Amdahl's Law

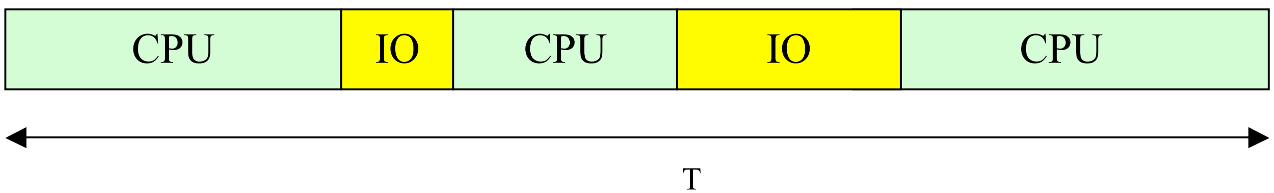
How is system performance altered when some component is changed?

Example 1:

Program execution time is made up of 75% CPU time and 25% I/O time. Which is the better enhancement:

(a) Increasing the CPU speed by 50% or (b) reducing I/O time by half?

Execution model: No overlap between CPU and I/O operations



Program execution time $T = T_{cpu} + T_{io}$

$T_{cpu} / T = 0.75$ and $T_{io} / T = 0.25$

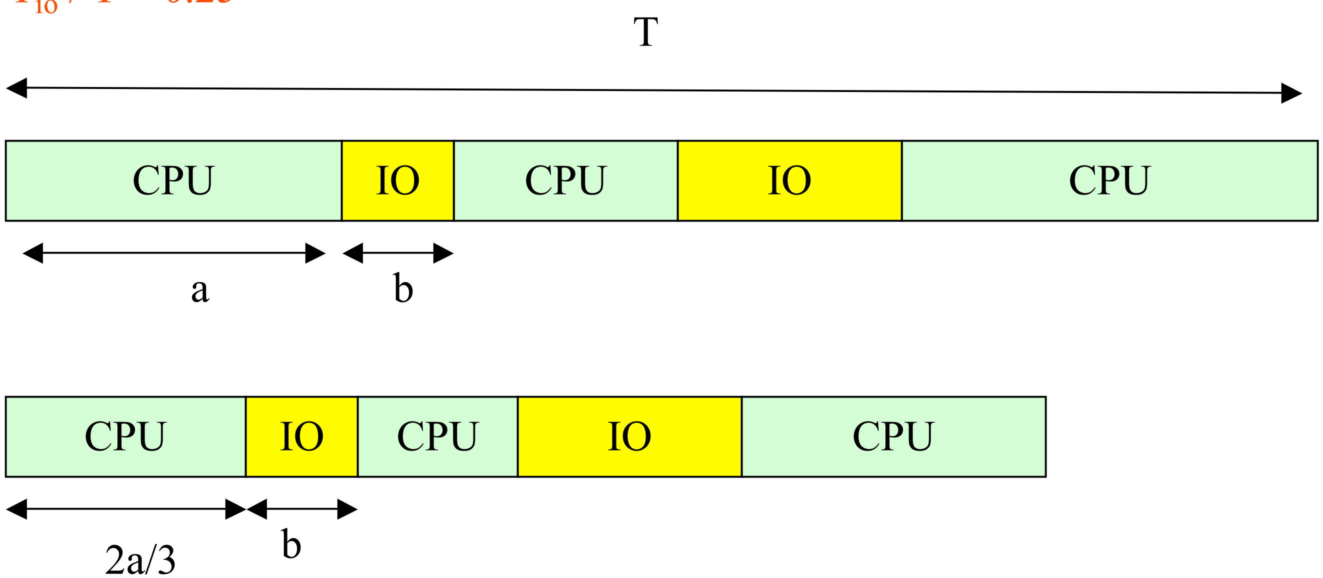
Amdahl's Law

(a) Increasing the CPU speed by 50%

Program execution time $T = T_{cpu} + T_{io}$ $\quad T_{old} = T$

$T_{cpu} / T = 0.75$

$T_{io} / T = 0.25$



Program execution time $T_{new} = T_{cpu} / 1.5 + T_{io}$

$T_{new} = T_{cpu} / 1.5 + T_{io} = 0.75\,T / 1.5 + 0.25T = 0.75T$

For a 50% improvement in CPU speed: Execution time decreases by 25%

Speedup $= T_{old} / T_{new} = T / 0.75T = 1.33$

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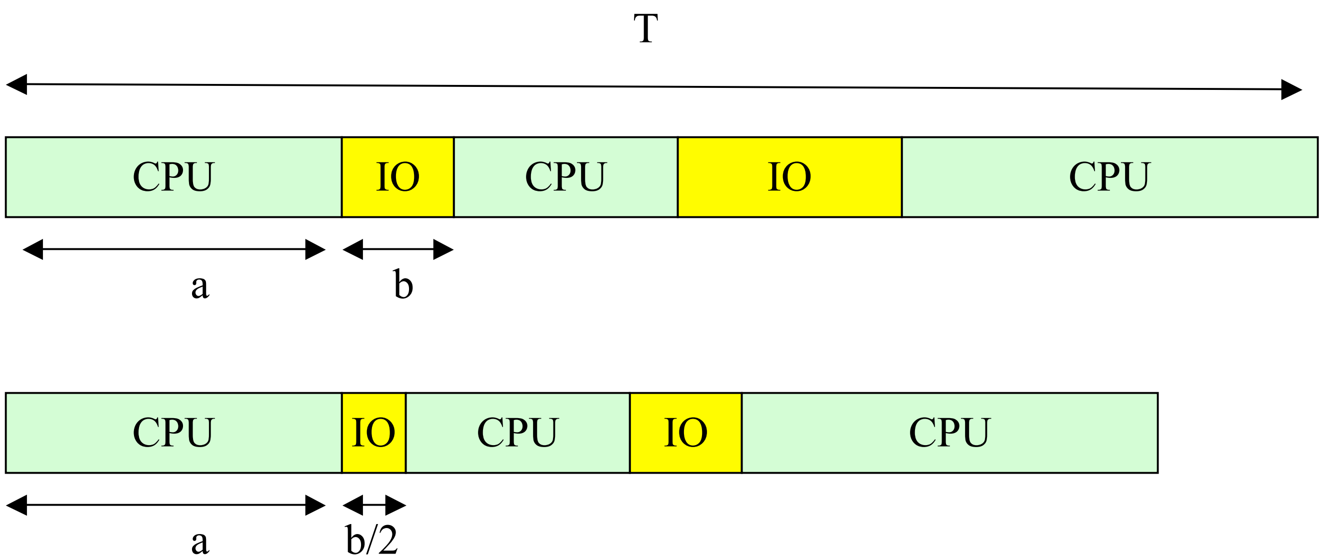
(b) Halve the IO Time

Program execution time $T = T_{cpu} + T_{io}$ $T_{old} = T$

$T_{cpu} / T = 0.75$

$T_{io} / T = 0.25$



Program execution time $T_{new} = T_{cpu} + T_{io} / 2$

$T_{new} = 0.75\,T + 0.25T/2 = 0.875T$

For a 100% improvement in IO speed: Execution time decreases by 12.5%

Speedup $= T_{old} / T_{new} = T / 0.875T = 1.14$

Amdahl's Law

Limiting Cases

- CPU speed improved infinitely so T_{CPU} tends to zero
  $T_{new} = T_{IO} = 0.25T$ Speedup limited to 4

- IO speed improved infinitely so T_{IO} tends to zero
  $T_{new} = T_{CPU} = 0.75T$ Speedup limited to 1.33

Amdahl's Law

Example 2: Parallel Programming (Multicore execution)

A program made up of 10% serial initialization and finalization code. The remainder is a fully parallelizable loop of N iterations.

INITIALIZATION CODE

```
for (j = 0; j < N; j++) {
a[j] = b[j] + c[j];
d[j] = d[j] * c;
}
```

FINALIZATION CODE

$$T = T_{INIT} + T_{LOOP} + T_{FINAL} = T_{SERIAL} + T_{LOOP}$$

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Each iteration can be executed in parallel with the other iterations

Assuming p = 4

a[0]	←	b[0]	+	c[0]
a[1]	←	b[1]	+	c[1]
a[23]	←	b[23]	+	c[23]
a[24]	←	b[24]	+	c[24]
a[25]	←	b[25]	+	c[25]
a[26]	←	b[26]	+	c[26]
a[48]	←	b[48]	+	c[48]
a[49]	←	b[49]	+	c[49]
a[50]	←	b[50]	+	c[50]
a[51]	←	b[51]	+	c[51]
a[73]	←	b[73]	+	c[73]
a[74]	←	b[74]	+	c[74]
a[75]	←	b[75]	+	c[75]
a[76]	←	b[76]	+	c[76]
a[98]	←	b[98]	+	c[98]
a[99]	←	b[99]	+	c[99]

```
for (j = 0; j < 25; j++) {
a[j] = b[j] + c[j];
d[j] = d[j] * c;
}
```

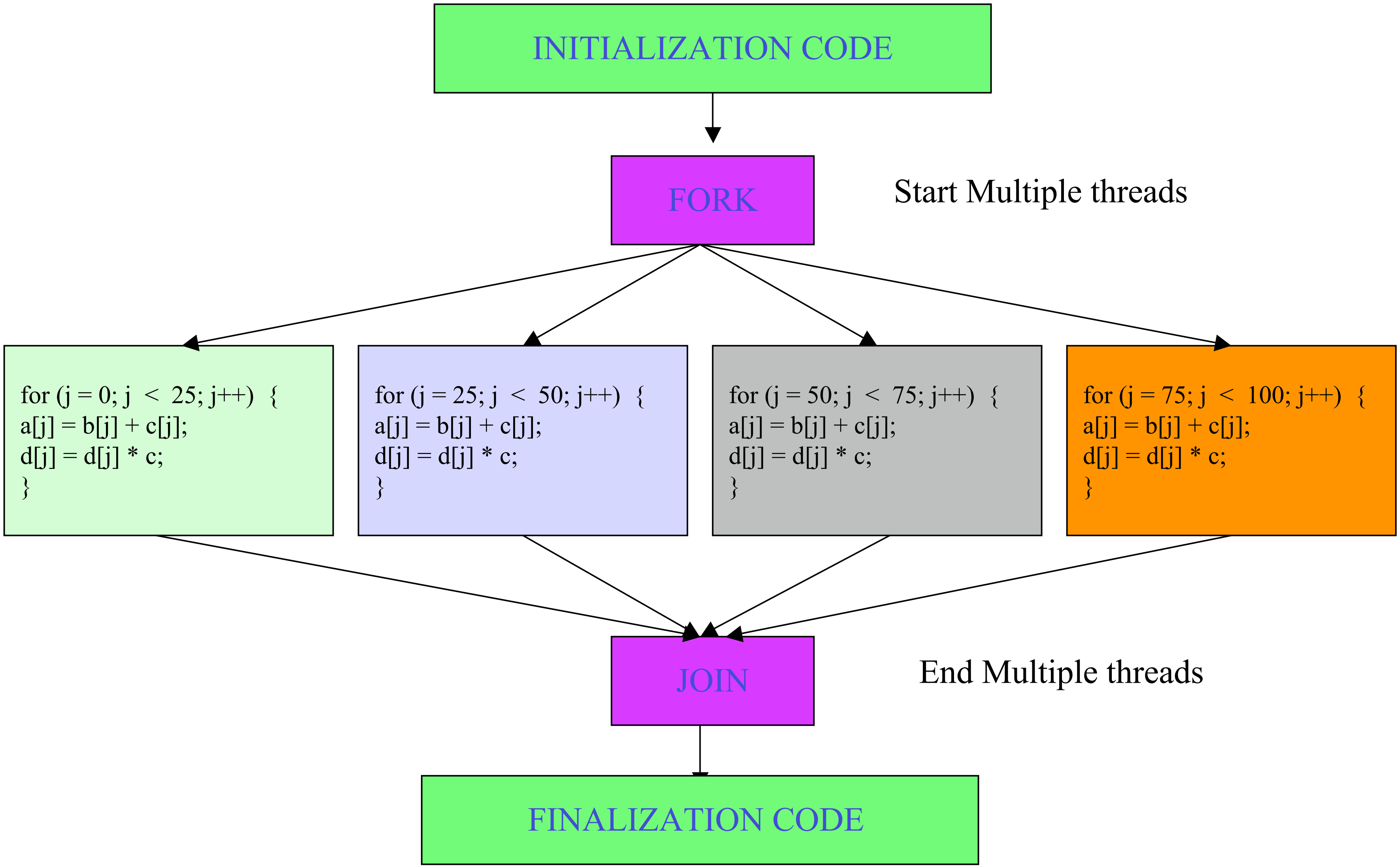
```
for (j = 25; j < 50; j++) {
a[j] = b[j] + c[j];
d[j] = d[j] * c;
}
```

```
for (j = 50; j < 75; j++) {
a[j] = b[j] + c[j];
d[j] = d[j] * c;
}
```

```
for (j = 75; j < 100; j++) {
a[j] = b[j] + c[j];
d[j] = d[j] * c;
}
```

Amdahl's Law

Example 2: Parallel Programming (Multicore execution)



Amdahl's Law

Performance Model

Assume

- System Calls for FORK/JOIN incur zero overhead
- Execution time for parallel loop scales linearly with the number of iterations in the loop
  - With p processors executing the loop in parallel

    Each processor executes N/p iterations

    Parallel time for executing the loop is : T_{LOOP} / p

Sequential time: $T_{SEQ} = T$ $\quad\quad T = T_{SERIAL} + T_{LOOP}$

$T_{SERIAL} = 0.1\,T$ $\quad\quad T_{LOOP} = 0.9T$

Parallel Time with p processors: $T_p = T_{SERIAL} + T_{LOOP} / p$

$= 0.1T + 0.9T/p$

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Performance Model

Parallel Time with p processors: $T_p = T_{SERIAL} + T_{LOOP} / p$

$T_p = 0.1T + 0.9T/p$

$p = 2$: $T_p = 0.1T + 0.9T/p = 0.55\ T$ Speedup $= T/0.55T = 1.8$

$p = 4$: $T_p = 0.1T + 0.9T/p = 0.325\ T$ Speedup $= T/0.325T = 3.0$

$p = 8$: $T_p = 0.1T + 0.9T/p = 0.2125\ T$ Speedup $= T/0.2125T = 4.7$

$p = 16$: $T_p = 0.1T + 0.9T/p = 0.15625\ T$ Speedup $= T/0.15625T = 6.4$

Limiting Case: p so large that T_{LOOP} is negligible (assume 0)

$T_p = 0.1T$ and Maximum Speedup is 10!!

Program with a fraction f of serial (non-parallelizable) code will have a maximum speedup of 1/f

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Diminishing Returns

- Adding more processors leads to successively smaller returns in terms of speedup
- Using 16 processors does not results in an anticipated 16-fold speedup
- The Non-parallelizable sections of code takes a larger percentage of the execution time as the loop time is reduced
- Maximum Speedup is theoretically limited by fraction f of serial code
  - So even 1% serial code implies speedup of 100 at best!

Q: In the light of this pessimistic assessment:

Why is multicore alive and well and even becoming the dominant paradigm?

Amdahl's Law

Why is multicore alive and well and even becoming the dominant paradigm?

1. Throughput Computing: Run large numbers of independent computations (e.g. Web or Database transactions) on different cores

2. Scaling Problem Size:
- Use parallel processing to solve larger problem sizes in a given amount of time
- Different from solving a small problem even faster

In many situations scaling the problem size (N in our example) does not imply a proportionate increase in the serial portion.

, Serial fraction f drops as problem size is increased

Examples:
- Opening a file is a fixed serial overhead independent of problem size
  - The fraction it represents decreases as the problem size is increased

- Parallel IO is routinely available today while it used to be a serialized overhead

- Sophisticated parallel algorithms / compiler techniques are able to parallelize what used to be considered intrinsically serial in the past

Amdahl's Law Summary

- How is system performance altered when some component of the design is changed?

- Performance Gains (Speedup) by enhancing some design feature
  - Base design time: T_{base}
  - Several design components $C_1, C_2 .. C_n$
  - Component C_k takes fraction f_k of the total time
  - Suppose C_k speeded up by factor S; others remain the same
  - Enhanced design time: $T_{enhanced}$

	Base Design	Enhanced Design
– Time for C_k :	T_{base} x f_k	T_{base} x f_k /S
– Time for rest:	T_{base} x$(1 - f_k)$	T_{base} $(1 - f_k)$
– Total Time:	T_{base}	$T_{base} (f_k /S + 1- f_k)$

Speedup = T_{base} / $T_{enhanced}$ = T_{base} / $T_{base} (f_k / S + 1 - f_k)$

=

$1 / ((1 - f_k) + f_k / S)$

- As S becomes large Speedup tends to 1/(1-f) asymptotically