the following trees:

2-3-4 Trees: Insertion (cont’d)
Insert an integer $N$ into an empty tree.
Insertion: The 2-state Case

If $T$ is not a leaf, then $N$ is inserted into $T$ when $N \geq X$. If $T$ is a leaf, then $T$ changes to a 3-state tree containing $N$ and $X$ in the proper order, with all empty subtrees.
If \( T \) is not a leaf, then \( N \) is inserted into \( \text{BST when } X \). N

If \( T \) is a leaf, then \( T \) changes to a 4-state tree containing \( N \). X, and Y

Insertion: The 3-state Case
In the proper order, with all empty subtrees.

\[ \text{If } T \text{ is a leaf, then } T \text{ changes to a 4-state tree containing } N', X', \text{ and } Y. \]

Insertion: The 4-state Case
Illustrates one of the cases. The other case is simply the mirror image.

- If P is a 2-state tree, then there are two cases:
  - parent tree, P, can only be a 2-state or a 3-state tree.
  - As we consistently split all 4-state trees on the way down the tree, the following way before N is inserted.

- If T has a parent tree, P, then T merges with its parent tree in the

Insertion: The 4-state Case (cont’d)
Illustrates one of the cases. If P is a 3-state tree, then there are three cases... the following diagram

Insertion: The 4-state Case (cont'd)