Part III: Relational Logic Homework Problems

Exercise 1:
Rosen, section 6.1 problem 2: For the relation "divides" on the set \{1,2,3,4,5,6\}:

1. List all the ordered pairs in the relation.
2. Display the relation graphically.
3. Display the relation in tabular form.

Solution:
(solution set will be posted later)

Exercise 2:
Rosen, section 6.1 problem 4: Determine whether, for the domain of all people, the following relations are reflexive, symmetric, antisymmetric, transitive.

1. taller?
2. bornSameDay?
3. sameFirstName?
4. commonGrandparent?

Solution:
(solution set will be posted later)

Exercise 3:
While Rosen discusses binary relations (subsets of pairs), one can also have unary relations (subsets of the domain – e.g. isPrime? for numbers), and ternary relations (subsets of triples – e.g. isAChildOf? over the domain of people.)

Suppose you wanted to encode addition as a ternary relation "addsTo?" over (triples of) numbers – describe briefly what triples \([x,y,z]\) would be in the relation; give an example of a triple in the relation addsTo? and a triple not in the relation addsTo?.

(Note that in general, you can represent any \(k\)-ary function as a \((k+1)\)-ary relation.)

Solution:
(solution set will be posted later)

Exercise 4:
Are each of the following formulas true for all interpretations ("valid")? (Remember that the relation-names are just names in the formula; don’t assume the name has to have any bearing on their interpretation.) (Don’t be misled by the relation-names used in the formula; they can be interpreted as any relation over any domain you specify.)

- For arbitrary \(a,b\) in the domain, \((\text{atLeastAsWiseAs}(a,b) \lor \text{atLeastAsWiseAs}(b,a))\)
- For arbitrary \(a\) in the domain, \((\text{prime}(a) \rightarrow (\text{odd}(a) \rightarrow \text{prime}(a)))\)
For arbitrary $a,b$ in the domain, $(\text{betterThan}(a,b) \rightarrow \neg\text{betterThan}(b,a))$

For each, if it is true or false under all interpretations, prove that. (For these small examples, a truth table will probably be easier than using boolean algebra or an inference-system proof.) Otherwise, give and interpretation in which it is true, and one in which it is false.

**Solution:**
(solution set will be posted later)

**Exercise 5:**
Suppose we wanted to represent the count of neighboring pirates with a binary relation, such that when location $A$ has two neighboring pirates, $\text{piratesNextTo}(A,2)$ will be true (and presumably, $\text{piratesNextTo}(A,1)$ would not be true in that particular relation, analagous to some the propositional WaterWorld domain axiom A2.)

If we only allow binary relations to be subsets a domain crossed with itself, then what must the domain be for this new relation $\text{piratesNextTo}$?

If we further introduced another relation, $\text{isNumber}$, what is a formula that would help distinguish intended interpretations from unintended interpretations? That is, give a formula that is true under all our intended interpretations of $\text{piratesNextTo}$ but is not true for some ”nonsense” interpretations we want to exclude. (This will be a formula without an analog in the WaterWorld domain axioms\(^1\).

**Solution:**
(solution set will be posted later)

\(^1\)http://cnx.rice.edu/modules/m10528/latest/