Testing
Four Problems

1. Sum of Consecutive Odd Numbers -- Speed of Algorithms

2. Sum of Consecutive Cubes -- Speed of Algorithms

3. Formula for Generating Prime Numbers -- Cryptography

4. Formula for Counting Prime Numbers -- Cryptography
Sum of Consecutive Odd Numbers

Example

- $1 = 1^2$

Conclusion

- The sum of consecutive odd numbers is equal to a square.
Sum of Consecutive Odd Numbers

Examples

\[ 1 = 1^2 \]
\[ 1 + 3 = 4 = 2^2 \]

Conclusion

The sum of consecutive odd numbers is equal to a square.
Sum of Consecutive Odd Numbers

Examples

\[1 = 1^2\]
\[1 + 3 = 4 = 2^2\]
\[1 + 3 + 5 = 9 = 3^2\]

Conclusion

*The sum of consecutive odd numbers is equal to a square.*
Sum of Consecutive Odd Numbers

Examples

\[ 1 = 1^2 \]
\[ 1 + 3 = 4 = 2^2 \]
\[ 1 + 3 + 5 = 9 = 3^2 \]
\[ 1 + 3 + 5 + 7 = 16 = 4^2 \]
Sum of Consecutive Odd Numbers

Examples

\[ 1 = 1^2 \]

\[ 1 + 3 = 4 = 2^2 \]

\[ 1 + 3 + 5 = 9 = 3^2 \]

\[ 1 + 3 + 5 + 7 = 16 = 4^2 \]

Conclusion

*The sum of the first \( n \) odd numbers is equal to \( n^2 \)*

\[ 1 + 3 + \cdots + 2n - 1 = n^2 \]
Sum of Consecutive Cubes

Example

• $1^3 = 1^2$

Conclusion

• *The sum of consecutive cubes is equal to a square.*
Sum of Consecutive Cubes

Example

\[1^3 = 1^2\]

\[1^3 + 2^3 = 1 + 8 = 9 = 3^2\]

Conclusion

The sum of consecutive cubes is equal to a square.
Sum of Consecutive Cubes

Example

\[ 1^3 = 1^2 \]
\[ 1^3 + 2^3 = 1 + 8 = 9 = 3^2 \]
\[ 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36 = 6^2 \]

Conclusion

The sum of consecutive cubes is equal to a square.
**Sum of Consecutive Cubes**

*Example*

\[ 1^3 = 1^2 \]

\[ 1^3 + 2^3 = 1 + 8 = 9 = 3^2 \]

\[ 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36 = 6^2 \]

\[ 1^3 + 2^3 + 3^3 + 4^3 = 1 + 8 + 27 + 64 = 100 = 10^2 \]
Sum of Consecutive Cubes

Example

\[1^3 = 1^2\]
\[1^3 + 2^3 = 1 + 8 = 9 = 3^2\]
\[1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36 = 6^2\]
\[1^3 + 2^3 + 3^3 + 4^3 = 1 + 8 + 27 + 64 = 100 = 10^2\]

Conclusion

The sum of consecutive cubes is equal to a square.

\[1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2\]
Formula for Generating Prime Numbers

\[ F(n) = n^2 + n + 41 \]

\[ F(1) = 1^2 + 1 + 41 = 43 \]

Conclusion

The function \( F(n) \) generates a prime number for every value of \( n \).
Formula for Generating Prime Numbers

\[ F(n) = n^2 + n + 41 \]

\[ F(1) = 1^2 + 1 + 41 = 43 \]
\[ F(2) = 2^2 + 2 + 41 = 4 + 2 + 41 = 47 \]

Conclusion

The function \( F(n) \) generates a prime number for every value of \( n \).
Formula for Generating Prime Numbers

\[ F(n) = n^2 + n + 41 \]

- \[ F(1) = 1^2 + 1 + 41 = 43 \]
- \[ F(2) = 2^2 + 2 + 41 = 4 + 2 + 41 = 47 \]
- \[ F(3) = 3^2 + 3 + 41 = 9 + 3 + 41 = 53 \]

Conclusion

The function \( F(n) \) generates a prime number for every value of \( n \).
Formula for Generating Prime Numbers

\[ F(n) = n^2 + n + 41 \]

\[ F(1) = 1^2 + 1 + 41 = 43 \]
\[ F(2) = 2^2 + 2 + 41 = 4 + 2 + 41 = 47 \]
\[ F(3) = 3^2 + 3 + 41 = 9 + 3 + 41 = 53 \]
\[ F(4) = 4^2 + 4 + 41 = 16 + 4 + 41 = 61 \]

Conclusion

The function \( F(n) \) generates a prime number for every value of \( n \).
Formula for Counting Prime Numbers

\[ \text{PrimePi}(n) = \text{number of primes less than or equal to } n \]

\[ \text{Li}(n) = \int_0^n \frac{dt}{\ln(t)} \]

Theorem

\[ \frac{n}{\text{Li}(n)} \leq \frac{n}{\text{PrimePi}(n)} \leq \ln(n) \quad \text{for } n > 20 \]
Four Problems

1. The sum of the first \( n \) odd numbers is equal to \( n^2 \).
   
   \[
   1 + 3 + \cdots + 2n - 1 = n^2
   \]

2. The sum of consecutive cubes is equal to a square.
   
   \[
   1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2
   \]

3. The function \( F(n) = n^2 + n + 41 \) generates a prime number for every value of \( n \).

4. \[
   \frac{n}{\text{Li}(n)} \leq \frac{n}{\text{PrimePi}(n)} \leq \ln(n) \quad \text{for } n > 20.
   \]