Design Patterns for Self-Balancing Trees

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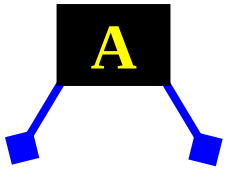
Motivations

- Classic self-balancing tree structures
  - 2-3-4 tree (see next slide)
  - red-black tree (binary tree equivalent of 2-3-4 tree)
  - B-tree (generalized 2-3-4 tree)
  - Difficult and complex. Where's the code?
- What's the proper abstraction?
  - Need to decouple algorithms from data structures.

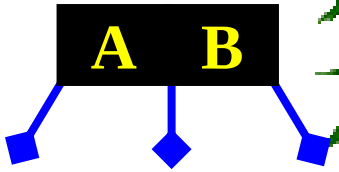
A 2-3-4 Tree is...

- **Empty**
  - **0-State: no data element + no sub-trees.**

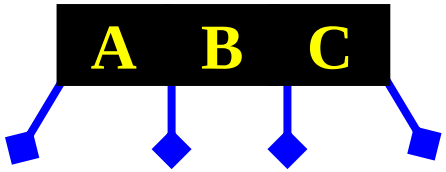
- **Non-Empty, in 3 possible states:**
  - **1-State: 1 data element + 2 sub-trees.**



  - **2-State: 2 data elements + 3 sub-trees.**



  - **3-State: 3 data elements + 4 sub-trees.**

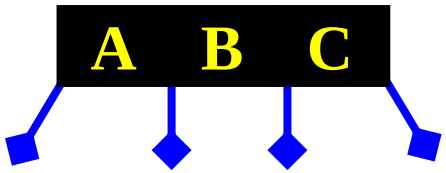


Variant *vs.* Invariant Operations

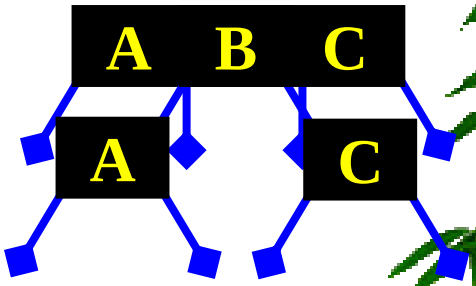
- **Self-balancing insertion is not an intrinsic (invariant) operation of a tree.**
- **What are the invariant operations?**
  - **Gettors & Constructors.**
  - **Constructive and Destructive operations:**
    - ***Constructive*: Splice a tree into another.**
    - ***Destructive*: Split a tree into a 2-state.**

Splittin' and Splicin'

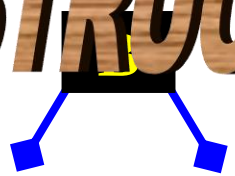
Splice:



Split Up:

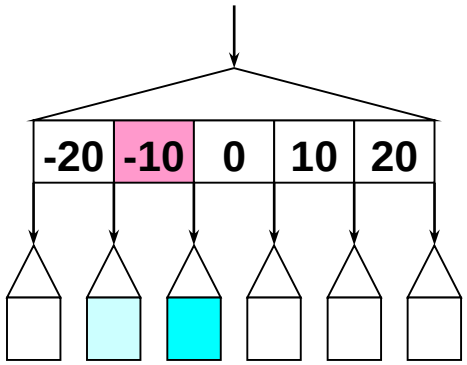


Intrinsic operations on the tree STRUCTURE, not the data!

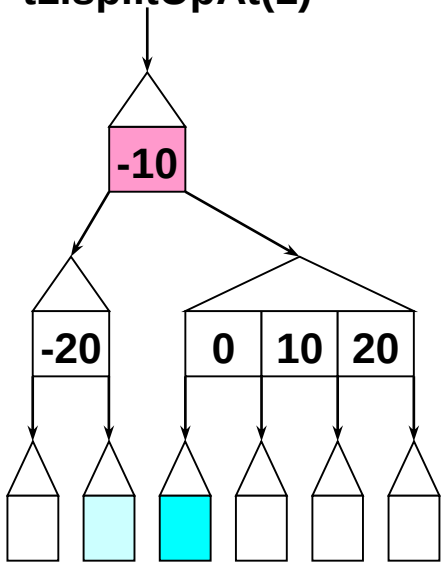


Structural Operations

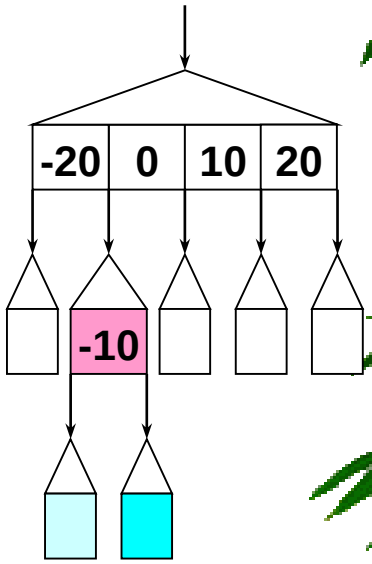
t1



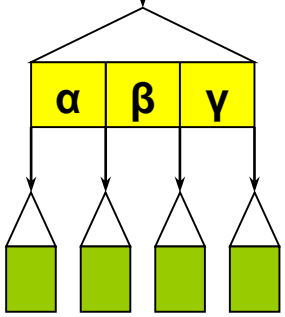
t1.splitUpAt(1)



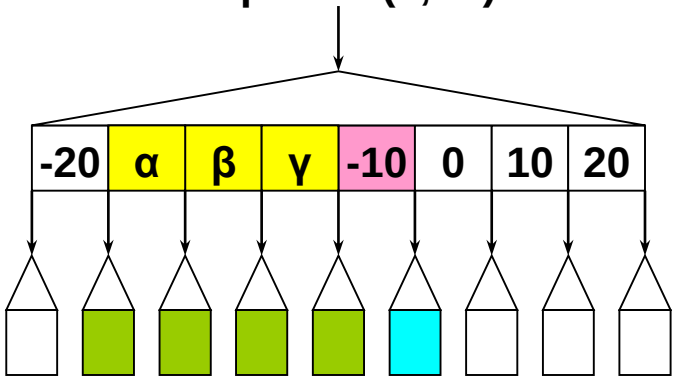
t1.splitDownAt(1)



t2

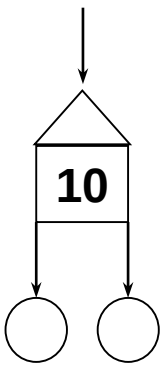


t1.spliceAt(1, t2)

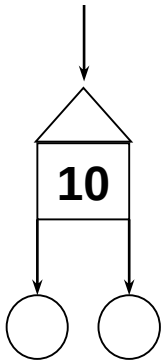


Con/De-struction

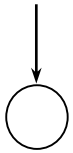
t1



t1.splitUpAt(0)

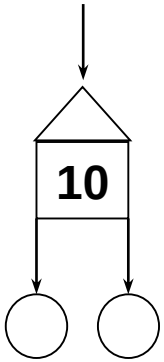


t1.splitDownAt(0)

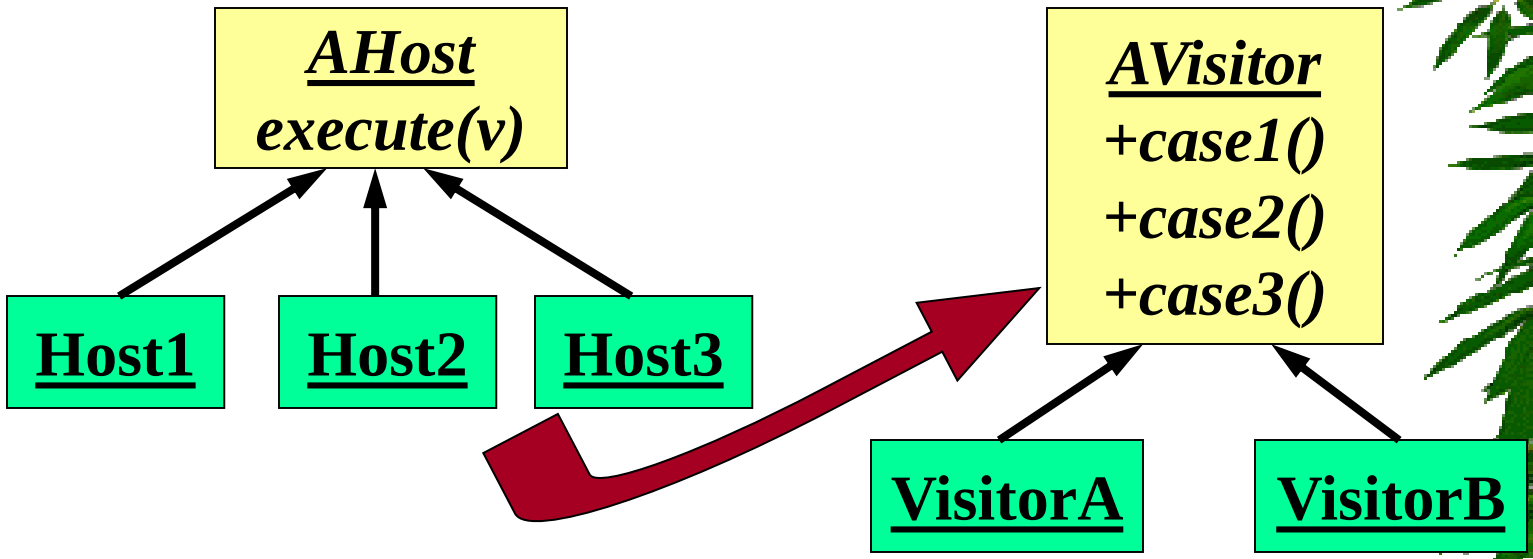


t2

t2.spliceAt(0, t1)



Visitor Design Pattern

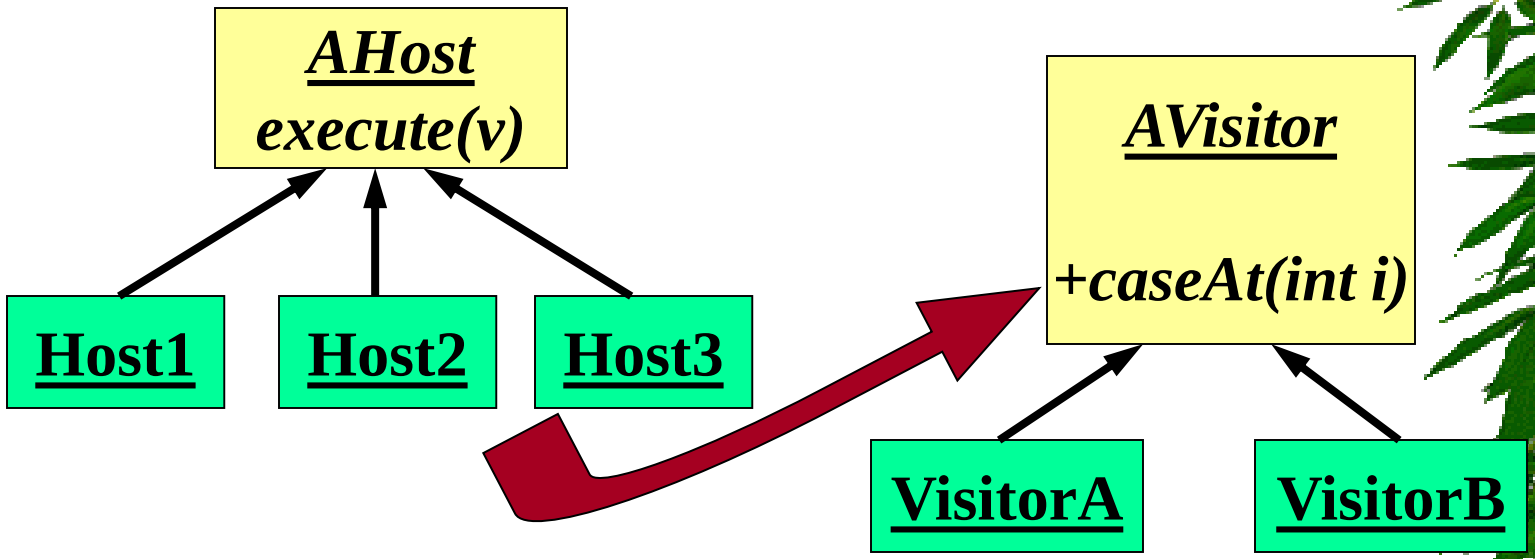


Invariant: $Host_i$ calls $case_i$ of the visitor.

Fixed # of methods → fixed # of hosts

Non-Extensible!

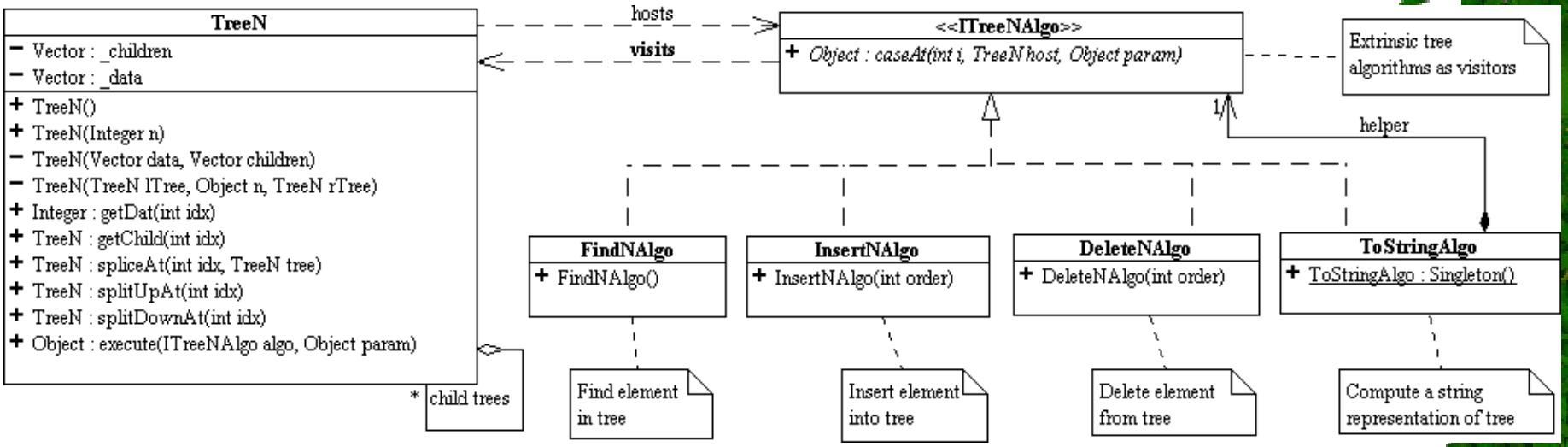
Generalized Visitors



Invariant: $Host_i$ calls caseAt(i) of the visitor.

Unbounded # of hosts!

TreeN and Algorithms



toString() Algorithm

```
public class ToStringAlgo implements ITreeNAlgo {
   // Constructors omitted

   public Object caseAt(int idx, TreeN host
      switch(idx) {
         case 0: {  return "[ ]";  }
         default: {
            String sData= "", sTrees="";
            for(int i = 0;i<idx;i++) {
              sData += host.getDat(i)+" ";
              sTrees += host.getChild(i).execute(toStringHelp,"|  ")+"\n";
             }
             sTrees += host.getChild(idx).execute(toStringHelp,"    ").toString();
             return sData +"\n"+sTrees;
          }
       }
   }
   ITreeNAlgo toStringHelp =  …see next slide….
}
```

Empty Tree

Non-Empty Tree

“Prefix” data

ToString() Helper

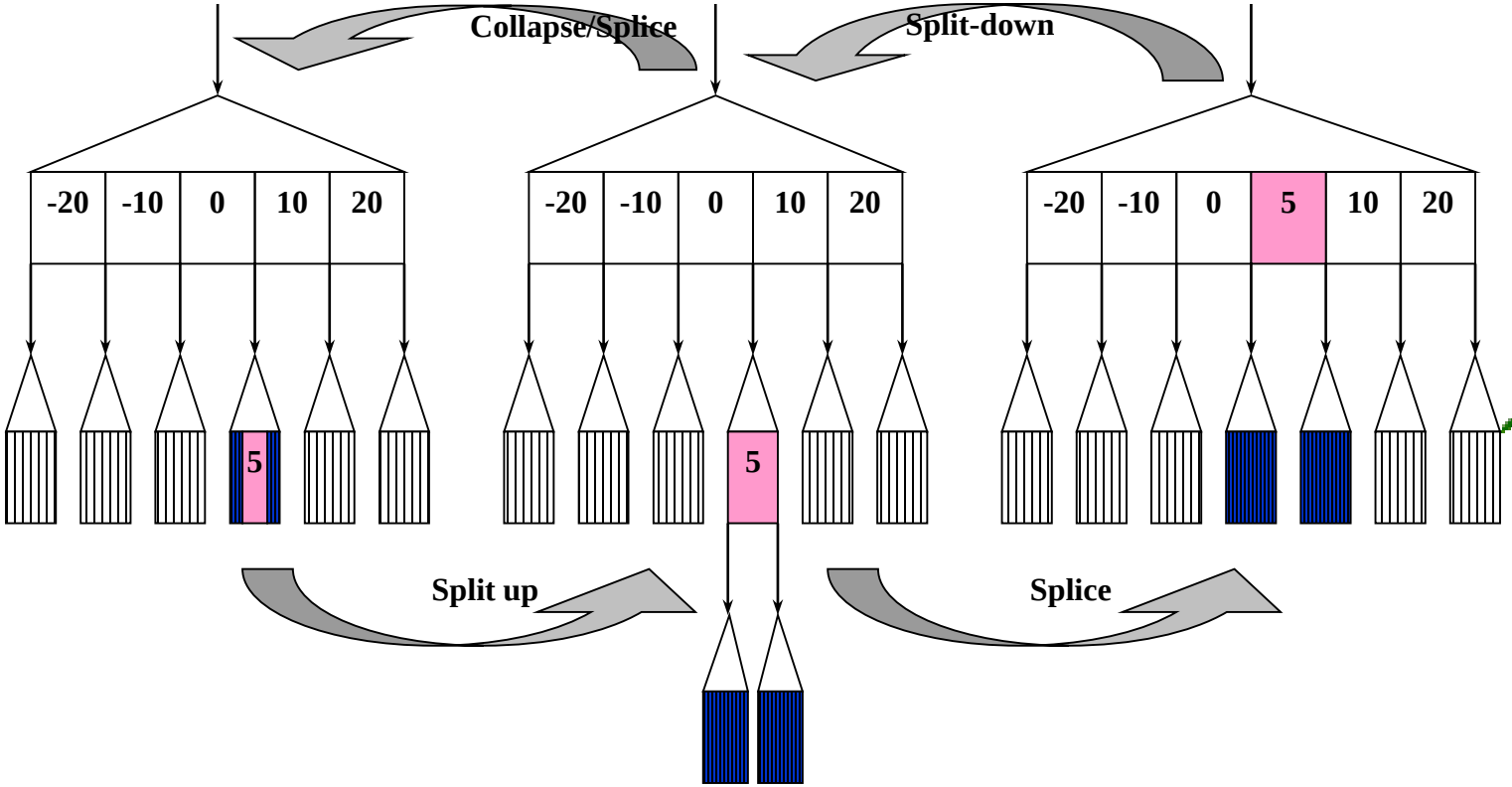
```
private final static ITreeNAlgo toStringHelp = new ITreeNAlgo() {
   public Object caseAt(int idx, TreeN host,
      switch(idx) {
         case 0: {  return "|_[ ]"; }
         default: {
            String sData= "", sTrees="";
            for(int i = 0;i<idx;i++) {
               sData += host.getDat(i)+" ";
               sTrees += prefix
                           + (String) host.getChild(i).execute(this, prefix+"|  ")+"\n";
            }
            sTrees += prefix
                        + host.getChild(idx).execute(this, prefix+"    " ).toString();
            return "|_ "+sData +"\n"+sTrees;
         }
      }
   }
};
```

Empty Tree

Non-Empty Tree

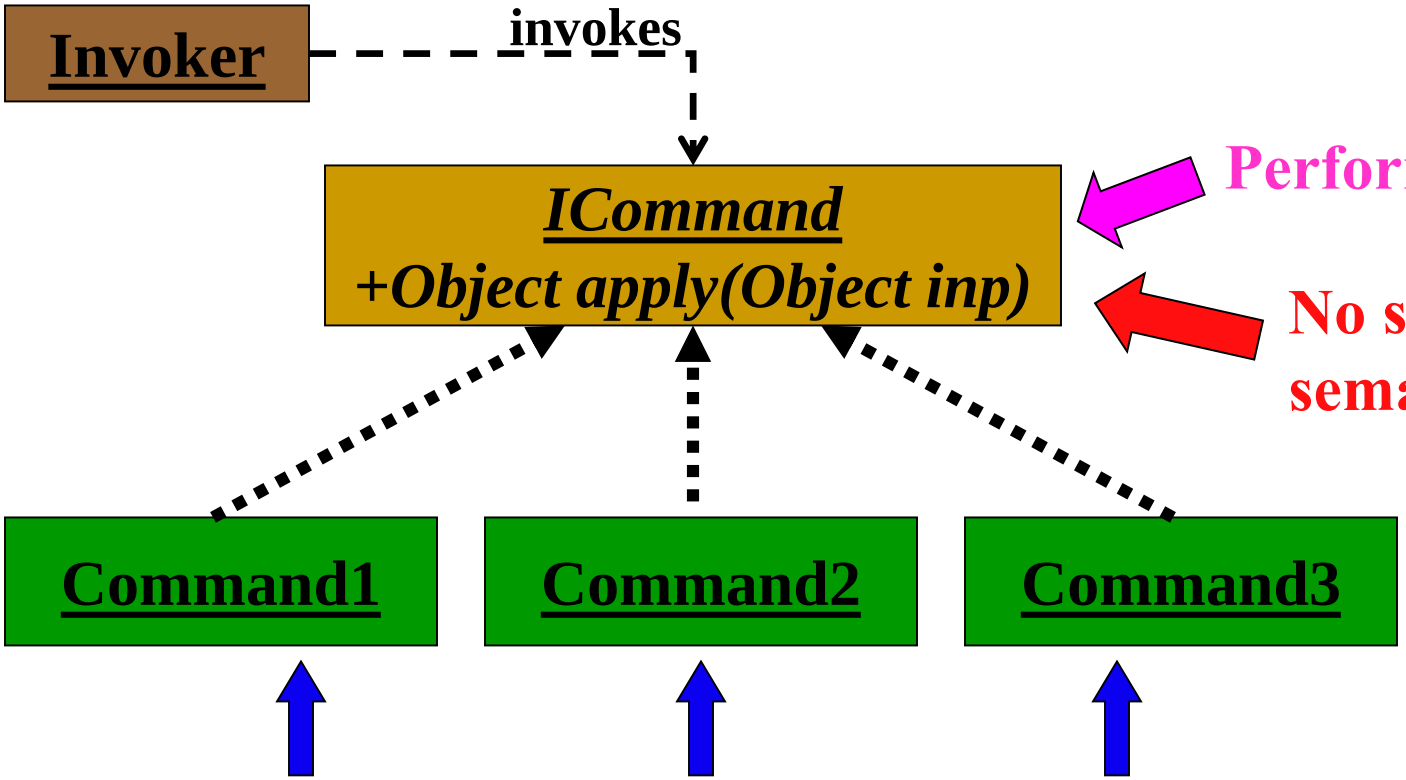
“Prefix” data

Vertical Data Transport



No net height change except at root and leaves!

Command Design Pattern



Well-defined, but unrelated semantics.

Commands = Lambda Functions

Anonymous inner classes provide closures for lambdas!

Insertion Heuristics

- Insertion must take place at the leaf.
- Tree must grow only at the root.

Must transport data
from the leaves to the root
without affecting the height balance.

Problem: If a child node is too wide, it needs to split up and splice into its parent, but…

- The child node does not know *where* to splice into its parent
- The child does not even have a reference to its parent.

Solution: Pass a command (lambda) forward from the parent to the child during the recursive call.

Split-up and Splice (Apply)

```
class SplitUpA                                    reeNAlgo {
    int _order;
    public SplitUpAndApply(int order) {  order = order;}

    public Object caseAt(int i, TreeN host
        if(i <= _order) return host;
        else {
            host.splitUpAt(i / 2);
            return ((ILambda)param).apply(host); }
    }
}
```

Max width of node

Not too wide? → no-op

Too wide? → Split up then apply lambda

The lambda splices this

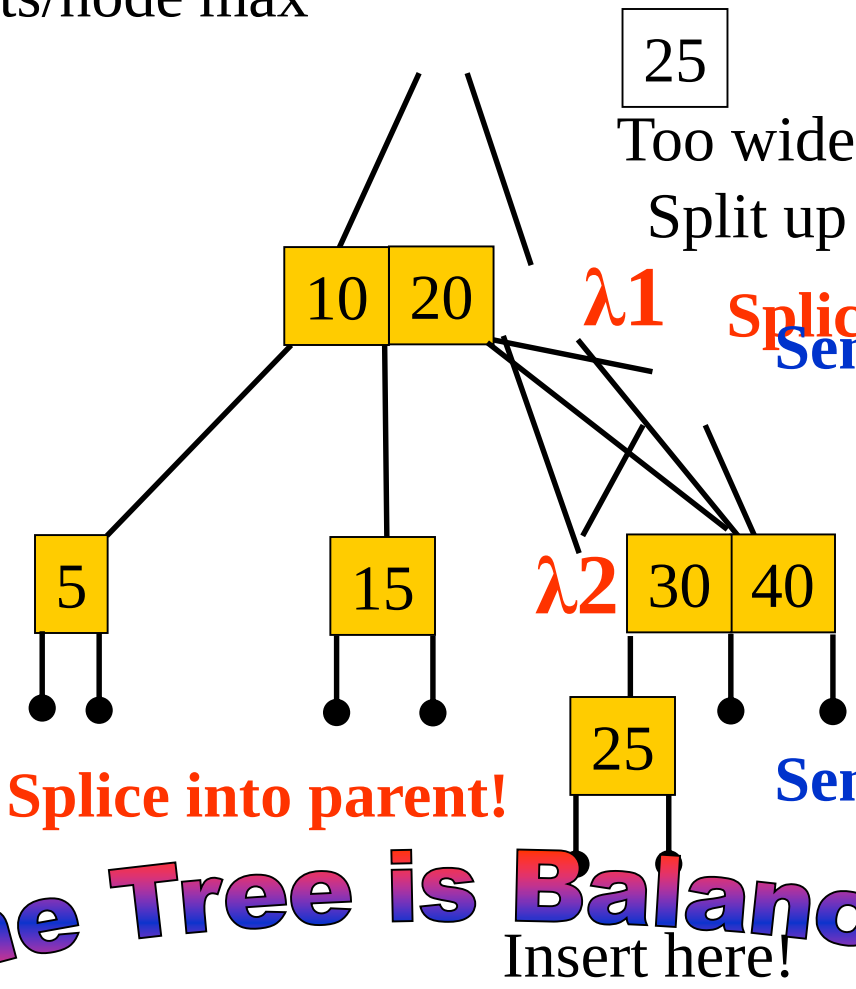
Lambda/commands enable decoupled communication!

Insertion Algorithm

- Find insertion point at the leaf and splice new data in.
- Use Split-and-Apply visitor to transport excess data upwards.
  - Visitor passed as parameter to recursive call.
  - Non-root: split-and-splice
  - Root node: split-and-no-op will cause entire tree to grow in height.
  - Abstract the splice/no-op as a command passed to the visitor!

Insertion Dynamics

2 elements/node max



Too wide!

Split up

λ1

Splice into parent!

Compare and create splicing lambda

Send lambda to child!

Too wide!

Split up

λ2

Compare and create splicing lambda

Send lambda to child!

Splice into parent!

The Tree is Balanced!

Insert here!

```
public Object caseAt(int s, final Tre
    switch(s) {
        case 0: {  return host.spliceAt(0
        default: {
            host.execute(new ITreeN
                public Object caseAt(
                    switch(s_help) {
                        case 0: { return ((ILambda)cmd).apply(new TreeN((Integer)key)) ;}
                        default: {
                            final int[] x={0};  // hack to get around final
                            for(; x[0] < s_help; x[0]
                                int d = h.getDat(x[0]).
                                if (d >= ((Integer)key).
                                    if (d == ((Integer
                                    else break; } }
                            h.getChild(x[0]).execute(t
                                public Object appl
                                    return h.splice
                            return h.execute(splitU
        new ILambda() {
            public Object appl
    return host;  } } }
```

Non-Empty tree? → create a recursive helper

Empty tree? → splice into parent

Empty tr Splice new tre in here!

Find the insertion/splice location

Recur into the child, passing on the splicing lambda

Run the given splicing lambda

Root has no parent to splice into

The beauty of closures!

O(log n) insertion!

Deletion Heuristics

- Deletion only well-defined at leaf.
- Data might exist anywhere in the tree.
- Tree can only shorten at root.

→ Push "candidate" data down from the root to the leaves.

→ Bubble "excess" data back to the root.

Must transport data
from the root to the leaves and
from the leaves to the root
without affecting the height balance.

Deletion Algorithm

- Identify candidate data
  - split down at candidate and collapse with children.
  - If root is a 2-node, then tree will shorten.
- Data to delete will appear as 2-node below leaves.
- Use Split-and-Apply to transport excess data upwards.

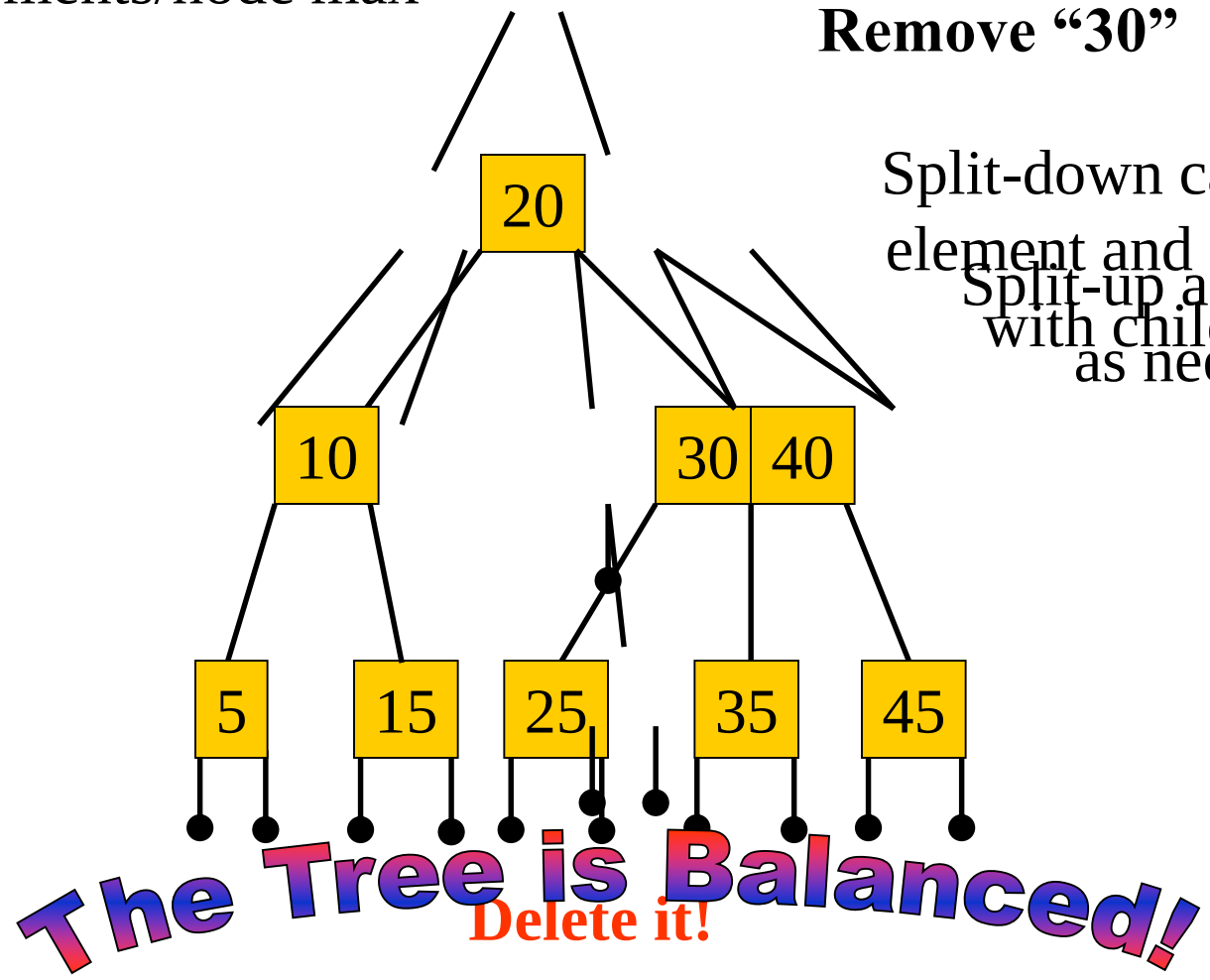
Deletion Dynamics

2 elements/node max

Remove “30”

Split-down candidate element and collapse

Split-up and splice with children as needed



Conclusions

- Proper abstraction leads to
  - Decoupling
  - Simplicity
  - Flexibility & extensibility
- Generalized Visitors open up new possibilities.
- Self-balancing trees teach
  - Abstract decomposition
  - Design patterns
  - Component-frameworks
  - Lambda calculus
  - Proof-of-correctness & complexity analysis