Principles of Parallel Algorithm Design: Concurrency and Mapping

John Mellor-Crummey

Department of Computer Science
Rice University

johnmc@rice.edu

Last Thursday

- Introduction to parallel algorithms
  - —tasks and decomposition
  - —threads and mapping
  - —threads versus cores
- Decomposition techniques - part 1
  - —recursive decomposition
  - —data decomposition

Owner Computes Rule

- Each datum is assigned to a thread
- Each thread computes values associated with its data
- Implications
  - —input data decomposition
    - all computations using an input datum are performed by its thread
  - —output data decomposition
    - an output is computed by the thread assigned to the output data

Topics for Today

- Decomposition techniques - part 2
  - —exploratory decomposition
  - —hybrid decomposition
- Characteristics of tasks and interactions
- Mapping techniques for load balancing
  - —static mappings
  - —dynamic mappings
- Methods for minimizing interaction overheads

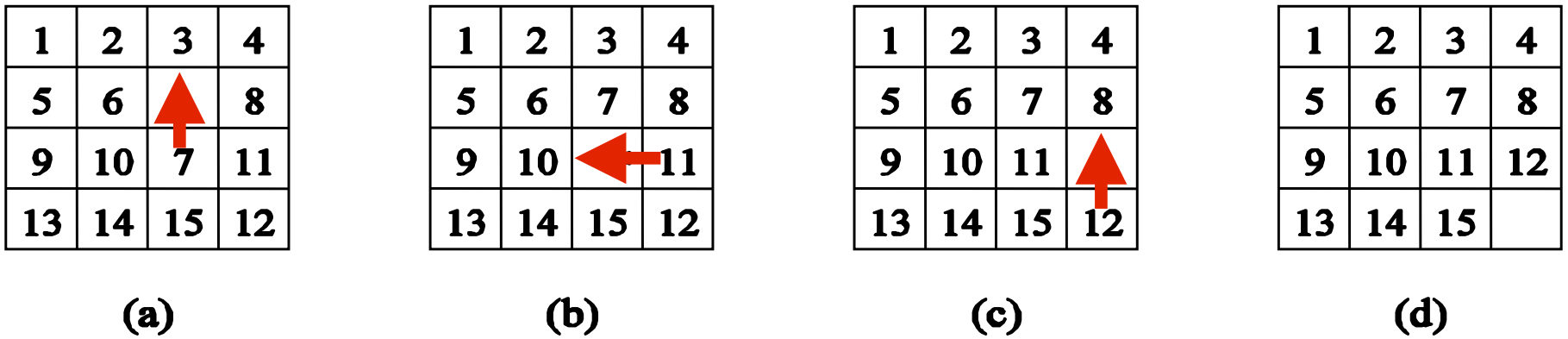
Exploratory Decomposition

- Exploration (search) of a state space of solutions
  - —problem decomposition reflects shape of execution
- Examples
  - —discrete optimization
    - 0/1 integer programming
  - —theorem proving
  - —game playing

Exploratory Decomposition Example

Solving a 15 puzzle

- Sequence of three moves from state (a) to final state (d)



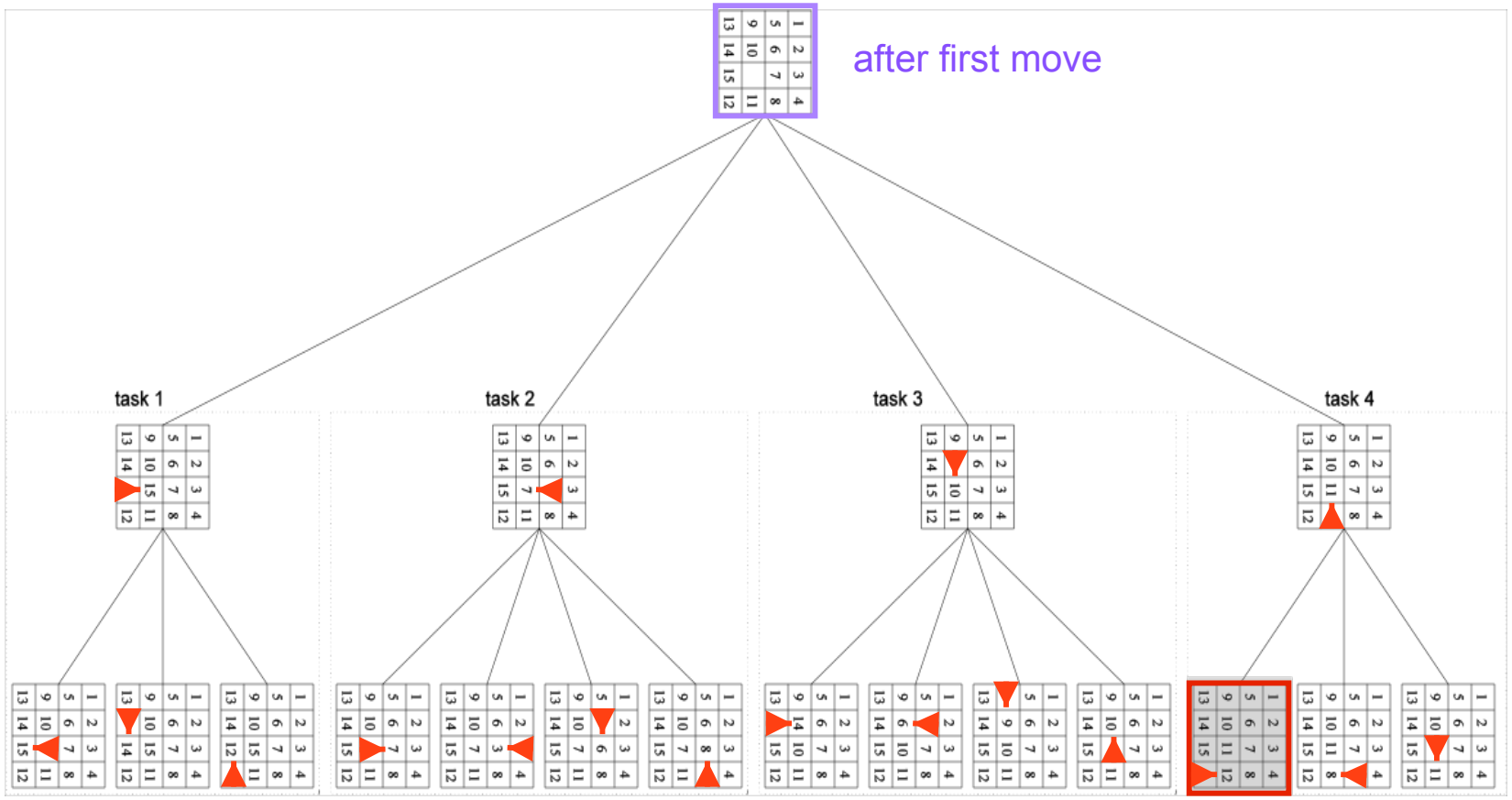
- From an arbitrary state, must search for a solution

Exploratory Decomposition: Example

Solving a 15 puzzle

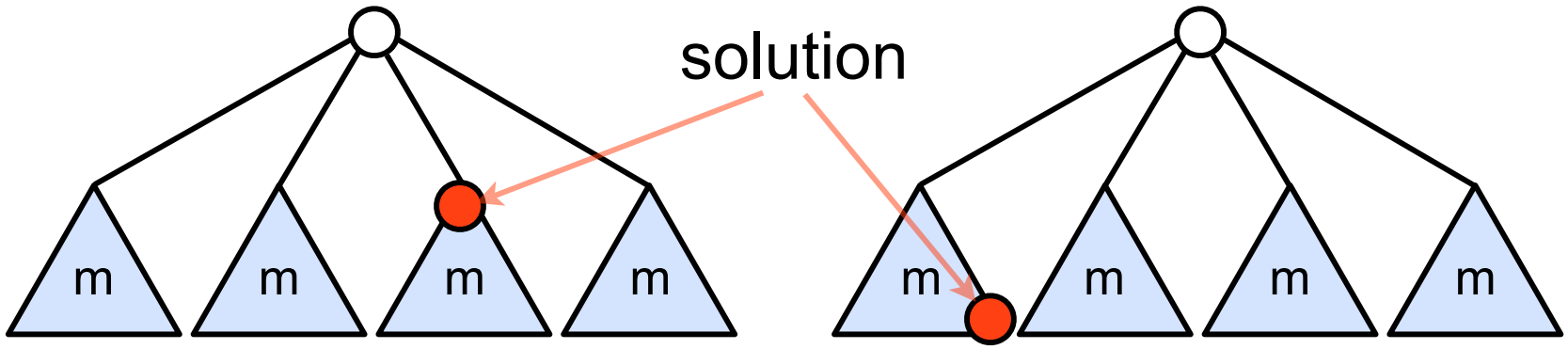
Search

- generate successor states of the current state
- explore each as an independent task



Exploratory Decomposition Speedup

- **Parallel formulation may perform a different amount of work**



total serial work = 2m + 1
total parallel work = 4

total serial work = m
total parallel work = 4m

- **Can cause super- or sub-linear speedup**

Speculative Decomposition

- Dependencies between tasks are not always known a-priori
  - —makes it impossible to identify independent tasks
- Conservative approach
  - —identify independent tasks only when no dependencies left
- Optimistic (speculative) approach
  - —schedule tasks even when they may potentially be erroneous
- Drawbacks for each
  - —conservative approaches
    - may yield little concurrency
  - —optimistic approaches
    - may require a roll-back mechanism if a dependence is encountered

Speculative Decomposition in Practice

Discrete event simulation

- **Data structure: centralized time-ordered event list**
- **Simulation**
  - **extract next event in time order**
  - **process the event**
  - **if required, insert new events into the event list**
- **Optimistic event scheduling**
  - **assume outcomes of all prior events**
  - **speculatively process next event**
  - **if assumption is incorrect, roll back its effects and continue**

Time Warp

David Jefferson. "Virtual Time,"
ACM TOPLAS, 7(3):404-425, July 1985

Speculative Decomposition in Practice

Time Warp OS http://bit.ly/twos-94

- **A new operating system for military simulations**
  - —expensive computational tasks
  - —composed of many interacting subsystems
  - —highly irregular temporal behavior
- **Optimistic execution and process rollback**
  - —don't treat rollback as a special case for handling exceptions, breaking deadlock, aborting transactions, …
  - —use rollback as frequently as other systems use blocking
- **Why a new OS?**
  - —rollback forces a rethinking of all OS issues
    - scheduling, synchronization, message queueing, flow control, memory management, error handling, I/O, and commitment
  - —building Time Warp on top of an OS would require two levels of synchronization, two levels of message queues, …

Optimistic Simulation

The CODES Project

Enabling C

Sam Lang
Chris Caro

atory

The goal of **e systems**
by providing **orage.** We
will develop ads. These
models will b a discrete-
event simulat ulation run
time of mass and using
our new high r nature of
exascale sto s "what if"
exploration o

To enable these capabilities requires a number of innovations across the fronts of modeling, simulation engine design and design of experiments (DOE). On the modeling and simulation front, ROSS.Net enables, (i) optimistic parallel simulation engine (called ROSS which stands for Rensselaer's Optimistic Simulation System) which leverages memory-efficient reversible computation instead of using traditional state-saving to support rollback recovery (ii) systemic memory-efficient methodology for model construction using a combination of library interfaces to key data structures and algorithms, and (iii) measurement.

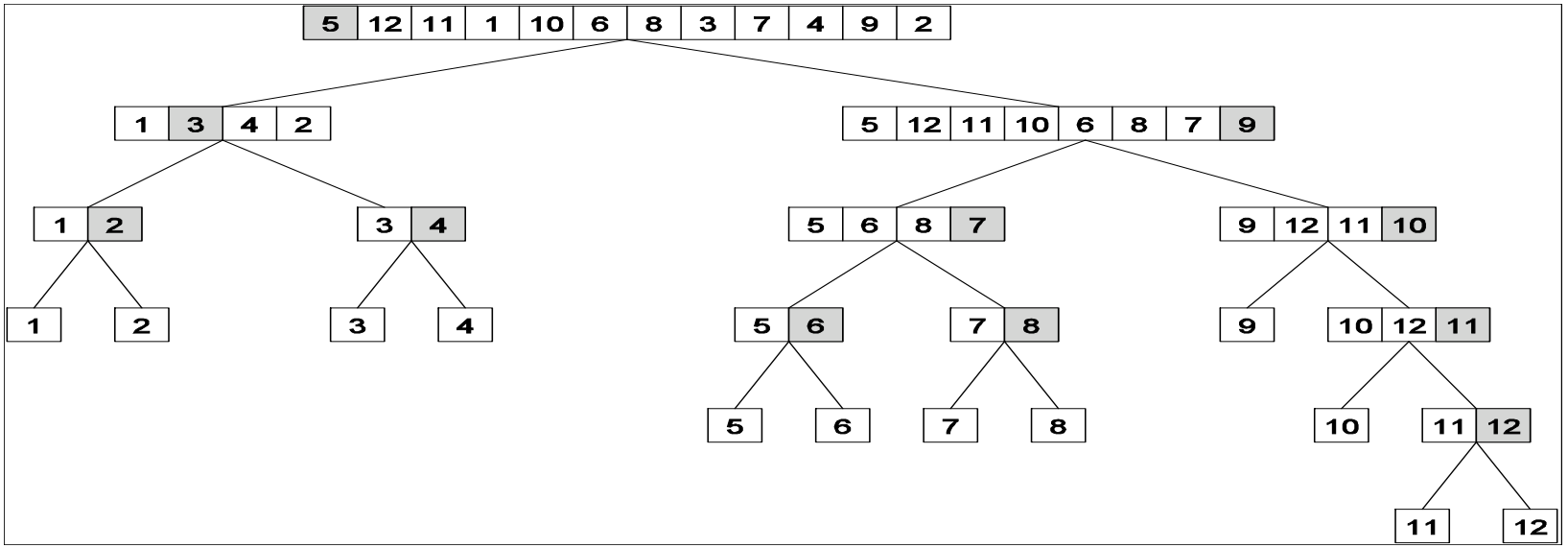
David Bauer et al. "ROSS.NET: Optimistic Simulation Framework For Large-scale Internet Models," *Proc. of the 2003 Winter Simulation Conference*

Hybrid Decomposition

Use multiple decomposition strategies together

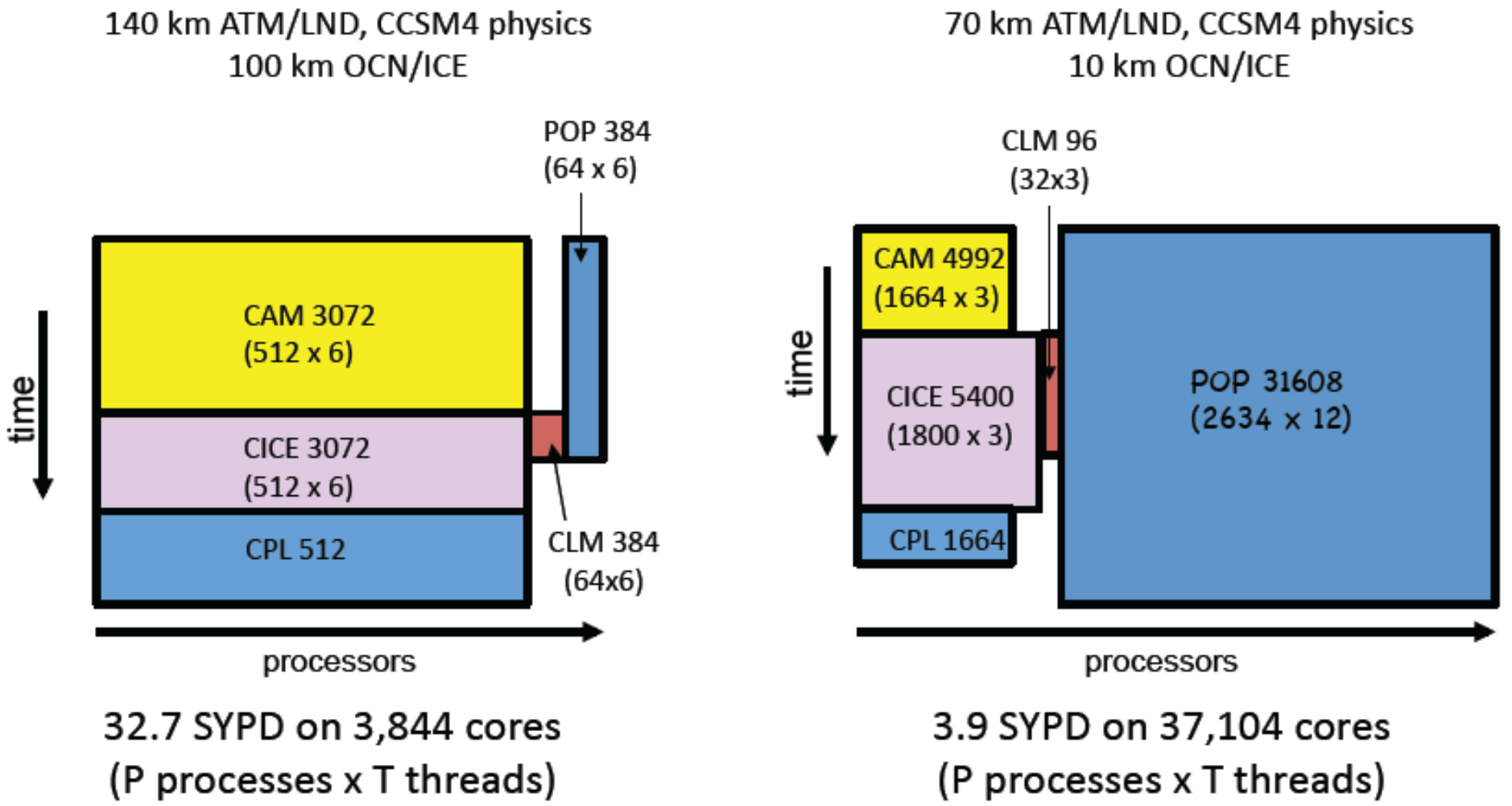
Often necessary for adequate concurrency

- **Quicksort**
  - —recursive decomposition alone limits concurrency



Hybrid Decomposition for Climate Simulation

Data decomposition within atmosphere, ocean, land, and sea-ice tasks



Performance Limiters: Left is CAM; Right is POP.

Figure courtesy of Pat Worley (ORNL)

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- Decomposition techniques - part 2
  - data decomposition
  - exploratory decomposition
  - hybrid decomposition
- Characteristics of tasks and interactions
- Mapping techniques for load balancing
  - static mappings
  - dynamic mappings
- Methods for minimizing interaction overheads
- Parallel algorithm design templates

Characteristics of Tasks

- **Key characteristics**
  - —generation strategy
  - —associated work
  - —associated data size
- **Impact choice and performance of parallel algorithms**

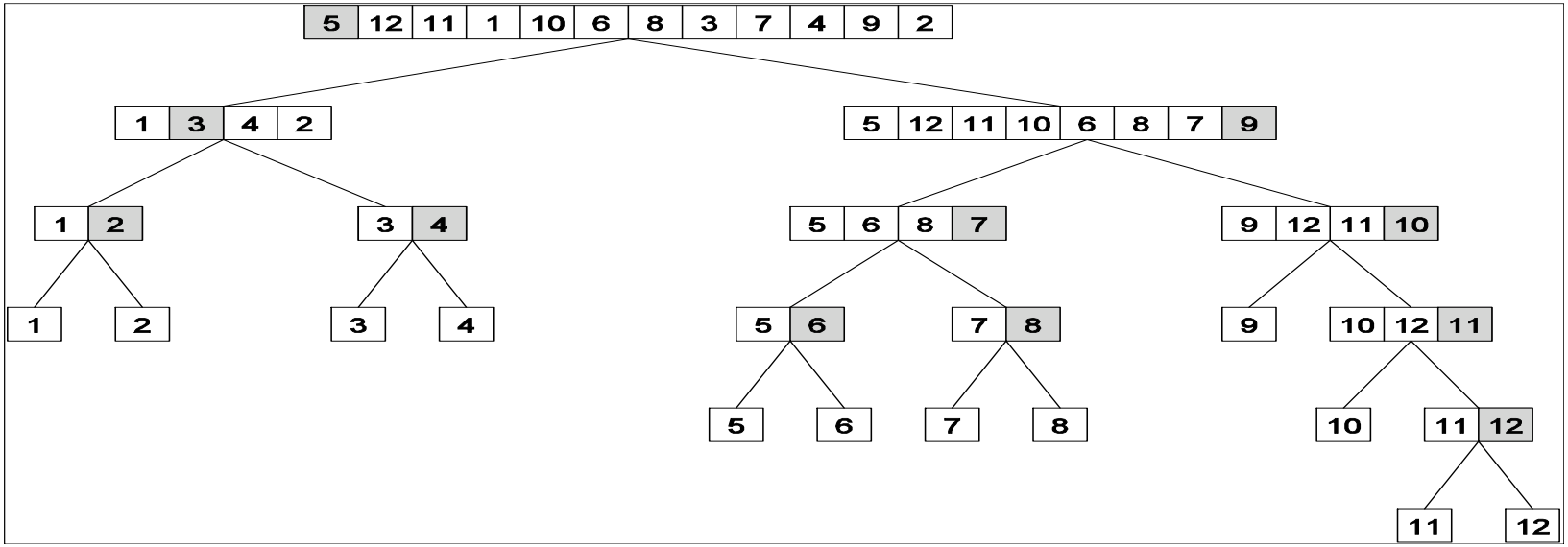
Task Generation

- **Static task generation**
  - —identify concurrent tasks a-priori
  - —typically decompose using data or recursive decomposition
  - —examples
    - matrix operations
    - graph algorithms on static graphs
    - image processing applications
    - other regularly structured problems
- **Dynamic task generation**
  - —identify concurrent tasks as a computation unfolds
  - —typically decompose using exploratory or speculative decompositions
  - —examples
    - puzzle solving
    - game playing

Task Size

- **Uniform: all the same size**
- **Non-uniform**
  - **sometimes sizes known or can be estimated *a-priori***
  - **sometimes not**
    - **example: tasks in quicksort**

      size of each partition depends upon pivot selected



Size of Data Associated with Tasks

- Data may be small or large compared to the computation
  - size(input) < size(computation), *e.g.*, 15 puzzle
  - size(input) = size(computation) > size(output), *e.g.*, min
  - size(input) = size(output) < size(computation), *e.g.*, sort

- Implications
  - small data: task can easily migrate to another thread
  - large data: ties the task to a thread
    - possibly can avoid communicating the task context
      reconstruct/recompute the context elsewhere

Characteristics of Task Interactions

Orthogonal classification criteria

- **Static vs. dynamic**
- **Regular vs. irregular**
- **Read-only vs. read-write**
- **One-sided vs. two-sided**

Characteristics of Task Interactions

- **Static interactions**
  - —tasks and interactions are known a-priori
  - —simpler to code
- **Dynamic interactions**
  - —timing or interacting tasks cannot be determined a-priori
  - —harder to code
    - especially using two-sided message passing APIs

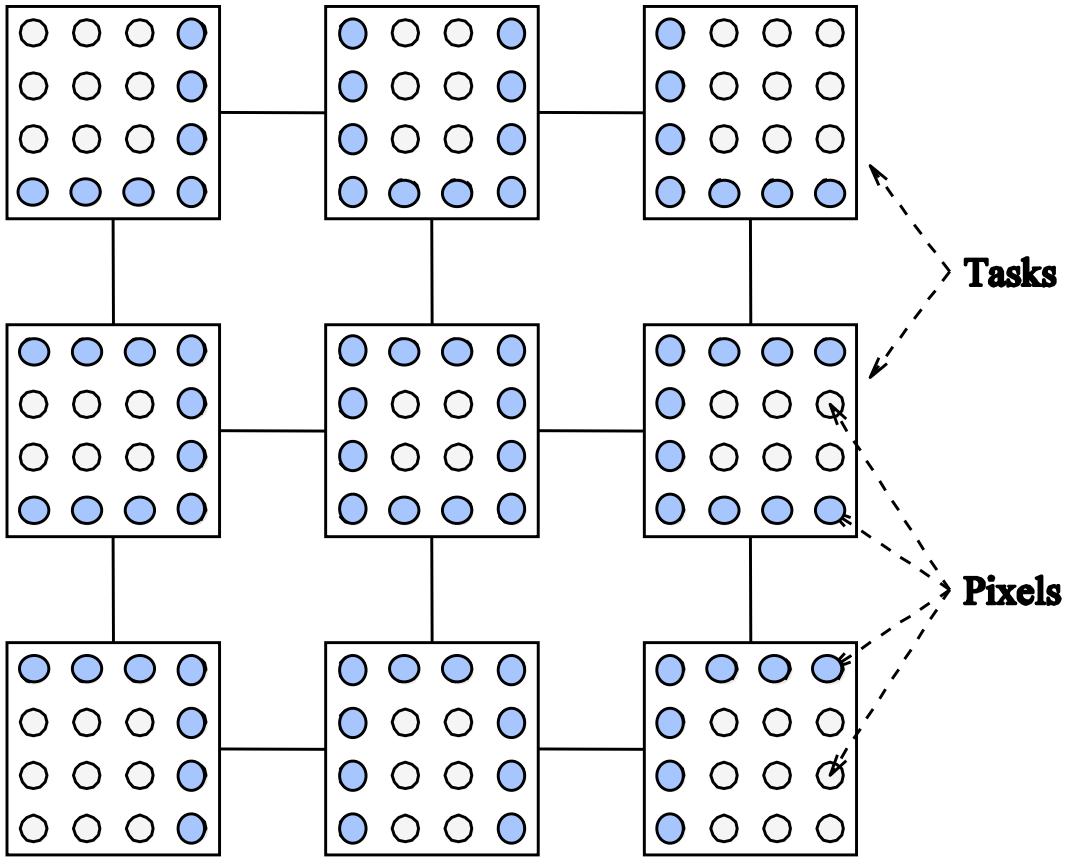
Characteristics of Task Interactions

- **Regular interactions**
  - —interactions have a pattern that can be described with a function
    - e.g. mesh, ring
  - —regular patterns can be exploited for efficient implementation
    - e.g. schedule communication to avoid conflicts on network links
- **Irregular interactions**
  - —lack a well-defined topology
  - —modeled by a graph

Static Regular Task Interaction Pattern

Image operations, e.g., edge detection

Nearest neighbor interactions on a 2D mesh



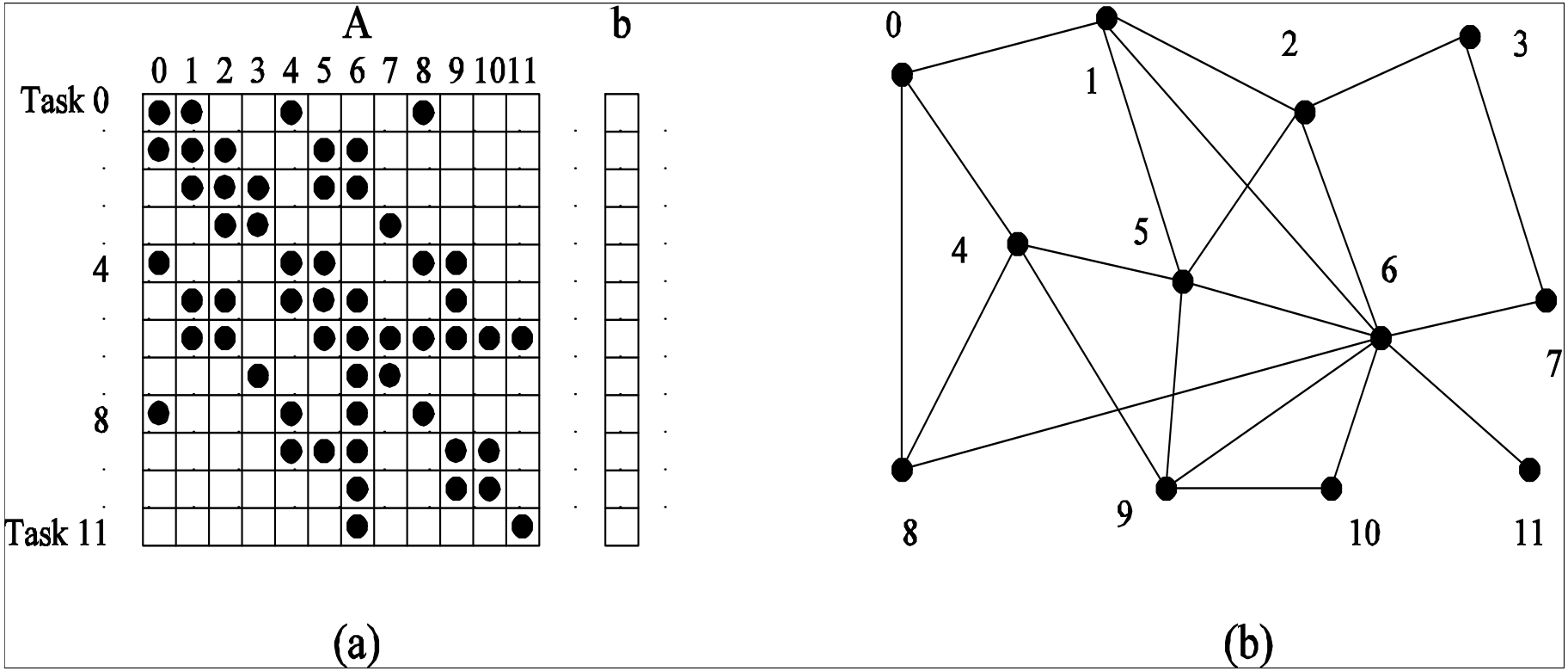
Sobel Edge Detection Stencils

$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

$$\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

Static Irregular Task Interaction Pattern

Sparse matrix-vector multiply



Characteristics of Task Interactions

- **Read-only interactions**
  - —tasks only read data associated with other tasks
- **Read-write interactions**
  - —read and modify data associated with other tasks
  - —harder to code: requires synchronization
    - need to avoid read-write and write-write ordering races

Characteristics of Task Interactions

- One-sided
  - —initiated & completed independently by 1 of 2 interacting tasks
    - READ or WRITE
    - GET or PUT
- Two-sided
  - —both tasks coordinate in an interaction
    - SEND and RECV

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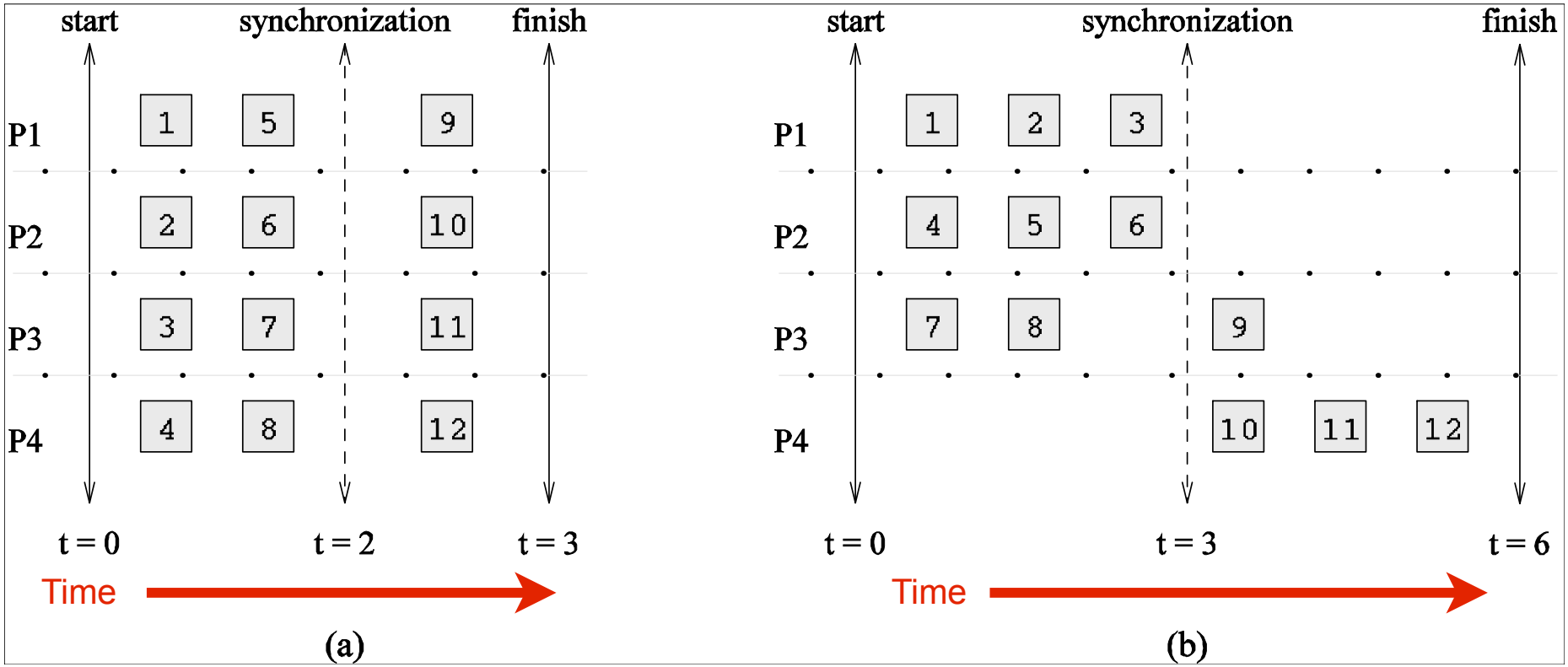
Mapping Techniques

Map concurrent tasks to threads for execution

- **Overheads of mappings**
  - —serialization (idling)
  - —communication
- **Select mapping to minimize overheads**
- **Conflicting objectives: minimizing one increases the other**
  - —assigning all work to one thread
    - minimizes communication
    - significant idling
  - —minimizing serialization introduces communication

Mapping Techniques for Minimum Idling

- **Must simultaneously minimize idling and load balance**
- **Balancing load alone does not minimize idling**



Mapping Techniques for Minimum Idling

Static vs. dynamic mappings

- **Static mapping**
  - —*a-priori* mapping of tasks to threads or processes
  - — requirements
    - – a good estimate of task size
    - – even so, computing an optimal mapping may be NP hard
      - e.g., even decomposition analogous to bin packing
- **Dynamic mapping**
  - — map tasks to threads or processes at runtime
  - — why?
    - – tasks are generated at runtime, or
    - – their sizes are unknown

Factors that influence choice of mapping

- size of data associated with a task
- nature of underlying domain

Schemes for Static Mapping

- Data partitionings
- Task graph partitionings
- Hybrid strategies

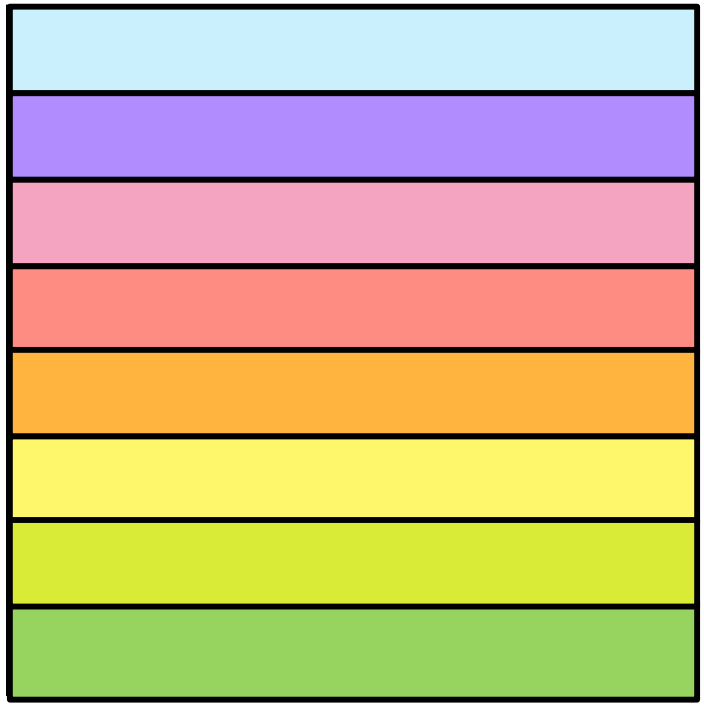
Mappings Based on Data Partitioning

Partition computation using a combination of

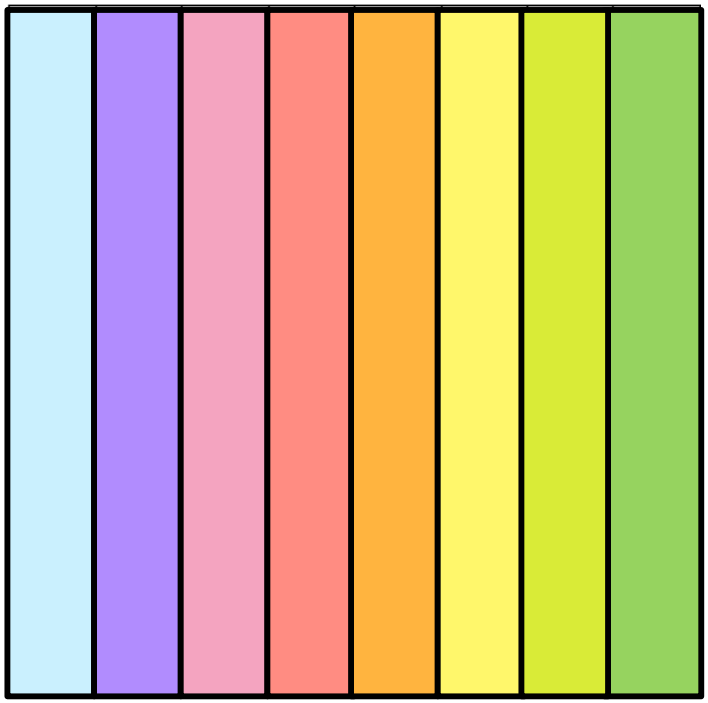
- **—data partitioning**
- **—*owner-computes* rule**

Example: 1-D block distribution for dense matrices

row-wise distribution

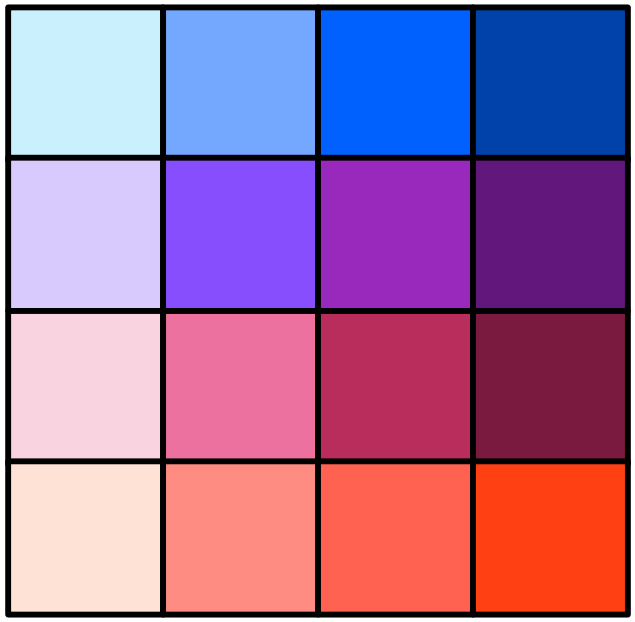


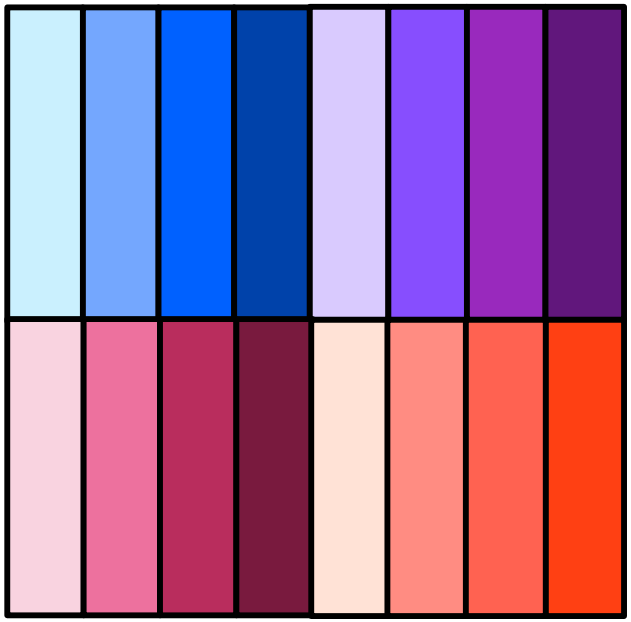
column-wise distribution



Block Array Distribution Schemes

Multi-dimensional block distributions





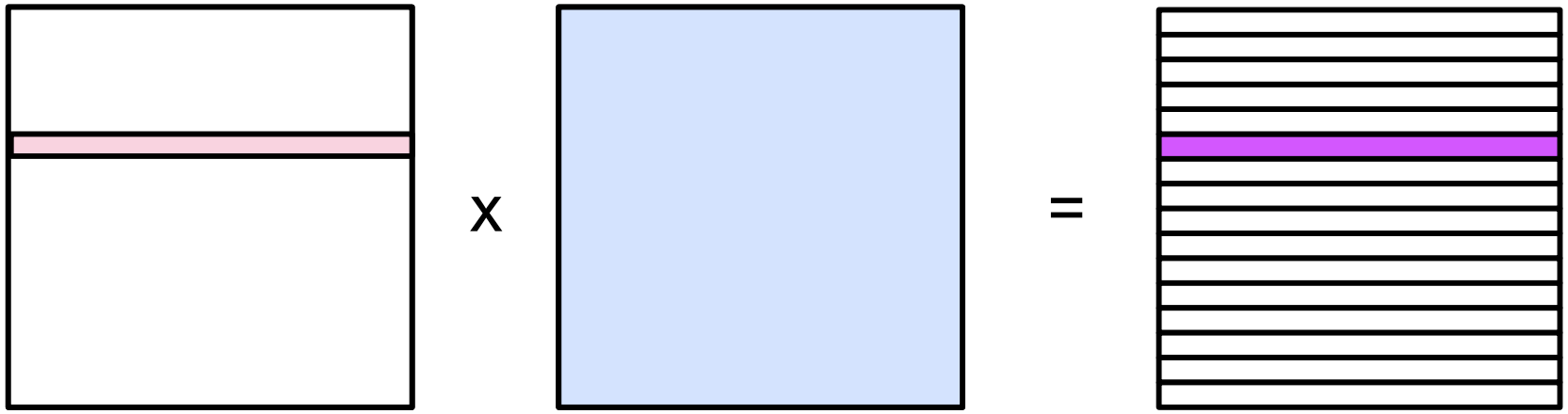
Multi-dimensional partitioning enables larger # of threads

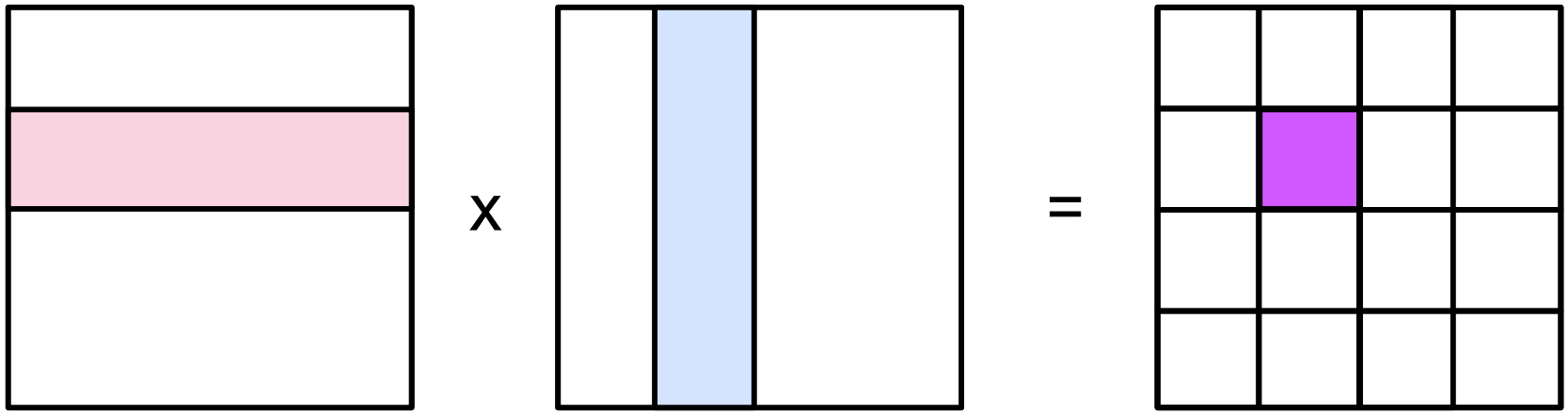
Block Array Distribution Example

Multiplying two dense matrices C = A x B

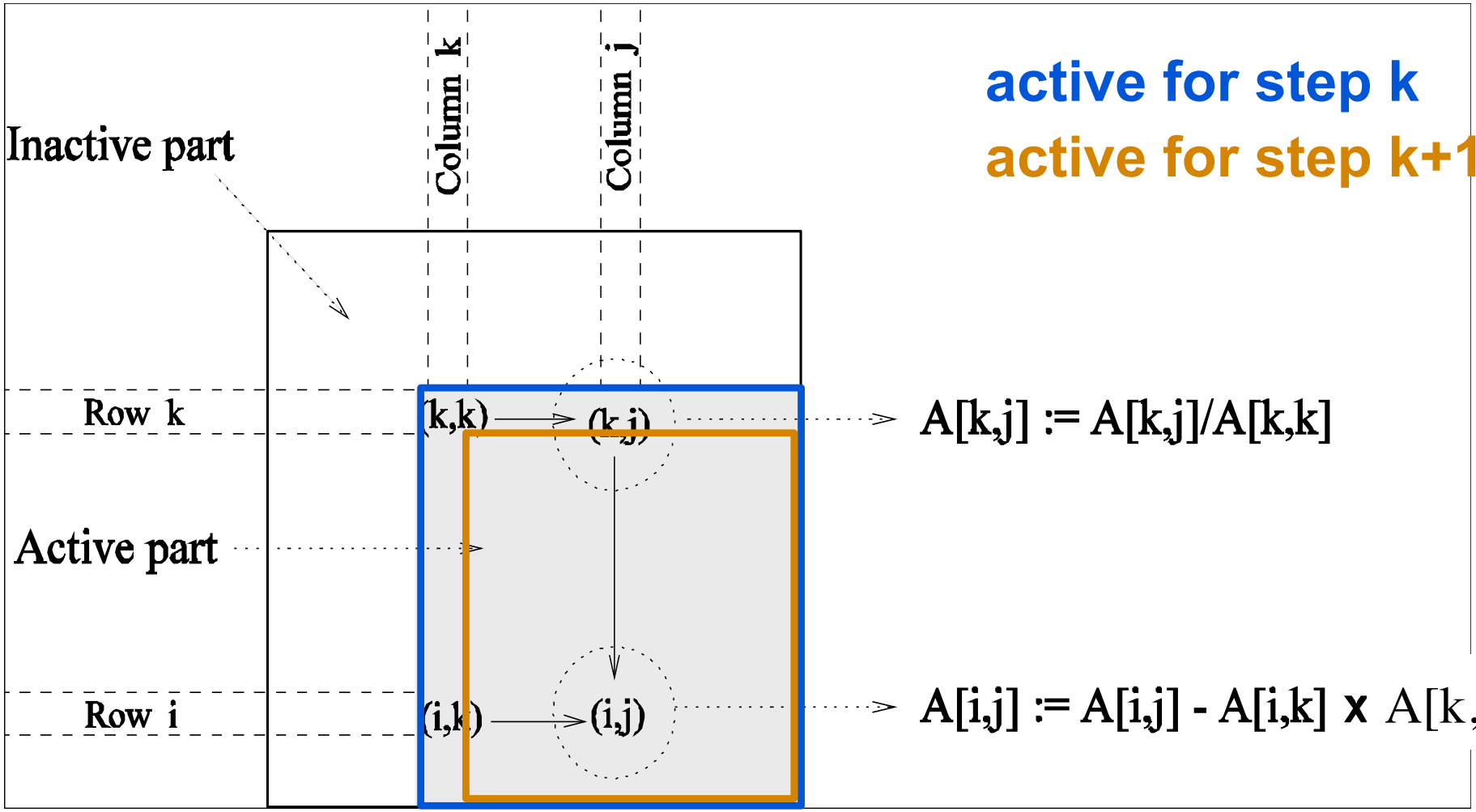
- Partition the output matrix C using a block decomposition
- Give each task the same number of elements of C
  - each element of C corresponds to a dot product
  - even load balance
- Obvious choices: 1D or 2D decomposition
- Select to minimize associated communication overhead

Data Usage in Dense Matrix Multiplication





Consider: Gaussian Elimination



Active submatrix shrinks as elimination progresses

Imbalance and Block Array Distributions

- **Consider a block distribution for Gaussian Elimination**
  - **amount of computation per data item varies**
  - **a block decomposition would lead to significant load imbalance**

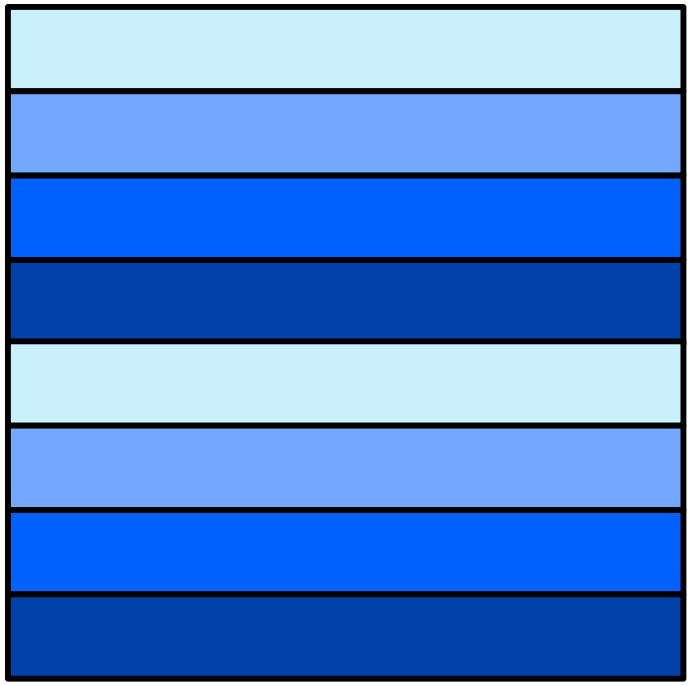
Block Cyclic Distribution

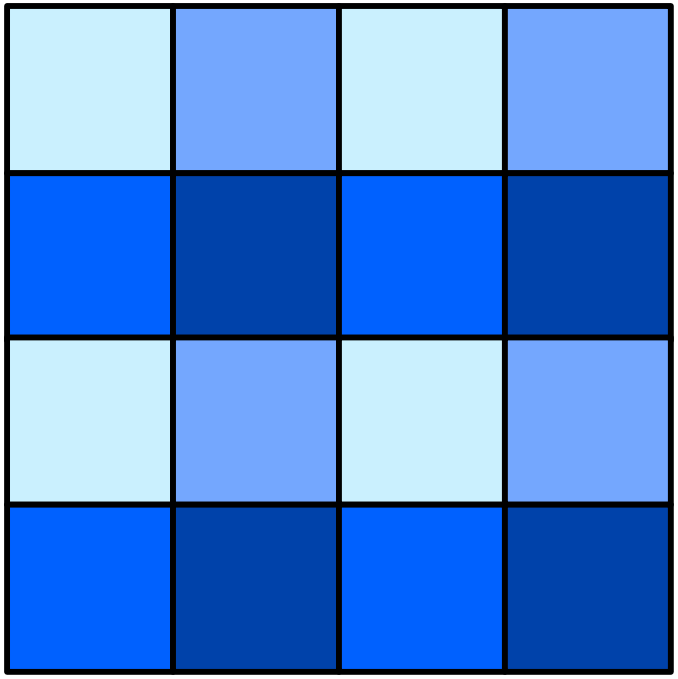
Variant of the block distribution scheme that can be used to alleviate the load-imbalance and idling

Steps

1. **partition an array into many more blocks than the number of available threads or processes**
2. **round-robin assignment of blocks to threads or processes**
   - **each thread or process gets several non-adjacent blocks**

Block-Cyclic Distribution





1D block-cyclic **2D block-cyclic**

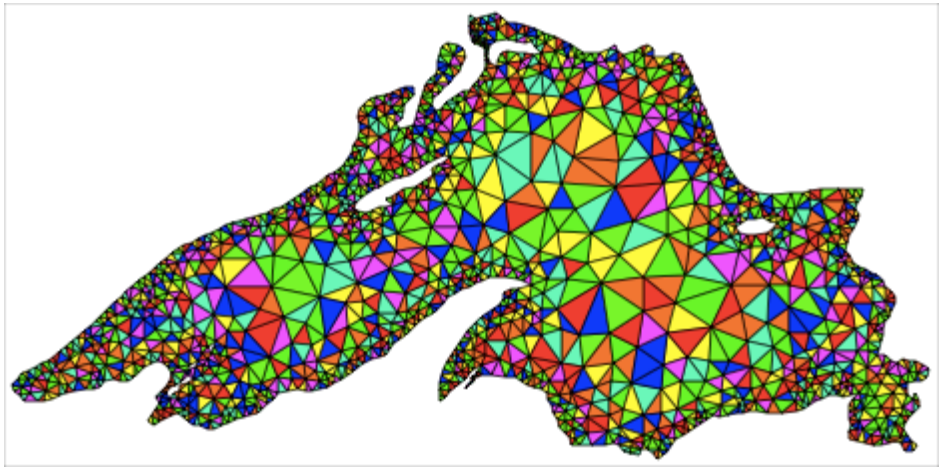
- **Cyclic distribution: special case with block size = 1**
- **Block distribution: special case with block size is *n/p***
  - **—*n* is the dimension of the matrix; *p* is the # of threads**

Decomposition by Graph Partitioning

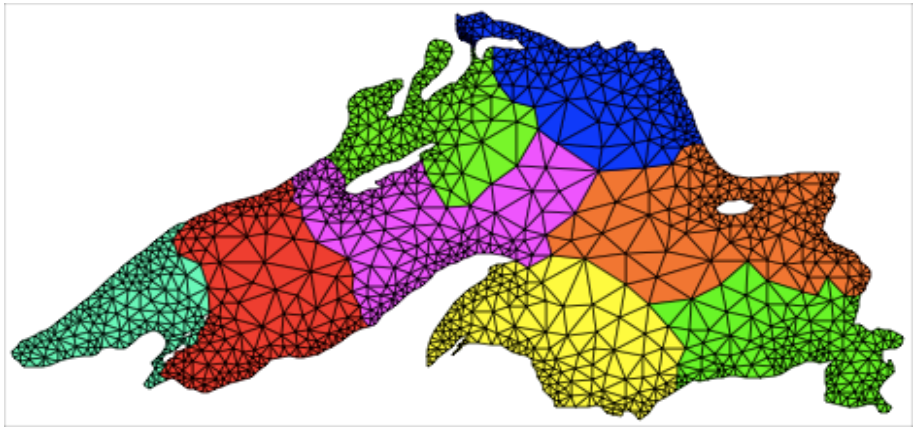
Sparse-matrix vector multiply

- **Graph of the matrix is useful for decomposition**
  - **work ~ number of edges**
  - **communication for a node ~ node degree**
- **Goal: balance work & minimize communication**
- **Partition the graph**
  - **assign equal number of nodes to each thread**
  - **minimize edge count of the graph partition**

Partitioning a Graph of Lake Superior



Random Partitioning



Partitioning for minimum edge-cut

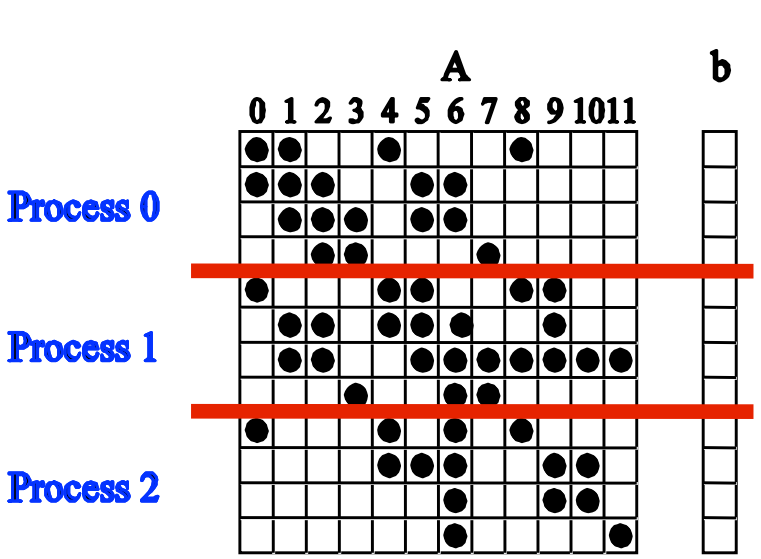
Mappings Based on Task Partitioning

Partitioning a task-dependency graph

- Optimal partitioning for general task-dependency graph
  - NP-hard problem
- Excellent heuristics exist for structured graphs

Mapping a Sparse Matrix

Sparse matrix-vector product



17 items to communicate

C0 = (4,5,6,7,8)

C1 = (0,1,2,3,8,9,10,11)

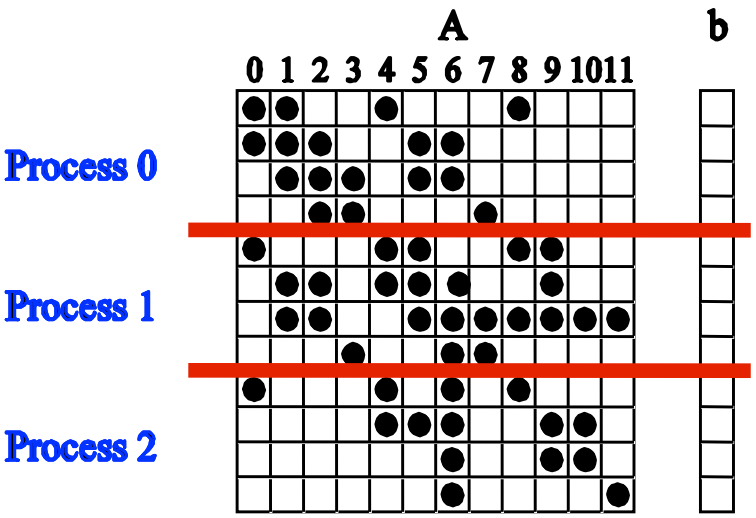
C2 = (0,4,5,6)

sparse matrix structure

mapping
partitioning

Mapping a Sparse Matrix

Sparse matrix-vector product



sparse matrix structure

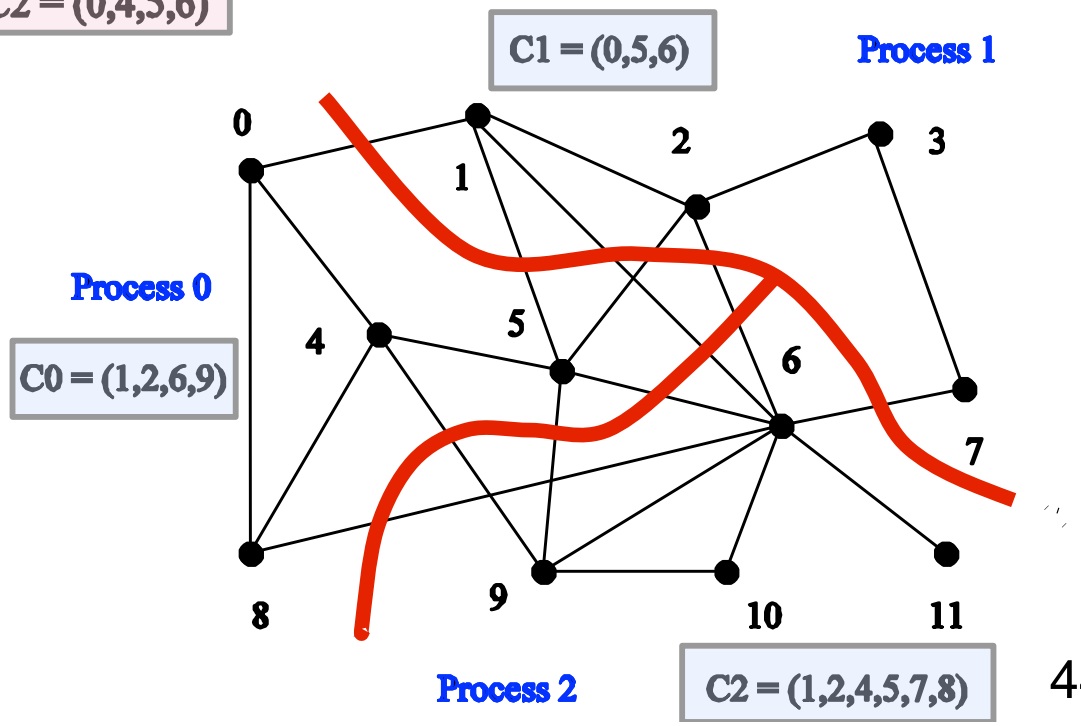
17 items to communicate

C0 = (4,5,6,7,8)

C1 = (0,1,2,3,8,9,10,11)

C2 = (0,4,5,6)

13 items to communicate

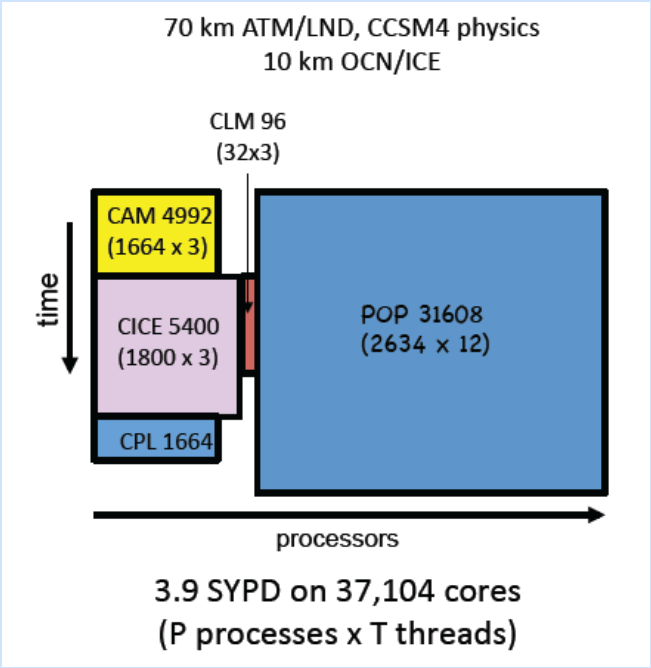


mapping
partitioning

Hierarchical Mappings

- **Sometimes a single-level mapping is inadequate**
- **Hierarchical approach**
  - **use a task mapping at the top level**
  - **data partitioning within each task**

Example:
Hybrid Decomposition
+ Data Partitioning for
Community Earth System Model



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Schemes for Dynamic Mapping

- Dynamic mapping AKA dynamic load balancing
  - —load balancing is the primary motivation for dynamic mapping
- Styles
  - —centralized
  - —distributed

Centralized Dynamic Mapping

- Threads types: masters or slaves
- General strategy
  - —when a slave runs out of work → request more from master
- Challenge
  - —master may become bottleneck for large # of threads
- Approach
  - —chunk scheduling: thread picks up several of tasks at once
  - —however
    - large chunk sizes may cause significant load imbalances
    - gradually decrease chunk size as the computation progresses

Distributed Dynamic Mapping

- All threads as peers
- Each thread can send or receive work from other threads
  - —avoids centralized bottleneck
- Four critical design questions
  - —how are sending and receiving threads paired together?
  - —who initiates work transfer?
  - —how much work is transferred?
  - —when is a transfer triggered?
- Ideal answers can be application specific
- Cilk uses a distributed dynamic mapping: “work stealing”

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Minimizing Interaction Overheads (1)

"Rules of thumb"

- Maximize data locality
  - don't fetch data you already have
  - restructure computation to reuse data promptly
- Minimize volume of data exchange
  - partition interaction graph to minimize edge crossings
- Minimize frequency of communication
  - try to aggregate messages where possible
- Minimize contention and hot-spots
  - use decentralized techniques (avoidance)

Minimizing Interaction Overheads (2)

Techniques

- Overlap communication with computation
  - use non-blocking communication primitives
    - overlap communication with your own computation
    - one-sided: prefetch remote data to hide latency
  - multithread code
    - overlap communication with another thread's computation
- Replicate data or computation to reduce communication
- Use group communication instead of point-to-point primitives
- Issue multiple communications and overlap their latency
  (reduces exposed latency)

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Parallel Algorithm Model

- **Definition: ways of structuring a parallel algorithm**
- **Aspects of a model**
  - **decomposition**
  - **mapping technique**
  - **strategy to minimize interactions**

Common Parallel Algorithm Templates

- **Data parallel**
  - each task performs similar operations on different data
  - typically statically map tasks to threads or processes
- **Task graph**
  - use task dependency graph relationships to promote locality, or reduce interaction costs
- **Master-slave**
  - one or more master threads generate work
  - allocate it to worker threads
  - allocation may be static or dynamic
- **Pipeline / producer-consumer**
  - pass a stream of data through a sequence of workers
  - each performs some operation on it
- **Hybrid**
  - apply multiple models hierarchically, or
  - apply multiple models in sequence to different phases

Topics for Tuesday

- Threaded programming models
- Introduction to Cilk Plus
  - —tasks
  - —algorithmic complexity measures
  - —scheduling
  - —performance and granularity
  - —task parallelism examples

References

- Adapted from slides “Principles of Parallel Algorithm Design” by Ananth Grama
- Based on Chapter 3 of “Introduction to Parallel Computing” by Ananth Grama, Anshul Gupta, George Karypis, and Vipin Kumar. Addison Wesley, 2003