Syntax Analysis, VII

The Canonical LR(1) Table Construction

Comp 412

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A table-driven LR(1) parser is a bottom-up shift-reduce parser

- Grammatical knowledge is encoded in two tables: *Action* & *Goto*
  - They encode the handle-finding automaton
  - They are constructed by an LR(1) parser generator

- Why two tables?
  - *Reduce* needs more information & more complex information than *shift*
  - *Goto* holds that extra information
Table-Driven LR Parsers

A table-driven LR(1) parser is a bottom-up shift-reduce parser

Back to the Meta Issue

• The compiler writer creates a grammar at design time
• The parser generator builds ACTION and GOTO tables at build time
• The compiler uses those tables to parse at compile time
Building LR(1) Tables

How do we generate the ACTION and GOTO tables?

• Use the grammar to build a model of the Control DFA
• Encode actions & transitions in ACTION & GOTO tables
• If construction succeeds, the grammar is LR(1)
  — “Succeeds” means defines each table entry uniquely

The Big Picture

• Model the state of the parser with LR(1) items
• Use two functions goto\((s, X)\) and closure\((s)\)
  — goto() is analogous to move() in the subset construction
  — Given a partial state, closure() adds all the items implied by the partial state
• Build up the states and transition functions of the DFA
• Use this information to fill in the ACTION and GOTO tables
To understand the algorithms, we need to understand the data structure that they use: LR(1) items

An LR(1) item is a pair \([P, \delta]\), where

- \(P\) is a production \(A \rightarrow \beta\) with a \(\bullet\) at some position in the RHS
- \(\delta\) is a single symbol lookahead \((symbol = word \ or \ EOF)\)

The LR(1) table construction algorithm models a configuration of the parser, or a state of the parser, as a set of LR(1) items

- Each item represents a specific production by which the parser might reduce in the future
- Each item’s placeholder represents the progress toward that reduction

The construction ties builds up a set of configurations that model all the possible behaviors implied by the grammar
LR(1) Items

An LR(1) item is a pair \([P, \delta]\), where

- \(P\) is a production \(A \rightarrow \beta\) with a • at some position in the RHS
- \(\delta\) is a single symbol lookahead \((\text{symbol} \neq \text{word or EOF})\)

The • in an item indicates the position of the top of the stack

\([A \rightarrow \bullet \beta \gamma, a]\) means that the input seen so far is consistent with the use of \(A \rightarrow \beta \gamma\) immediately after the symbol on top of the stack.
We call an item like this a possibility.

\([A \rightarrow \beta \bullet \gamma, a]\) means that the input sees so far is consistent with the use of \(A \rightarrow \beta \gamma\) at this point in the parse, and that the parser has already recognized \(\beta\) (that is, \(\beta\) is on top of the stack).
We call an item like this a partially complete item.

\([A \rightarrow \beta \gamma \bullet, a]\) means that the parser has seen \(\beta \gamma\), and that a lookahead symbol of \(a\) is consistent with reducing to \(A\).
This item is complete.

\(\text{LR}(k)\) parsers rely on items with a lookahead of \(\leq k\) symbols.
That leads to \(\text{LR}(k)\) items, with correspondingly longer \(\delta\).
LR(1) Items

The production $A \rightarrow \beta$, where $\beta = B_1 B_2 B_3$ with lookahead $a$, can give rise to 4 items

\[
[A \rightarrow \bullet B_1 B_2 B_3, a], [A \rightarrow B_1 \bullet B_2 B_3, a], [A \rightarrow B_1 B_2 \bullet B_3, a], & [A \rightarrow B_1 B_2 B_3 \bullet, a]
\]

The set of LR(1) items for a grammar is \textit{finite}.

What’s the point of all these lookahead symbols?

• Carry them along to help choose the correct reduction

• Lookaheads are bookkeeping, unless item has $\bullet$ at right end
  – Has no direct use in $[A \rightarrow \beta \bullet, a]$
  – In $[A \rightarrow \beta \bullet, a]$, a lookahead of $a$ implies a reduction by $A \rightarrow \beta$
  – For \{ $[A \rightarrow \beta \bullet, a], [B \rightarrow \gamma \bullet, b]$ \}, $a \Rightarrow \text{reduce to } A$; $\text{FIRST}(\delta) \Rightarrow \text{shift}$

$\Rightarrow$ Limited right context is enough to pick the actions
LR(1) Items

The set of LR(1) items for a grammar is finite

Consider the SheepNoise grammar

<table>
<thead>
<tr>
<th>Prod</th>
<th>Goal</th>
<th>SheepNoise</th>
<th>SheepNoise</th>
<th>baa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Goal</td>
<td>→ SheepNoise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>SheepNoise</td>
<td>→ SheepNoise</td>
<td>baa</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>baa</td>
<td></td>
</tr>
</tbody>
</table>

It gives rise to 7 LR(1) items

[Goal → • SheepNoise]  [Goal → SheepNoise •]  0
[Goal → • SheepNoise baa]  [Goal → SheepNoise • baa]  [Goal → SheepNoise baa •]  1
[Goal → • baa]  [Goal → baa •]  2

7 is finite. Why does this matter?
A configuration of the LR(1) parser is represented as a set of LR(1) items—the collection of productions by which the parser might reduce in the future, given the context seen so far, with the placeholder showing the progress in recognizing each production. The number of such states is, again, finite.
LR(1) Items: Why should you know this stuff?

Debugging a grammar

• When you build an LR(1) parser, it is possible (likely) that the initial grammar is not LR(1)
• The tools will provide you with debugging output
• To the right is a sample of bison’s output for the if-then-else grammar

The state is described by its LR(1) items:

<table>
<thead>
<tr>
<th>LR(1) Items</th>
<th>Debugging a grammar</th>
</tr>
</thead>
<tbody>
<tr>
<td>goal</td>
<td>stmt_list</td>
</tr>
<tr>
<td>stmt_list</td>
<td>stmt_list stmt</td>
</tr>
<tr>
<td></td>
<td>stmt</td>
</tr>
<tr>
<td>stmt</td>
<td>IF EXPR THEN stmt</td>
</tr>
<tr>
<td></td>
<td>IF EXPR THEN stmt</td>
</tr>
<tr>
<td></td>
<td>ELSE stmt</td>
</tr>
<tr>
<td></td>
<td>OTHER</td>
</tr>
</tbody>
</table>

That period is the •
LR(1) Table Construction

High-level overview

1. Build the Canonical Collection of Sets of LR(1) Items, \( I \)
   a. Begin in an appropriate state, \( s_0 \)
      - \([S' \rightarrow S, \text{EOF}]\), along with any equivalent items
      - Derive equivalent items as \( \text{closure}(s_0) \)
   b. Repeatedly compute, for each \( s_k \), and each \( X \), \( \text{goto}(s_k, X) \)
      - If the set is not already in the collection, add it
      - Record all the transitions created by \( \text{goto}(\) )
      This eventually reaches a fixed point

2. Fill in the table from the Canonical Collection of Sets of LR(1) items

The sets in the canonical collection form the states of the Control DFA.
The construction traces the DFA’s transitions

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LR(1) Table Construction

High-level overview

1 Build the Canonical Collection of Sets of LR(1) Items, \( I \)
   a Begin in an appropriate state, \( s_0 \)
      ◆ \( [S’ \rightarrow \cdot S, EOF] \), along with any equivalent items
      ◆ Derive equivalent items as \( closure(s_0) \)
   b Repeatedly compute, for each \( s_k \), and each \( X \), \( goto(s_k, X) \)
      ◆ If the set is not already in the collection, add it
      ◆ Record all the transitions created by \( goto() \)

This eventually reaches a fixed point

2 Fill in the table from the Canonical Collection of Sets of LR(1) items

Let’s build the tables for the left-recursive SheepNoise grammar \( \text{(S’ is Goal)} \)

<table>
<thead>
<tr>
<th></th>
<th>Goal</th>
<th>SheepNoise</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>SheepNoise</td>
<td>SheepNoise baa</td>
</tr>
<tr>
<td>2</td>
<td>SheepNoise</td>
<td>baa</td>
</tr>
</tbody>
</table>

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Computing Closures

\( \text{Closure}(s) \) adds all the \textit{possibilities} for the items already in \( s \)

- Any item \([A \rightarrow \beta \cdot B \delta, a]\) where \( B \in NT \) implies \([B \rightarrow \cdot \tau, x]\) for each production that has \( B \) on the \textit{lhs}, and each \( x \in \text{FIRST}(\delta a) \)
- Since \( \beta B \delta \) is valid, any way to derive \( \beta B \delta \) is valid, too

The Algorithm

\begin{verbatim}
\textbf{Closure}( s )
while ( s is still changing )
    \forall \text{items } [A \rightarrow \beta \cdot B \delta, a] \in s
        \text{lookahead} \leftarrow \text{FIRST}(\delta a) // \delta \text{ might be } \varepsilon
    \forall \text{productions } B \rightarrow \tau \in P
        \forall \text{ } b \in \text{lookahead}
            \text{if } [B \rightarrow \cdot \tau, b] \not\in s
                then \text{s} \leftarrow \text{s} \cup \{ [B \rightarrow \cdot \tau, b] \}
\end{verbatim}

- Classic fixed-point method
- Halts because \( s \subset I \), the set of all items (finite)
- Worklist version is faster
- \textbf{Closure} “fills out” a state \( s \)

Generate new lookaheads. See note on p. 128
Computing Closures

Generating Closures is the place where a human is most likely to make a mistake

• With everything going on in the construction, it is easy to lose track of $\delta a$ and the fact that it refers to the item, not the current production

$\textbf{Closure}(s)$

while ( $s$ is still changing )

$\forall$ items $[A \rightarrow \beta \cdot B \delta, a] \in s$

$\forall$ productions $B \rightarrow \tau \in P$

$\forall b \in \text{FIRST}(\delta a)$ // $\delta$ might be $\varepsilon$

if $[B \rightarrow \cdot \tau, b] \not\in s$

then $s \leftarrow s \cup \{[B \rightarrow \cdot \tau, b]\}$

• The lookahead computation is a great example of why these table constructions should be done by computers, not human beings
Example From SheepNoise

Initial step builds the item $[Goal \rightarrow \bullet \text{SheepNoise}, \text{EOF}]$ and takes its Closure($)$

Closure($ [Goal \rightarrow \bullet \text{SheepNoise}, \text{EOF}]$)

<table>
<thead>
<tr>
<th>Item</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[Goal \rightarrow \bullet \text{SheepNoise}, \text{EOF}]$</td>
<td>Original item</td>
</tr>
<tr>
<td>$[\text{SheepNoise} \rightarrow \bullet \text{SheepNoise} \ baa, \text{EOF}]$</td>
<td>ITER 1, PR 1, $\delta \text{a is EOF}$</td>
</tr>
<tr>
<td>$[\text{SheepNoise} \rightarrow \bullet \ baa, \text{EOF}]$</td>
<td>ITER 1, PR 2, $\delta \text{a is EOF}$</td>
</tr>
<tr>
<td>$[\text{SheepNoise} \rightarrow \bullet \text{SheepNoise} \ baa, baa]$</td>
<td>ITER 2, PR 1, $\delta \text{a is baa EOF}$</td>
</tr>
<tr>
<td>$[\text{SheepNoise} \rightarrow \bullet \ baa, baa]$</td>
<td>ITER 2, PR 2, $\delta \text{a is baa EOF}$</td>
</tr>
</tbody>
</table>

So, $S_0$ is

$$\{ [Goal \rightarrow \bullet \text{SheepNoise}, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise} \ baa, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \ baa, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise} \ baa, baa], [\text{SheepNoise} \rightarrow \bullet \ baa, baa] \}$$

<table>
<thead>
<tr>
<th>Symbol</th>
<th>FIRST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal</td>
<td>{ baa }</td>
</tr>
<tr>
<td>SheepNoise</td>
<td>{ baa }</td>
</tr>
<tr>
<td>baa</td>
<td>{ baa }</td>
</tr>
<tr>
<td>EOF</td>
<td>{ EOF }</td>
</tr>
</tbody>
</table>

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Computing Gotos

$Goto(s,x)$ computes the state that the parser would reach if it recognized an $x$ while in state $s$

- $Goto( \{ [A\rightarrow\beta \cdot X\delta,a] \}, X )$ produces $\{ [A\rightarrow\beta X \cdot \delta,a] \}$  
  \textit{(obviously)}
- It finds all such items & uses $Closure()$ to fill out the state

The Algorithm

\begin{verbatim}
Goto( s, X )
    new ← Ø
    \forall items [A→β • Xδ,a] ∈ s
    new ← new ∪ { [A→βX • δ,a] }
    return Closure( new )
\end{verbatim}

- $Goto( )$ models a transition in the automaton
- Straightforward computation
- $Goto( )$ is \underline{not} a fixed-point method (but it calls $Closure( )$)

$Goto$ in this construction is analogous to $Move$ in the subset construction.
Example from SheepNoise

Assume that $S_0$ is

\[
\{ [\text{Goal} \rightarrow \bullet \text{SheepNoise,EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise baa,EOF}],
[\text{SheepNoise} \rightarrow \bullet \text{ baa,EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise baa, baa}],
[\text{SheepNoise} \rightarrow \bullet \text{ baa, baa}] \}
\]

\textbf{Goto}( S_0, \text{ baa } )

- Loop produces

\begin{tabular}{|l|l|}
\hline
\textbf{Item} & \textbf{Source} \\
\hline
[\text{SheepNoise} \rightarrow \text{ baa } \bullet, \text{ EOF}] & \text{Item 3 in } s_0 \\
[\text{SheepNoise} \rightarrow \text{ baa } \bullet, \text{ baa}] & \text{Item 5 in } s_0 \\
\hline
\end{tabular}

- \textbf{Closure} adds nothing since $\bullet$ is at end of rhs in each item

In the construction, this produces $s_2$

\[
\{ [\text{SheepNoise} \rightarrow \text{ baa } \bullet, \{\text{EOF}, \text{ baa}\}] \}
\]

New, but \textit{obvious}, notation for two distinct items

\[
[\text{SheepNoise} \rightarrow \text{ baa } \bullet, \text{ EOF}] \& [\text{SheepNoise} \rightarrow \text{ baa } \bullet, \text{ baa}]
\]
Building the Canonical Collection

Start from $s_0 = \text{Closure}( [S' \rightarrow \bullet S, \text{EOF}] )$

Repeatedly construct new states, until all are found

**The Algorithm**

\[
s_0 \leftarrow \text{Closure} \left( \{ [S' \rightarrow \bullet S, \text{EOF}] \} \right) \\
S \leftarrow \{ s_0 \} \\
k \leftarrow 1
\]

\[\text{while (} S \text{ is still changing) } \]
\[
\forall s_j \in S \text{ and } \forall x \in (T \cup NT) \\
\quad s_k \leftarrow \text{Goto}(s_j, x) \\
\quad \text{record } s_j \rightarrow s_k \text{ on } x \\
\quad \text{if } s_k \notin S \text{ then} \\
\quad \quad S \leftarrow S \cup \{ s_k \} \\
\quad \quad k \leftarrow k + 1
\]

- Fixed-point computation
- Loop adds to $S$ (monotone)
- $S \subseteq 2^{\text{ITEMS}}$, so $S$ is finite
- **Worklist version is faster because it avoids duplicated effort**

This membership / equality test requires careful and/or clever implementation.
Example from SheepNoise

Starts with $S_0$

$S_0 : \{ \[\text{Goal} \rightarrow \bullet \text{SheepNoise}, \text{EOF}\], \[\text{SheepNoise} \rightarrow \bullet \text{SheepNoise \ baa, EOF}\],
\[\text{SheepNoise} \rightarrow \bullet \text{baa, EOF}\], \[\text{SheepNoise} \rightarrow \bullet \text{SheepNoise \ baa, baa}\], \[\text{SheepNoise} \rightarrow \bullet \text{baa, baa} \] \}$

Iteration 1 computes

$S_1 = \text{Goto} (S_0, \text{SheepNoise}) =
\{ \[\text{Goal} \rightarrow \text{SheepNoise } \bullet, \text{EOF}\], \[\text{SheepNoise} \rightarrow \text{SheepNoise } \bullet \text{ baa, EOF}\],
\[\text{SheepNoise} \rightarrow \text{SheepNoise } \bullet \text{ baa, baa} \] \}$

$S_2 = \text{Goto} (S_0, \text{baa}) = \{ \[\text{SheepNoise} \rightarrow \text{baa } \bullet, \text{EOF}\],
\[\text{SheepNoise} \rightarrow \text{baa } \bullet, \text{baa} \] \}$

Iteration 2 computes

$S_3 = \text{Goto} (S_1, \text{baa}) = \{ \[\text{SheepNoise} \rightarrow \text{SheepNoise } \text{ baa } \bullet, \text{EOF}\],
\[\text{SheepNoise} \rightarrow \text{SheepNoise } \bullet, \text{baa} \] \}$

Nothing more to compute, since $\bullet$ is at the end of every item in $S_3$.
Example from SheepNoise

\[ S_0 : \{ [Goal \rightarrow \bullet \text{SheepNoise, EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise baa, EOF}], \]
\[ [\text{SheepNoise} \rightarrow \bullet \text{ baa, EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise baa, baa}], \]
\[ [\text{SheepNoise} \rightarrow \bullet \text{ baa, baa}] \} \]

\[ S_1 = \text{Goto}(S_0, \text{SheepNoise}) = \]
\[ \{ [\text{Goal} \rightarrow \text{SheepNoise } \bullet], [\text{SheepNoise} \rightarrow \text{SheepNoise } \bullet \text{ baa, EOF}], \]
\[ [\text{SheepNoise} \rightarrow \text{SheepNoise } \bullet \text{ baa, baa}] \} \]

\[ S_2 = \text{Goto}(S_0, \text{ baa}) = \{ [\text{SheepNoise} \rightarrow \text{ baa } \bullet, \text{ EOF}], [\text{SheepNoise} \rightarrow \text{ baa } \bullet, \text{ baa}] \} \]

\[ S_3 = \text{Goto}(S_1, \text{ baa}) = \{ [\text{SheepNoise} \rightarrow \text{SheepNoise baa } \bullet, \text{ EOF}], \]
\[ [\text{SheepNoise} \rightarrow \text{SheepNoise baa } \bullet, \text{ baa}] \} \]

\[ \begin{array}{|c|c|c|}
\hline
\text{State} & SN & baa \\
\hline
S_0 & S_1 & S_2 \\
\hline
S_1 & - & S_3 \\
\hline
S_2 & - & - \\
\hline
S_3 & - & - \\
\hline
\end{array} \]

\textit{Goto Relationships}
Filling in the ACTION and GOTO Tables

The Table Construction Algorithm

\[ \forall \text{ set } S_x \in S \]
\[ \forall \text{ item } i \in S_x \]
\[ \text{if } i \text{ is } [A \rightarrow \beta \cdot \delta, b] \text{ and goto}(S_x, a) = S_k, a \in T \]
\[ \text{then ACTION}[x, a] \leftarrow \text{"shift } k\" \]
\[ \text{else if } i \text{ is } [S' \rightarrow S \cdot \text{EOF}] \]
\[ \text{then ACTION}[x, \text{EOF}] \leftarrow \text{"accept"} \]
\[ \text{else if } i \text{ is } [A \rightarrow \beta \cdot a] \]
\[ \text{then ACTION}[x, a] \leftarrow \text{"reduce } A \rightarrow \beta\" \]

\[ \forall n \in NT \]
\[ \text{if } \text{goto}(S_x, n) = S_k \]
\[ \text{then GOTO}[x, n] \leftarrow k \]

Many items generate no table entry

→ Placeholder before a NT does not generate an ACTION table entry
→ Closure( ) instantiates FIRST(X) directly for [A \rightarrow \beta \cdot X \delta, a]
Example from SheepNoise

\[ S_0 : \{ [\text{Goal} \rightarrow \text{SheepNoise}, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \text{baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \text{baa}, \text{baa}] \} \]

\[ S_1 = \text{Goto}(S_0, \text{SheepNoise}) = \{ [\text{Goal} \rightarrow \text{SheepNoise \ baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise \ baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise \ baa, baa}] \} \]

\[ S_2 = \text{Goto}(S_0, \text{baa}) = \{ [\text{SheepNoise} \rightarrow \text{baa \ baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \text{baa \ baa}] \} \]

\[ S_3 = \text{Goto}(S_1, \text{baa}) = \{ [\text{SheepNoise} \rightarrow \text{SheepNoise \ baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise \ baa}, \text{baa}] \} \]

\[ \text{before } T \Rightarrow \text{shift } k \]

so, \text{ACTION}[s_0,\text{baa}] \text{ is “shift } S_2” \text{ (clause 1)}

(items define same entry)
Example from SheepNoise

\[ S_0 : \{ [Goal \rightarrow \bullet \text{SheepNoise}, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise} \text{baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise} \text{baa}, \text{baa}], [\text{SheepNoise} \rightarrow \bullet \text{baa}, \text{baa}] \} \]

\[ S_1 = Goto(S_0, \text{SheepNoise}) = \{ [Goal \rightarrow \text{SheepNoise} \bullet, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise} \bullet \text{baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise} \bullet \text{baa}, \text{baa}] \} \]

\[ S_2 = Goto(S_0, \text{baa}) = \{ [\text{SheepNoise} \rightarrow \text{baa} \bullet, \text{EOF}], [\text{SheepNoise} \rightarrow \text{baa} \bullet, \text{baa}] \} \]

\[ S_3 = Goto(S_1, \text{baa}) = \{ [\text{SheepNoise} \rightarrow \text{SheepNoise} \text{baa} \bullet, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise} \text{baa} \bullet, \text{baa}] \} \]

so, ACTION[S_1, baa] is "shift S_3" (clause 1)
Example from SheepNoise

$S_0 : \{ [\text{Goal} \rightarrow \bullet \text{SheepNoise}, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise \ baa}, \text{EOF}],$

$[\text{SheepNoise} \rightarrow \bullet \text{baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise \ baa, baa}],$

$[\text{SheepNoise} \rightarrow \bullet \text{baa, baa}] \}$

$S_1 = \text{Goto}(S_0, \text{SheepNoise}) =$

$\{ [\text{Goal} \rightarrow \text{SheepNoise} \bullet, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise} \bullet \text{baa}, \text{EOF}],$

$[\text{SheepNoise} \rightarrow \text{SheepNoise} \bullet \text{baa, baa}] \}$

$S_2 = \text{Goto}(S_0, \text{baa}) =$

$\{ [\text{SheepNoise} \rightarrow \text{baa} \bullet, \text{EOF}],$

$[\text{SheepNoise} \rightarrow \text{baa} \bullet, \text{baa}] \}$

$S_3 = \text{Goto}(S_1, \text{baa}) =$

$\{ [\text{SheepNoise} \rightarrow \text{SheepNoise} \text{baa} \bullet, \text{EOF}],$

$[\text{SheepNoise} \rightarrow \text{SheepNoise} \text{baa} \bullet, \text{baa}] \}$

so, $\text{ACTION}[S_1, \text{EOF}]$ is “accept” (clause 2)
Example from SheepNoise

\[ S_0 : \{ [\text{Goal} \rightarrow \bullet \text{SheepNoise}, \text{EOF}], [\text{SheepNoise} ightarrow \bullet \text{SheepNoise baa}, \text{EOF}], \\
[\text{SheepNoise} ightarrow \bullet \text{baa}, \text{EOF}], [\text{SheepNoise} ightarrow \bullet \text{SheepNoise baa}, \text{baa}], \\
[\text{SheepNoise} ightarrow \bullet \text{baa}, \text{baa}] \} \]

\[ S_1 = \text{Goto}(S_0, \text{SheepNoise}) = \\
\{ [\text{Goal} \rightarrow \text{SheepNoise} \bullet, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise} \bullet \text{baa}, \text{EOF}], \\
[\text{SheepNoise} \rightarrow \text{SheepNoise} \bullet \text{baa}, \text{baa}] \} \]

\[ S_2 = \text{Goto}(S_0, \text{baa}) = \{ [\text{SheepNoise} \rightarrow \text{baa} \bullet, \text{EOF}], \\
[\text{SheepNoise} \rightarrow \text{baa} \bullet, \text{baa}] \} \]

\[ S_3 = \text{Goto}(S_1, \text{baa}) = \{ [\text{SheepNoise} \rightarrow \text{SheepNoise baa} \bullet, \text{EOF}], \\
[\text{SheepNoise} \rightarrow \text{SheepNoise baa} \bullet, \text{baa}] \} \]

so, \text{ACTION}[S_2, \text{EOF}] is “reduce 2” (clause 3) (baa, too)

ACTION[S_3, EOF] is “reduce 1” (clause 3) (baa, too)
Building the Goto Table

\[ S_0 : \{ [Goal \rightarrow \bullet \text{SheepNoise}, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise baa}, \text{EOF}], \\
[\text{SheepNoise} \rightarrow \bullet \text{baa}, \text{EOF}], [\text{SheepNoise} \rightarrow \bullet \text{SheepNoise baa, baa}], \\
[\text{SheepNoise} \rightarrow \bullet \text{baa, baa}] \} \]

\[ S_1 = \text{Goto}(S_0, \text{SheepNoise}) = \\
\{ [Goal \rightarrow \text{SheepNoise} \bullet, \text{EOF}], [\text{SheepNoise} \rightarrow \text{SheepNoise} \bullet \text{baa}, \text{EOF}], \\
[\text{SheepNoise} \rightarrow \text{SheepNoise} \bullet \text{baa, baa}] \} \]

\[ S_2 = \text{Goto}(S_0, \text{baa}) = \{ [\text{SheepNoise} \rightarrow \text{baa} \bullet, \text{EOF}], \\
[\text{SheepNoise} \rightarrow \text{baa} \bullet, \text{baa}] \} \]

\[ S_3 = \text{Goto}(S_1, \text{baa}) = \{ [\text{SheepNoise} \rightarrow \text{SheepNoise baa} \bullet, \text{EOF}], \\
[\text{SheepNoise} \rightarrow \text{SheepNoise baa} \bullet, \text{baa}] \} \]

### The Goto table holds just the entries for nonterminal symbols.
*(the baa column went into Action)*

<table>
<thead>
<tr>
<th>State</th>
<th>SN</th>
<th>baa</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_0)</td>
<td>(S_1)</td>
<td>(S_2)</td>
</tr>
<tr>
<td>(S_1)</td>
<td>—</td>
<td>(S_3)</td>
</tr>
<tr>
<td>(S_2)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(S_3)</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

**Goto Relationships**
ACTION & GOTO Tables

Here are the tables for the left-recursive SheepNoise grammar

The tables

<table>
<thead>
<tr>
<th>ACTION TABLE</th>
<th>GOTO TABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>State</td>
<td>EOF</td>
</tr>
<tr>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>accept</td>
</tr>
<tr>
<td>2</td>
<td>reduce 2</td>
</tr>
<tr>
<td>3</td>
<td>reduce 1</td>
</tr>
</tbody>
</table>

The grammar

0 Goal → SheepNoise
1 SheepNoise → SheepNoise baa
2 baa

Remember, this is the left-recursive SheepNoise; EaC2e shows the right-recursive version.
What can go wrong?

What if a set $s$ contains $[A \rightarrow \beta \cdot a \gamma, b]$ and $[B \rightarrow \beta \cdot , a]$?

- First item generates “shift”, second generates “reduce”
- Both define $\text{ACTION}[s,a]$ — cannot do both actions
- This is a fundamental ambiguity, called a **shift/reduce error or conflict**
- Modify the grammar to eliminate it
- Shifting will often resolve it correctly

What if a set $s$ contains $[A \rightarrow \gamma \cdot , a]$ and $[B \rightarrow \gamma \cdot , a]$?

- Each generates “reduce”, but with a different production
- Both define $\text{ACTION}[s,a]$ — cannot do both reductions
- This is a fundamental ambiguity, called a **reduce/reduce error or conflict**
- Modify the grammar to eliminate it

*In either case, the grammar is not LR(1)*
Implementing the Construction
Building the Canonical Collection

Start from \( s_0 = \text{closure}( [S'\rightarrow \cdot S, \text{EOF}] ) \)
Repeatedly construct new states, until all are found

The algorithm

\[
\begin{align*}
\ s_0 & \leftarrow \text{closure}( [S'\rightarrow \cdot S, \text{EOF}] ) \\
S & \leftarrow \{ s_0 \} \\
k & \leftarrow 1 \\
\text{while } (S \text{ is still changing}) & \\
\ & \forall s_j \in S \text{ and } \forall x \in (T \cup NT) \\
\ & s_k \leftarrow \text{goto}(s_j, x) \\
\ & \text{record } s_j \rightarrow s_k \text{ on } x \\
\text{if } s_k \notin S \text{ then } & \\
\ & S \leftarrow S \cup \{ s_k \} \\
\ & k \leftarrow k + 1
\end{align*}
\]

Remember this comment about implementing the equality test at the bottom of the algorithm to build the Canonical Collection of Sets of LR(1) Items?

- Only need to compare core items — the rest will follow
- Represent items as a triple \((R, P, L)\)
  - \(R\) is the rule or production
  - \(P\) is the position of the placeholder
  - \(L\) is the lookahead symbol
- Order items, then
  1. Compare set cardinalities
  2. Compare (in order) by \(R, P, L\)

This membership / equality test requires careful and/or clever implementation.