Undecidable Problems
Undecidable Problems

• Halting Problem

• Detecting Division by Zero

• Determining if Two Arbitrary Programs Always Generate the Same Output on the Same Input

• Finding Optimal Programs
Halting Problem

Halting Program

• \( H(P, I) \) -- prints YES, if \( P \) HALTS on input \( I \)
• \( H(P, I) \) -- prints NO, if \( P \) LOOPS FOREVER on input \( I \)
• Note: \( H(P, I) \) halts for all input \( P, I \).

Negation of Halting Program

• \( K(P) \)
  -- Run \( H(P, P) \)
  -- If Output is YES, then LOOP FOREVER
  -- If Output is NO, then HALT
Halting Problem (continued)

Paradox

• $K(K)$
  -- Run $H(K,K)$
  -- If Output is YES, then LOOP FOREVER
  -- If Output is NO, then HALT

• $H(K,K)$
  – If Output is YES, then $K(K)$ LOOPS FOREVER
  -- If Output is NO, then $K(K)$ HALTS

Therefore $H$ FAILS to solve the Halting Problem!
Detecting Division by Zero

Problem

- Given a program and some input, does the program ever divide by zero?

Observation

- Detecting division by zero is an Undecidable problem.
- If we could solve Division by Zero Problem, then we could solve the Halting Problem.
Theorem: Division by Zero is an Undecidable Problem

Proof: For every program:

• Replace every HALT command by a Division by Zero.
• Replace every division by;
  -- A Test to Determine if the Denominator is Zero
  -- If the Test is Positive:
    -- Perform an action equivalent to Division by Zero
    -- Jump around the Division
    Otherwise just Perform the Division
• New Programs Divides by Zero $\iff$ Old Program HALTS
• If we could solve the Division by Zero Problem we could also solve the HALTING Problem.
• Therefore we cannot solve the Division by Zero Problem.
More Undecidable Problems

Theorem: Determining whether two programs are equivalent is undecidable

Proof: For two programs to be equivalent they must at least either both HALT or both Loop on the same input. But the Halting Problem is Undecidable.

Theorem: Determining whether a program is optimal is undecidable.

Proof: To determine if a program is optimal, we must first determine equivalence, but equivalence is undecidable.