

Part II: Propositional Logic Homework Problems

ASIDE: Rice Students: If you like, you can play WaterWorld on OwlNet, in `/home/scheme/bin/waterworld`. To run Waterworld on your home computer, download (from owlnet) the directory `/home/scheme/plt/203/collects/waterworld/`, and (from `drscheme`) add it as a teachpack.

1 Propositional Logic

Exercise 1:

[practice]

Your friend Tracy argues: "It is bad to be depressed. Watching the news makes me feel depressed. Thus it's good to avoid watching the news."

Regardless of whether the premises and conclusion are true, show that the *argument* is not, by showing it doesn't hold for all domains. Replace "depressed" and "watching news" with expressions which leave the premises true, but the conclusion false (or at least, what most reasonable people would consider false).

Solution:

Lots of possible counterexamples. "It is bad to be depressed. Doing homework makes me depressed; so it's good to not do my homework." Or, "It is bad for people to be in physical pain. Childbirth causes pain. Therefore childbirth should be avoided by all people." If the original conclusion is really correct, Tracy needs to elucidate some of his unspoken assumptions.

The flaw seems to be along the lines of, "avoiding bad in the short run may not always be good in the long run" (or equivalently, sometimes you have to choose the lesser of two evils). No, you weren't asked to name a specific flaw, and reasonable people can differ on precisely what the flaw is. (And, formal logic is not particularly helpful here.) Nonetheless, uncovering hidden assumptions in arguments often helps understand the real issues involved.

ASIDE: For fun, pick up the front page of the daily newspaper, and see how many arguments use faulty rules of inference and/or rely on unspoken premises (which not all might agree with). In particular, political issues as spun to the mainstream press are often riddled with error, even though there are usually reasonable arguments on both sides which policy-makers and courts debate.

Exercise 2:

An acquaintance says the following to you: "Chris claims knowledge is more important than grades. But she spent yesterday doing an extra-credit assignment which she already knew how to do. Therefore, she's a hypocrite and deserves no respect."

Regardless of whether the premises and conclusion are true, show that the *argument* is not, by showing it doesn't hold for all domains. Replace "knowledge" and "grades" with expressions which give you true premises, but a false conclusion (or at least, what most reasonable people would consider false).

HINT: Exaggerate "knowledge" to something more important, and "grades" to something less important.

Solution:

(solution set will be posted later)

Exercise 3:

[practice]

Let p , q , and r be the following propositions:

- p : You get an A on the final exam
- q : You do every exercise in this book.
- r : You get an A in this class.

Write the following formulas using p , q , and r and logical connectives.

1. You get an A in this class, but you do not do every exercise in this book.
2. To get an A in this class, it is necessary for you to get an A on the final.
3. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

Solution:

1. $(r \wedge \neg q)$
2. $(r \rightarrow p)$
3. $((p \wedge q) \rightarrow r)$

Exercise 4:

Translate the following English sentences into propositional logic:

1. If the Astros win the series ("AW"), then pigs will fly ("PF").
2. Pigs will not fly, and/or bacon will be free ("BF").
3. The Astros will win the series, or bacon will be free (but not both).

Solution:

(solution set will be posted later)

Exercise 5:

[practice]

It just so happens that all the web pages in Logiconia which contain the word "Poppins" also contain the word "Mary". Write a formula expressing this. Use the proposition Poppins to represent the concept "the web page contains 'Poppins'" (and similar for Mary).

Solution:

$(\text{Poppins} \rightarrow \text{Mary})$

Exercise 6:

It further happens to be the case for Logiconian web pages: if such a page contains the word "weasel", it also contains either "words" or "eyed". Whenever a page contains the word "mongoose", it does *not* also contain the word "weasel". Finally, it is required to have the word "Logiconia" in it. Write a formula expressing these traits. (Your formula will involve six propositions – weasel, etc.. Try to find a formula which directly reflects the wording of the English above.)

If a web page in Logiconia does not contain "weasel", does it contain "mongoose"?

Let's go meta for a moment: What can you conclude about *this* web page? (Yes, this one you're looking at now – the one with the homework problems.) Why?

Solution:

(solution set will be posted later)

Exercise 7:

[practice]

Consider the particular board shown in the above figure (Figure 1) (.pdf)¹ (.ps)².

1. $Y - \text{safe}$, $Y - \text{has} - 0$, and $\neg Y - \text{has} - 2$ are among the formulas which are true for this board but not for all boards. (That is, they are not domain axioms or tautologies.) Give two other such formulas.
2. $V - \text{safe}$ might or might not be true for this board. Give two other such formulas.

Solution:

1. There are many simple answers, such as $Y - \text{has} - 1$, $\neg W - \text{has} - 1$, ...
2. There are many simple answers, such as A , $N - \text{has} - 1$, $J - \text{has} - 3$, ...

For each, there are also many such formulas composed with connectives such as \wedge and \vee .

Exercise 8:

In that same board (Figure 1) (.pdf)³ (.ps)⁴, is location W safe? What is your informal reasoning? (List all your small steps.) Similarly for location P.

Solution:

(solution set will be posted later)

Exercise 9:

Give a domain axiom of WaterWorld which is not explicitly shown in the WaterWorld domain axioms⁵. (Just show one that's omitted in the ellipses.)

Solution:

(solution set will be posted later)

¹<http://www.teachLogic.org/Base/Printables/hwA.pdf>

²<http://www.teachLogic.org/Base/Printables/hwA.ps>

³<http://www.teachLogic.org/Base/Printables/hwA.pdf>

⁴<http://www.teachLogic.org/Base/Printables/hwA.ps>

⁵<http://cnx.rice.edu/modules/m10528/latest/>

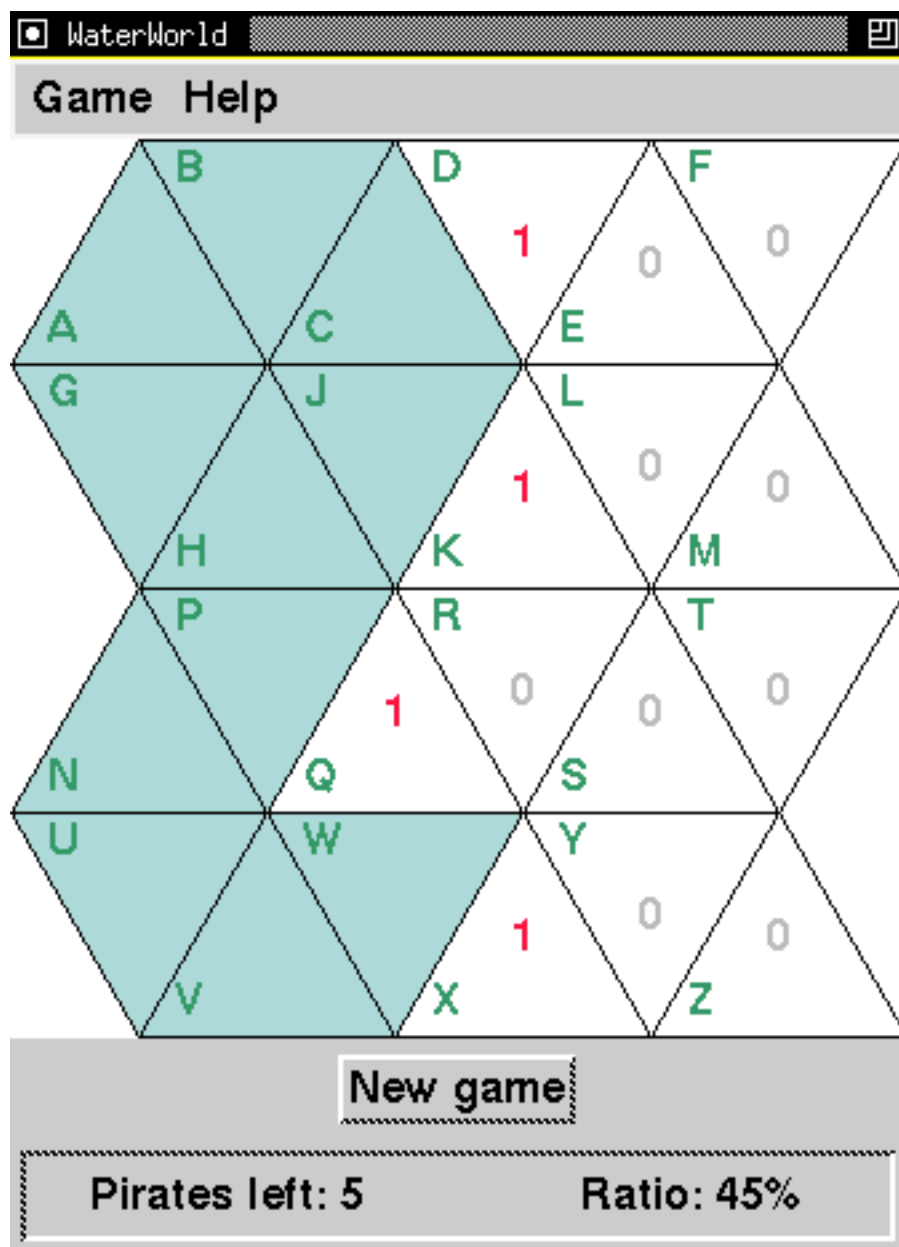


Figure 1: A sample WaterWorld board

Exercise 10:

Give one WFF which meets all three conditions:

- true in all WaterWorld boards ("A **theorem** of WaterWorld")
- not already listed as one of the WaterWorld domain axioms⁶, and
- not a tautology of propositional logic (can be made false in some truth assignment, though it may not be a truth assignment which satisfies the waterworld axioms).

This can either be a wimpy obvious formula, or can be some pattern you've noticed when playing, that requires several steps of inference.

Solution:

(solution set will be posted later)

2 Reasoning with Truth Tables

Exercise 11:

Write the truth-table for "xnor", the negation of exclusive-or. What is a more common name for this Boolean function?

Solution:

(solution set will be posted later)

Exercise 12:

Using truth-tables, show that $((a \vee c) \wedge (b \rightarrow c) \wedge (c \rightarrow a))$ is equivalent to $((b \rightarrow c) \wedge a)$, but that these are not equivalent to $((a \vee c) \wedge (b \rightarrow c))$.

Solution:

(solution set will be posted later)

Exercise 13:

Consider the following conditional code.

```
int i;
bool a,b;

...

if (a && (i > 0))
    return b;
else if (a && i <= 0)
    return false;
else if (a || b)
    return a;
else
    return (i > 0);
```

⁶<http://cnx.rice.edu/modules/m10528/latest/>

Simplify it by filling in the following blank with a single Boolean formula (not using a conditional `if`).

```
int i;
bool a,b;

...

return -----;
```

Solution:

(solution set will be posted later)

Exercise 14:

When writing a complicated conditional that involves multiple pieces of data, it is easy to incorrectly oversimplify. One strategy for avoid mistakes is to write such code in a two-step process. First, write a conditional with a case for every possible combination, as in a truth table. Second, simplify the conditional.

Using this approach, we might obtain the following code after the first step. Simplify this code.

```
list merge_sorted_lists(list list1, list list2)
{
    if (is_empty(list1) && is_empty(list2))
        return empty_list;
    else if (is_empty(list1) && !is_empty(list2))
        return list2;
    else if (!is_empty(list1) && is_empty(list2))
        return list1;
    else if (!is_empty(list1) && !is_empty(list2)) {
        if (first_element(list1) < first_element(list2))
            return make_list(first_element(list1),
                             merge_sorted_lists(rest_elements(list1),list2));
        else if (first_element(list1) >= first_element(list2))
            return make_list(first_element(list2),
                             merge_sorted_lists(list1,rest_elements(list2)));
    }
}
```

Solution:

(solution set will be posted later)

3 Reasoning with Equivalences

Exercise 15:

Using algebraic identities⁷ (.ps)⁸, show that $((a \vee c) \wedge (b \rightarrow c) \wedge (c \rightarrow a))$ is equivalent

⁷<http://cnx.rice.edu/modules/m10540/latest/>

⁸<http://www.teachLogic.org/Base/Printables/algebra-laws.ps>

to $((b \rightarrow c) \wedge a)$.

This is an algebraic hand-evaluation: a series of formulas joined by \equiv . Don't write just portions of previous formulas and mysteriously re-introduce the dropped parts later. For each step, mention which identity you used. It is also helpful if you underline the formula you are rewriting in the next step. You can use commutativity and associativity without using a separate line, but mention when you use it.

Some issues to think about (w/o turning in): in a previous exercise (Exercise 12) (.pdf)⁹ (.ps)¹⁰ we showed this same fact by using truth tables. Which approach appeals more to you? If trying to show equivalent to formulas with 10 propositions (instead of just 3) equivalent, which approach might you try? What about the rest of the previous problem – showing two formulas *non*-equivalent – it is possible to approach this with boolean algebra rather than truth tables. How? Is there some hybrid approach, that would convince you of the non/equivalence of formulas?

Solution:

(solution set will be posted later)

Exercise 16:

Using algebraic identities¹¹ (.ps)¹², rewrite the formula $((a \rightarrow (b \vee c)) \wedge \neg b)$ to one with fewer connectives.

Solution:

(solution set will be posted later)

Exercise 17:

Using algebraic identities¹³ (.ps)¹⁴, and the definition $\text{NOR}(\phi, \psi) = \neg(\phi \vee \psi)$, express the function \wedge in terms of NOR only.

Solution:

(solution set will be posted later)

Exercise 18:

Different search engines on the web have their own syntax for specifying searches. Some¹⁵ allow full Boolean queries. Some use implicit conjunctions, others implicitly use disjunctions, and they might or might not have further syntax for presenting more refined searches.

Read over the search syntax for the search language of eBay@@¹⁶.

1. Write a query (formula) for auctions which contain "border", do not contain "common", and contain at least one of "foreign" or "foriegn" [sic – misspellings are a great way to find underexposed auctions].

⁹<http://www.teachLogic.org/Base/Printables/hwA.pdf>

¹⁰<http://www.teachLogic.org/Base/Printables/hwA.ps>

¹¹<http://cnx.rice.edu/modules/m10540/latest/>

¹²<http://www.teachLogic.org/Base/Printables/algebra-laws.ps>

¹³<http://cnx.rice.edu/modules/m10540/latest/>

¹⁴<http://www.teachLogic.org/Base/Printables/algebra-laws.ps>

¹⁵<http://help.altavista.com/search/adv`help>

¹⁶<http://pages.ebay.com/help/buyerguide/search.html#commands>

2. Using the three connectives \wedge , \vee , and \neg , give a formula which doesn't have a *direct* translation into eBay's syntax. There might be some equivalent formula with a direct translation, but that's not part of this problem. (Strive for an example with the fewest connectives.) Test your answer on eBay¹⁷.

Solution:

(solution set will be posted later)

4 Reasoning with Inference rules

For proofs on this homework, Remember that each step must be justified by "premise", a listed inference rule¹⁸ (.ps)¹⁹ (a previous line number and, if ambiguous, substitutions for the inference rule's metavariables), or a WaterWorld domain axiom²⁰.

Exercise 19:

Fill in the blank reasons in the following proof that \vee commutes, that is, $(\chi \vee v) \vdash (v \vee \chi)$.

1. $(\chi \vee v)$ [premise]
2. subproof: $\chi \vdash (v \vee \chi)$
 - (a) χ [premise for subproof]
 - (b) $(v \vee \chi)$ [\vee Intro]
3. subproof: $v \vdash (v \vee \chi)$
 - (a) v [premise for subproof]
 - (b) $(v \vee \chi)$ [.....]
4. $(v \vee \chi)$ [.....]

Solution:

(solution set will be posted later)

Exercise 20:

Show that $(\tau \wedge \chi), (\tau \rightarrow v), (\chi \rightarrow \omega) \vdash (v \wedge \omega)$. (It may take around 8 steps.)

Solution:

(solution set will be posted later)

Exercise 21:

Using the inference rule RAA, prove $\neg\chi \vdash \neg(\chi \wedge v)$.

Solution:

(solution set will be posted later)

Exercise 22:

Show that $(\neg W - \text{safe} \vee \neg Y - \text{unsafe}) \vdash (W - \text{unsafe} \vee Y - \text{safe})$.

¹⁷<http://www.ebay.com/>

¹⁸<http://cnx.rice.edu/modules/m10529/latest/>

¹⁹<http://www.teachLogic.org/Base/Printables/inference-rules.ps>

²⁰<http://cnx.rice.edu/modules/m10528/latest/#waterworld-rules>

HINT: Use the \vee Elim inference rule to get the final result.

Solution:

(solution set will be posted later)

Exercise 23:

THIS PROBLEM IS REMOVED FROM HW03. It is incorrect as stated. Just ignore it, or you can patch it for a bit of extra-credit.²¹

For the following WaterWorld proof of $X - \text{has} - 1 \vdash (W - \text{unsafe} \vee Y - \text{unsafe})$, provide the missing steps and/or reasons:

1. $X - \text{has} - 1$ [.....]
2. [WaterWorld domain axiom]
3. [\rightarrow Elim]
4. subproof: $(W - \text{safe} \wedge Y - \text{unsafe}) \vdash (W - \text{unsafe} \vee Y - \text{unsafe})$
 - (a) $(W - \text{safe} \wedge Y - \text{unsafe})$ [premise for subproof]
 - (b) $Y - \text{unsafe}$ [.....]
 - (c) $(W - \text{unsafe} \vee Y - \text{unsafe})$ [\vee Intro]
5. subproof:
 - (a) $\neg(W - \text{safe} \wedge Y - \text{unsafe})$ [.....]
 - (b) [CaseElim (left), where $\phi = \dots$, and $\psi = \dots$]
 - (c) $W - \text{unsafe}$ [\wedge Elim]
 - (d) [\vee Intro]
6. [.....]

Solution:

(solution set will be posted later)

Exercise 24:

Given the above figure (Figure 2) (.pdf)²² (.ps)²³, and using any of the immediately obvious facts as premises, prove that location P is safe by using our proof system and the WaterWorld domain axioms.

While this proof is longer (about 15 steps), it's not too bad. To make life easier, you may also use the following:

- $\neg\phi \vdash \neg(\phi \wedge \psi)$
- $(Q - \text{has} - 1 \rightarrow ((P - \text{safe} \wedge R - \text{safe}) \vee (P - \text{safe} \wedge W - \text{safe}) \vee (R - \text{safe} \wedge W - \text{safe})))$.
- Not only is $Y - \text{safe}$ an allowable premise (reading off the given board), but so is $\neg Y - \text{unsafe}$.

Solution:

(solution set will be posted later)

²¹<http://www.owl.net.rice.edu/03spring/Homeworks/hw03-23ec.shtml>

²²<http://www.teachLogic.org/Base/Printables/hwA.pdf>

²³<http://www.teachLogic.org/Base/Printables/hwA.ps>

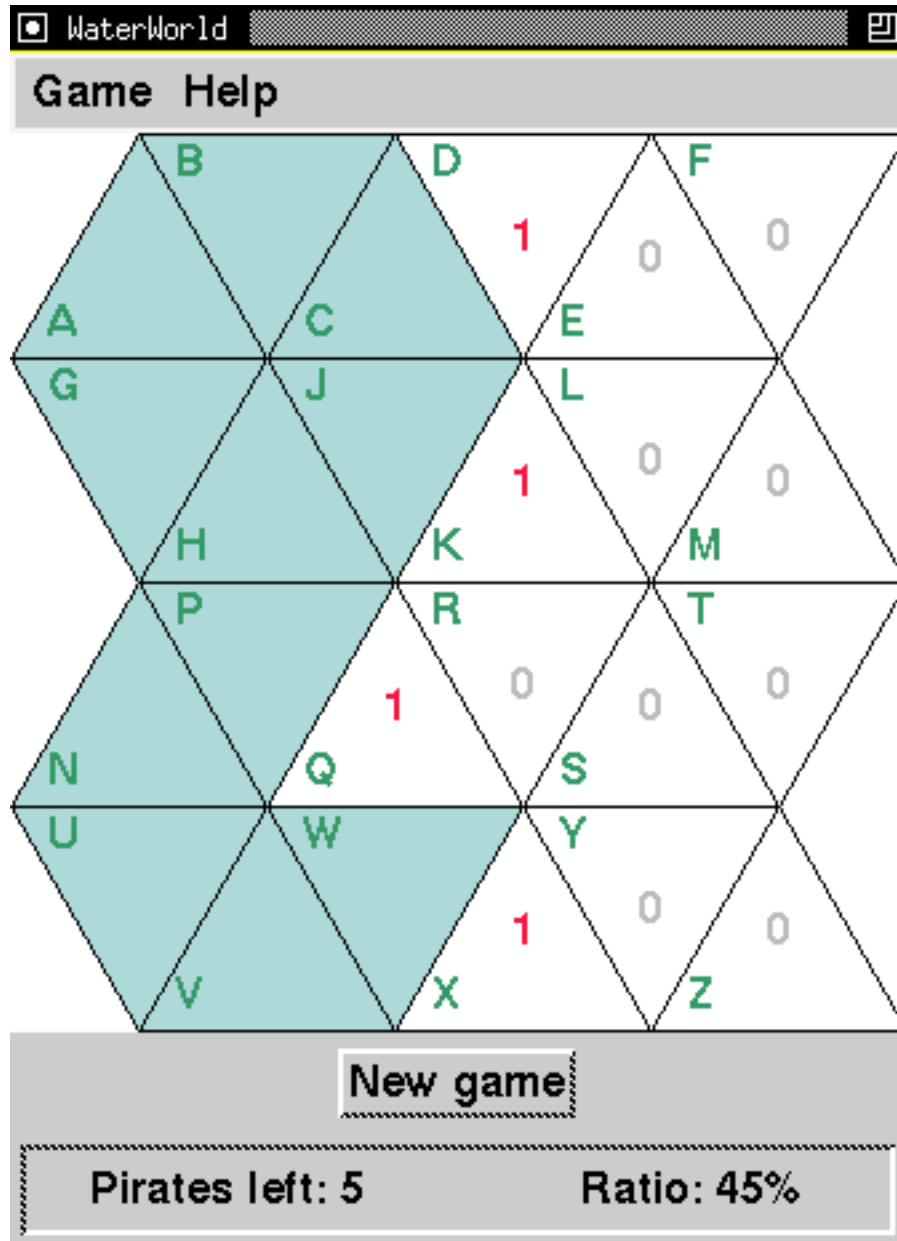


Figure 2: A sample WaterWorld board

Exercise 25:

Show that the \neg Elim inference rule is redundant in our system. In other words, without using \neg Elim, prove that $\neg\neg\phi \vdash \phi$.

Solution:

(solution set will be posted later)

Exercise 26:

Show that the \neg Intro inference rule is redundant in our system. In other words, without using \neg Intro, prove that $\phi \vdash \neg\neg\phi$.

Solution:

(solution set will be posted later)

Exercise 27:

State where on a board pirates could be positioned, so that: $(P - \text{has} - 1 \wedge U - \text{has} - 1 \wedge W - \text{has} - 1)$, but $X - \text{safe}$. Compare this with a previous theorem (example 11)²⁴ (.pdf)²⁵ (.ps)²⁶ $((B - \text{has} - 1 \wedge G - \text{has} - 1 \wedge J - \text{has} - 1) \rightarrow K - \text{unsafe})$, the same idea shifted down a couple of rows. Suppose we try to translate this proof so as to conclude $\neg X - \text{safe}$ (clearly untrue, by the above). What is the first step of the proof which doesn't hold when B,G,J,K are transliterated to P,U,W,X, respectively)? If you replace such steps with the closest true formulas you can ("translating" instead of "transliterating", if you will), where is the first line of the proof which fails and cannot be patched?

Solution:

(solution set will be posted later)

Exercise 28:

Sketch a board and an accompanying (simple) WFF where you know the formula is true, but it's not possible to prove it using the WaterWorld domain axioms²⁷.

HINT: Use a board where knowing the (total) number of pirates is needed to solve the puzzle.

Solution:

(solution set will be posted later)

Exercise 29:

The preceding difficulty (Exercise 28) (.pdf)²⁸ (.ps)²⁹ arises from the fact that we don't have any domain axioms dealing with the pirate count (even though we have propositions involving it).

This WaterWorld logic is incomplete, since we can write sentences that are true but don't have proofs. Clearly, this is a shortcoming of the WaterWorld proof system!

²⁴<http://cnx.rice.edu/modules/m10718/latest/#b1glj1:-k>

²⁵<http://www.teachLogic.org/Base/Printables/partIId.pdf>

²⁶<http://www.teachLogic.org/Base/Printables/partIId.ps>

²⁷<http://cnx.rice.edu/modules/m10528/latest/#waterworld-rules>

²⁸<http://www.teachLogic.org/Base/Printables/hwA.pdf>

²⁹<http://www.teachLogic.org/Base/Printables/hwA.ps>

ASIDE: If we'd never included propositions involving the number of remaining pirates, we'd have a complete proof system – but we wouldn't be able to fully express all the pertinent knowledge we use in our real-life reasoning about the board! In general, there is a balance between expressiveness, and desirable properties like completeness.

In this case though, it's easy to shore up the system by adding some domain axioms which are clearly independent from the rest. Give a new domain axiom which helps prove true formulas not previously provable, in boards where the total pirate count is 1. (Use "... " as needed, for repetitious parts). (There are many possible answers here.) Similarly for boards where the total pirate count is 2. (To think to yourself: about how many such new domain axioms are needed, just for boards with a pirate count of 5?)

ASIDE: Remember that our WaterWorld vocabulary only deals with the underlying state of the game, not with the view of the game. There are no propositions dealing with whether location A has been revealed or not; instead we recognize that some proofs use A – safe as a premise and some don't.

Solution:

(solution set will be posted later)