Parallel Computing Platforms
Network Topologies

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Topics for Today

Interconnection networks

- Taxonomy
- Metrics
- Topologies
- Characteristics
  - cost
  - performance
Interconnection Networks

• Carry data
  — between processors
  — between processors and memory

• Interconnect components
  — switches
  — links (wires, fiber)

• Interconnection network flavors
  — static networks: point-to-point communication links
    – AKA direct networks
  — dynamic networks: switches and communication links
    – AKA indirect networks
Static vs. Dynamic Networks

static/direct network

dynamic/indirect network

Sometimes, the processor and network interface are on the same chip, e.g., Blue Gene
Dynamic Network Switch

• Maps a fixed number of inputs to outputs
• Number of ports on a switch = degree of the switch
• Switch cost
  — grows as the square of switch degree
  — packaging cost grows linearly with the number of pins
• Key property: blocking vs. non-blocking
  — blocking
    – path from p to q may conflict with path from r to s for independent p, q, r, s
  — non-blocking
    – disjoint paths between each pair of independent sources and sinks
Network Interface

Processor node’s link to the interconnect

• Network interface responsibilities
  — packetizing communication data
  — computing routing information
  — buffering incoming/outgoing data

• Network interface connection
  — I/O: e.g., Peripheral Component Interface Express (PCIe)
  — memory: e.g., AMD HyperTransport, Intel QuickPath
    – higher bandwidth and tighter coupling than I/O bus

• Network performance
  — depends on relative speeds of I/O and memory links
Network Topologies

- Many network topologies
- Tradeoff: performance vs. cost
- Machines often implement hybrids of multiple topologies
  — why?
    - packaging
    - cost
    - available components
Metrics for Interconnection Networks

- **Degree**
  - number of links per node

- **Diameter**
  - longest distance between two nodes in the network

- **Bisection width**
  - min # of wire cuts to divide the network in 2 halves

- **Cost:**
  - ~ # links and switches
Network Topologies: Bus

• All processors access a common bus for exchanging data
• Used in simplest and earliest parallel machines
• Advantages
  — distance between any two nodes is $O(1)$
  — provides a convenient broadcast media
• Disadvantages
  — bus bandwidth is a performance bottleneck
Bus-based interconnect with local memory/cache

Since much of the data accessed by processors is local to the processor, cache is critical for the performance of bus-based machines.

Bus Replacement: Direct Connect

Intel Quickpath interconnect
(2009 - present)
Direct Connect: 4 Node Configurations

**4 Node Square**

- **I/O**
- **P0**
- **P1**
- **P2**
- **P3**

**4 Node fully connected**

- **I/O**
- **P0**
- **P1**
- **P2**
- **P3**

**4N SQ (2GT/s HyperTransport)**
- Diam 2 Avg Diam 1.00
- XFIRE BW 14.9GB/s

**4N FC (2GT/s HyperTransport)**
- Diam 1 Avg Diam 0.75
- XFIRE BW 29.9GB/s

**W/ HYPERTRANSPORT3**

**4N FC (4.4GT/s HyperTransport3)**
- Diam 1 Avg Diam 0.75
- XFIRE BW 65.8GB/s

“crossfire” bandwidth is the link-limited all-to-all bandwidth (data only)

Figure Credit: The Opteron CMP NorthBridge Architecture, Now and in the Future, AMD, Pat Conway, Bill Hughes, HOT CHIPS 2006
Direct Connect: 8 Node Configurations

**8N TL (2GT/s HyperTransport)**
- Diam 3 Avg Diam 1.62
- XFire BW 15.2GB/s

**8N 2x4 (4.4GT/s HyperTransport3)**
- Diam 2 Avg Diam 1.12
- XFire BW 72.2GB/s *(5X)*

**8N FC (4.4GT/s HyperTransport3)**
- Diam 1 Avg Diam 0.88
- XFire BW 94.4GB/s *(6X)*

Figure Credit: The Opteron CMP NorthBridge Architecture, Now and in the Future, AMD, Pat Conway, Bill Hughes, HOT CHIPS 2006
A $k \times k$ crossbar network uses a $k \times k$ grid of switches to connect $k$ inputs to $k$ outputs in a non-blocking manner.

B. Dally. From Hypercubes to Dragonflies. IAA Workshop, 2008.
Crossbars in Practice

• Generally difficult to scale for large values of p

• Examples:
  — Earth Simulator
    - custom 640-way single-stage
      S. Habata et al. Earth Simulator System
      total cable length ~1491 miles
  — crossbar as building block
    - Rice Terascale Cluster (retired in 2008): Myrinet 2000 interconnect
      16-way crossbar switches in 128-way Clos network
Assessing Network Alternatives

• Buses
  — excellent cost scalability
  — poor performance scalability

• Crossbars
  — excellent performance scalability
  — poor cost scalability

• Multistage interconnects
  — compromise between these extremes
Multistage Network

Schematic of processor-to-memory multistage interconnection network

(e.g., BBN Monarch)
Multistage Omega Network

• Organization
  — \(\log p\) stages
  — \(p\) inputs/outputs

• At each stage, input \(i\) is connected to output \(j\) if:

\[
j = \begin{cases} 
2i, & 0 \leq i \leq p/2 - 1 \\
2i + 1 - p, & p/2 \leq i \leq p - 1 
\end{cases}
\]

if \(p = 2^k\) then \(j = \text{left_rotate}(i)\)
Each Omega stage is connected in a perfect shuffle

A perfect shuffle interconnection for eight inputs and outputs
Omega Network Switches

- 2×2 switches connect perfect shuffles
- Each switch operates in two modes:
  - Pass-through
  - Cross-over
Multistage Omega Network

Omega network connecting eight inputs and eight outputs

Cost: \( \frac{p}{2} \times \log p \) switching nodes \( \rightarrow O(p \log p) \)
Omega Network Routing

• Let
  — $s =$ binary representation of the source processor
  — $d =$ binary representation of the destination processor or memory

• The data traverses the link to the first switching node
  
  if the most significant bit of $s$ and $d$ are the same
  route data in pass-through mode by the switch
  else
  use crossover path

• Strip off leftmost bit of $s$ and $d$

• Repeat for each of the $\log p$ switching stages
Example: \( s = 001 \rightarrow d = 100 \)

- **Stage 1**: leftmost bit \( s \neq d \) → **crossover**
- **Stage 2**: middle bit \( s = d \) → **pass-through**
- **Stage 3**: rightmost bit \( s \neq d \) → **crossover**
One of the messages (010 to 111 or 110 to 100) blocks at link AB
Butterfly and Flattened Butterfly

- Start with conventional butterfly k-ary n-fly
- Flatten routers in each row of the network into single router
- Flattened butterfly has better performance and path diversity

Clos Network

- Multistage non-blocking network with odd number of stages
  — uses fewer switches than a complete crossbar
- Equivalent to two back-to-back butterfly networks
  — last stage of input network fused w/ first stage of output network
- Input network
  — routes from any input to any middle stage switch
- Output network
  — routes from any middle stage switch to any output

Figure credit: http://en.wikipedia.org/wiki/File:Benesnetwork.png
Clos Network

- **Advantages**
  - provides many paths between each pair of nodes
  - path diversity enables Clos to route arbitrary traffic patterns without a loss of throughput

- **Disadvantages**
  - cost that is nearly double of a butterfly with equal capacity
  - greater latency than a butterfly
  - why higher cost and latency?
    - need to route packets to arbitrary middle stage & then destination
    - double number of long cables = double cost
    - doubles number of inter-router channels traversed: doubles latency

Figure credit: http://en.wikipedia.org/wiki/File:Benesnetwork.png
Clos Network Based on Crossbars

- Multistage non-blocking network with odd number of stages
  - uses fewer switches than a complete crossbar

- Input network
  - routes from any input to any middle stage switch

- Output network
  - routes from any middle stage switch to any output

Folded Clos Network

192 hosts, 64 16-way crossbar switches

8 hosts attach to each 16-port switch node

Linear Array

- Each node has two neighbors: left & right

- If connection between nodes at ends: 1D torus (ring)
Meshes and $k$-dimensional Meshes

- **Mesh**: generalization of linear array to 2D
  - nodes have 4 neighbors: north, south, east, and west.

- **$k$-dimensional mesh**
  - node have $2^k$ neighbors

---

2D mesh  
2D torus  
3D mesh
Constructing hypercubes from hypercubes of lower dimension
Hypercube Properties

- Distance between any two nodes is at most $\log p$.
- Each node has $\log p$ neighbors
- Distance between two nodes =
  
  # of bit positions that differ between node numbers
Comparing Network Performance

Uniform Random Traffic

Worst Case Traffic

Trees

Examples of complete binary tree networks

static tree network

dynamic tree network

processor node

switching node
Tree Properties

- Distance between any two nodes is no more than $2 \log p$
- Trees can be laid out in 2D with no wire crossings
- Problem
  - links closer to root carry more traffic than those at lower levels
- Solution: fat tree
  - widen links as depth gets shallower
    - copes with higher traffic on links near root
Fat Tree Network

Fat tree network for 16 processing nodes

- Can judiciously choose “fatness” of links
- take full advantage of technology and packaging constraints

“We prove that for any given amount of communications hardware, a fat-tree built from that amount of hardware can simulate every other network built from the same amount of hardware, using only slightly more time (a polylogarithmic factor greater). The basic assumption we make of competing networks is the following. In unit time, at most $O(a)$ bits can enter or leave a closed 3D region with surface area $a$."


This paper proves the universality result for off-line simulations only.
A Kautz graph $K_{M}^{N+1}$ is a directed graph of degree $M$ and dimension $N+1$, which has $(M+1)M^{N}$ vertices labeled by all possible strings $s_0 \ldots s_N$ of length $N+1$ which are composed of characters $s_i$ chosen from an alphabet $A$ containing $M+1$ distinct symbols, subject to the condition that adjacent characters in the string cannot be equal (i.e., $s_i \neq s_{i+1}$).

**Properties**

- Smallest diameter for any directed graph with $V$ vertices and degree $M$
- Diameter grows as $\log V$

Credit: https://en.wikipedia.org/wiki/Kautz_graph
SciCortex: A Kautz Graph Interconnect

SiCortex5832: 5832 cores (6-core MIPS), 972 nodes, diameter 6, 2916 links. (2003-2009).

Case Study: SGI Altix UV
SGI Altix UV (2010): Node

Figure credit: http://www.sgi.com/pdfs/4192.pdf
256-socket building block; fat tree (indirect)

Figure credit: http://www.sgi.com/pdfs/4192.pdf
SGI Altix UV - System

16,384 socket (131,072 core); torus (direct)

Figure credit: http://www.sgi.com/pdfs/4192.pdf
Dragonfly
The Trend in Routers

Low radix router
(small number of fat ports)

High radix router
(large number of skinny ports)
High Radix Routers

(a) Baseline design  (b) Fully buffered crossbar  (c) Hierarchical crossbar

B. Dally. From Hypercubes to Dragonflies. IAA Workshop, 2008.
Dragonfly: Three Level Network

- **Levels**
  - **router**
  - **group**
  - **system**

  A router has to connect to:
  - \( p \) terminals
  - \( a - 1 \) routers within the same group
  - \( h \) global channels to other groups

  \[
  \text{radix} = p + a - 1 + h
  \]

Dragonfly Scalability

network scale vs. router radix

Valiant’s Randomized Routing

Avoid hot spots with two step routing

• Message from \( s \rightarrow d \)
  — first sent to a randomly chosen intermediate processor \( i \)
  — then forward from \( i \) to destination \( d \)

• Reduces a worst case permutation route to two randomized routing steps
  — one with randomly picked source nodes
  — a second with randomly picked destination nodes

Global Adaptive Routing

- VAL gives optimal worst-case throughput
- MIN gives optimal benign traffic performance
- UGAL (Universal Globally Adaptive Load-balance)
  - [Singh ’05]
  - Routes benign traffic minimally
  - Starts routing like VAL if load imbalance in channel queues
  - In the worst-case, degenerates into VAL, thus giving optimal worst-case throughput
UGAL

1. $H_m =$ shortest path (SP) length
2. $q_m =$ congestion of the outgoing channel for SP
3. Pick $i$, a random intermediate node
4. $H_{nm} =$ non-min path ($s \rightarrow i \rightarrow d$) length
5. $q_{nm} =$ congestion of the outgoing channel for $s \rightarrow i \rightarrow d$
6. Choose SP if $H_m q_m \leq H_{nm} q_{nm}$; else route via $i$, minimally in each phase
Dragonfly Performance

UGAL-L uses local queue info at the current router node
UGAL-G uses queue info for all global channels in $G_s$, the group containing the source

adversarial traffic: each node in a group sends to randomly selected node in another group

Uniform random traffic

Adversarial traffic

Cost Comparison

Metrics for Interconnection Networks

• **Diameter**: longest distance between two nodes in the network
  — examples
    – linear array: \( p - 1 \)
    – mesh: \( 2(\sqrt{p} - 1) \)
    – tree and hypercube: \( O(\log p) \)
    – completely connected network: \( O(1) \)

• **Bisection Width**: min # of wire cuts to divide the network in 2 halves
  — examples
    – linear array and tree: 1
    – mesh: \( \sqrt{p} \)
    – hypercube: \( p/2 \)
    – completely connected network: \( p^2/4 \)

• **Cost**: \( \sim \) # links or switches (whichever is asymptotically higher)
  — other cost issues
    – ability to layout the network
    – length of wires
Points to Remember

• Indirect networks
  — high-radix routers are better
    – many thin links rather than fewer fat links
    – networks built from high-radix routers have lower latency and cost
  — Clos or flat butterfly have good cost and performance

• Direct networks
  — 3D Torus popular for very large networks
    – good bisection bandwidth - \(2p^{2/3}\)
    – modest number of links - 3p
    – low fixed degree - 6

• Hybrid configurations
  — SGI UV: QPI direct connect, fat tree (indirect), torus (direct)
  — balance latency vs. cost

• Current supercomputers
  — Cray XC30: dragonfly using high radix routers
  — Blue Gene/Q: 5D torus
References

• Adapted from slides “Parallel Programming Platforms” by Ananth Grama

• Based on Chapter 2 of “Introduction to Parallel Computing” by Ananth Grama, Anshul Gupta, George Karypis, and Vipin Kumar. Addison Wesley, 2003

