

Direct-Mapped Cache: Write Allocate with Write-Through Protocol

WRITE **data** to address $[x]_{n-m} [w]_m [d]_b$

Block Address $A = [x]_{n-m} [w]_m$

Block size in bytes: $B = 2^b$

Cache size in blocks: $M = 2^m$ (2^{b+m} bytes)

Memory size in blocks = 2^n (2^{b+n} bytes)

Compute cache index $w = A \bmod M$

if (Cache Hit)

1. Write **data** into byte **d** of $\text{cache}[w].\text{DATA}$
2. Store **data** into memory address $[x]_{n-m} [w]_m [d]_b$

if (Cache Miss)

1. Load block at memory block address **A** into $\text{cache}[w].\text{DATA}$
2. Update $\text{cache}[w].\text{TAG}$ to **x** ; $\text{cache}[w].V = \text{TRUE}$
3. Retry cache access

READ from address $[x]_{n-m} [w]_m [d]_b$

Cache Hit: Replace step 1 with Read word from the cache line and omit step 2

Direct-Mapped Cache: Write Allocate and Write Back

Write Allocate and Write-Back Protocol : write **data** to address $[x]_{n-m} [w]_m [d]_b$

Block Address $A = [x]_{n-m} [w]_m$

Compute cache index $w = A \bmod M$

if **Cache Hit**

Write data into byte **d** of block $\text{cache}[w].\text{DATA}$

Set $\text{cache}[w].D$ to **TRUE**

else **/* Cache Miss */**

Stall Processor

 if cache block is **dirty** **/* cache[w].D = TRUE */**

Store $\text{cache}[w].\text{DATA}$ into memory **block** at address $[\text{TAG}][w]$

Load memory **block** at address $[x][w]$

Update $\text{cache}[w].\text{TAG}$ to **x**, $\text{cache}[w].V = \text{TRUE}$ and $\text{cache}[w].D$ to **FALSE**

Retry cache Access

Direct-Mapped Cache: Reads in a Write Back Cache

Write-Back Protocol : read address $[x]_{n-m} [w]_m [d]_b$

If **cache hit** read data field of cache entry

If **cache miss**

replace current block writing it to **memory** if **dirty**

read in new block from **memory** and install in **cache**

Compute cache index $w = A \bmod M$

if **Cache Hit**

Read block $\text{cache}[w].\text{DATA}$; select word d of block

else **/* Cache Miss */**

Stall processor

if **cache block** is **dirty** **/* cache[w].D = TRUE */**

Store $\text{cache}[w].\text{DATA}$ into **memory** at **address** $[\text{TAG}][w]$

Read block at memory address A into $\text{cache}[w].\text{DATA}$

Update $\text{cache}[w].\text{TAG}$ to x , $\text{cache}[w].V$ to **TRUE**, $\text{cache}[w].D$ to **FALSE**

Retry cache access

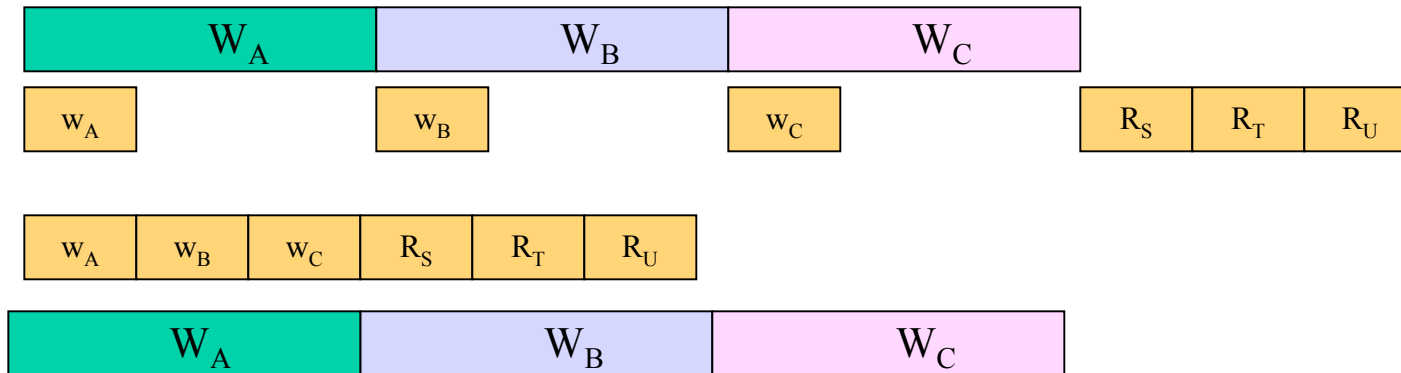
Direct-Mapped Cache: Write Allocate with Write-Through

Write Allocate and Write-Through Protocol: write **data** to address $[x]_{n-m} [w]_m [d]_b$

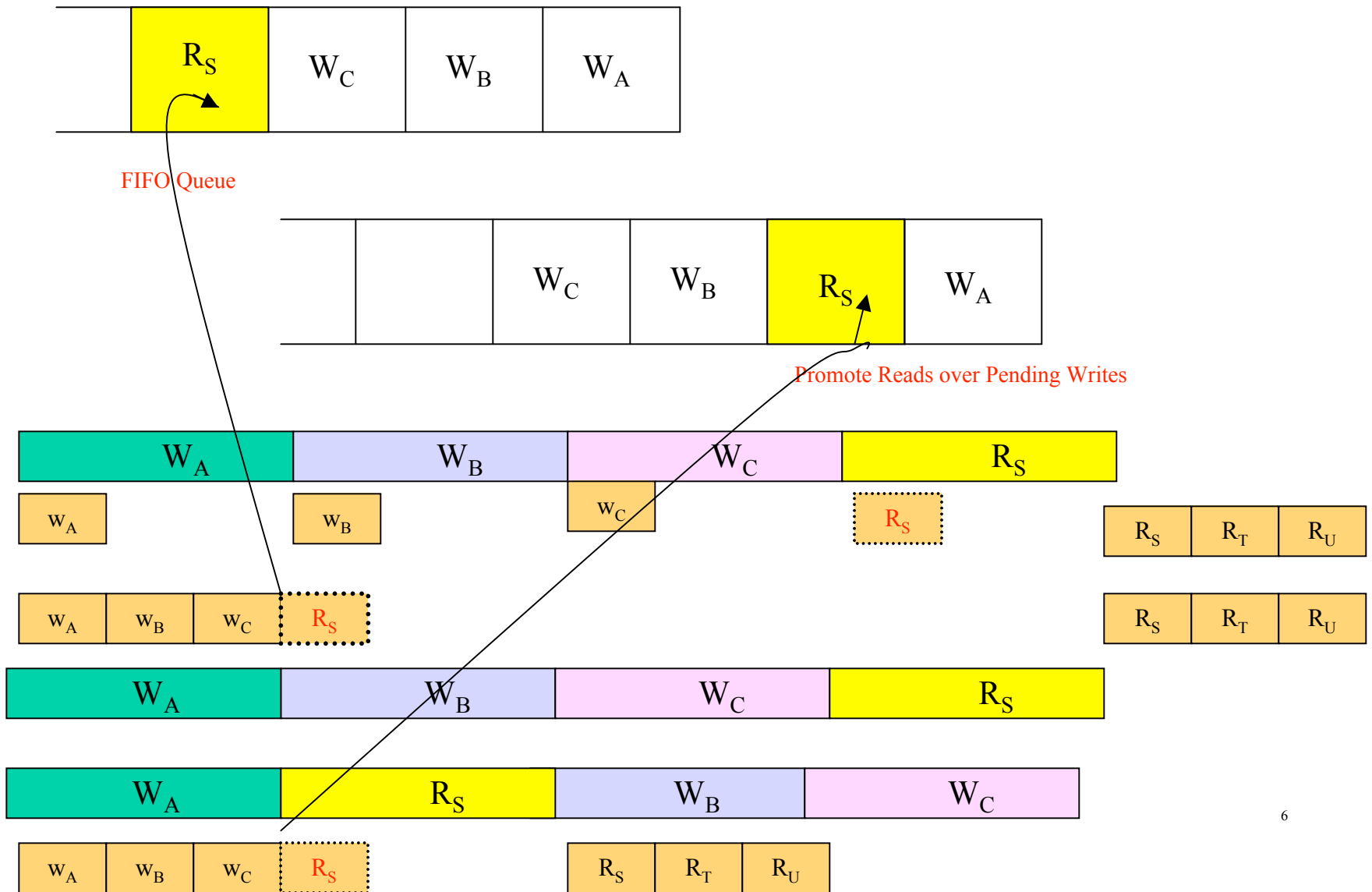
Block Address $A = [x]_{n-m} [w]_m$

- Synchronous Writes
- Writes proceed at the speed of main memory **not** at speed of cache

W_A W_B W_C R_S R_T R_U



Direct-Mapped Cache: Write Allocate with Write-Through



Direct-Mapped Cache: Write Allocate with Write-Through

Write Allocate and Write-Through Protocol: write **data** to address $[x]_{n-m} [w]_m [d]_b$

Block Address $A = [x]_{n-m} [w]_m$

- Writes proceed at the **speed of main memory** **not** at **speed of cache**
- To speed up writes use **asynchronous writes**:
 - Write into **cache** and **simultaneously** into a **write buffer**
 - **Execution** continues **concurrently** with **memory write** from buffer
 - Write buffer should be **deep** enough to buffer **burst of writes**
 - If **write buffer full** on write then **stall** processor till buffer frees up
 - Write buffer served in FCFS order : simple protocol
 - Allow (later) reads to overtake pending writes
 - **Read protocol** modified appropriately
 - On memory read **check** write buffer for a **write in transit**

Writes Summary

1. In a **write allocate** scheme with a **write through** policy:

→ **Write Hit**: Update both cache and main memory ($1W$)

Write Miss: Read in block to cache. Update cache and main memory ($1R + 1W$)

2. In a **write allocate** scheme with a **write back** policy:

→ **Write Hit**: Update cache only

Write Miss: Read in block to cache. Write evicted block if dirty. Update cache. ($1R + 1W$ if dirty block being replaced)

3. In a **no write allocate** scheme with a **write through** policy:

→ **Write Hit**: Update both cache and main memory ($1W$)

Write Miss: Update main memory only ($1W$)

4. In a **no write allocate** scheme with a **write back** policy:

→ **Write Hit**: Update cache only

Write Miss: Update main memory only ($1W$)



Set-Associative Organization

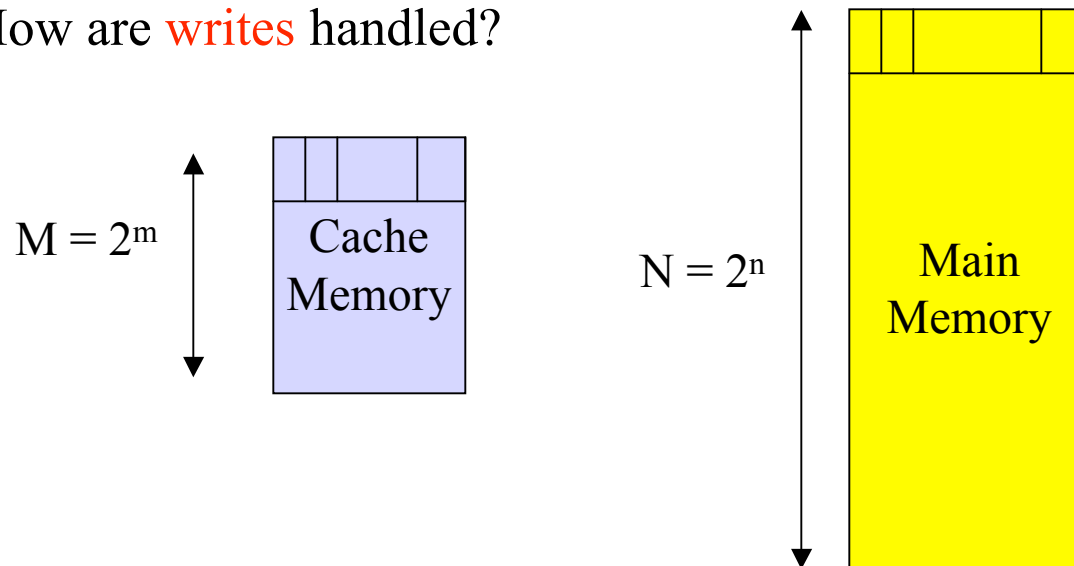
Cache Organization: Main memory address: $n+b$ bits

2^m cache blocks vs 2^n blocks of main memory, $n > m$

Block consists of 2^b consecutive bytes

Four Basic Questions:

1. **Where** in cache do we place a block of main memory?
2. How do we locate (**search**) for a memory reference in the cache?
3. Which block in the cache do we **replace**?
4. How are **writes** handled?



Set-Associative Cache: Motivation

Direct Mapped Cache:

1. **Only one** cache location to store any memory block

Conflict Misses: cache forces eviction even if other cache blocks unused

Improve **miss ratio** by providing **choice** of locations for each memory block

Fully Associative Cache:

1. **Any** cache location to store **any** memory block

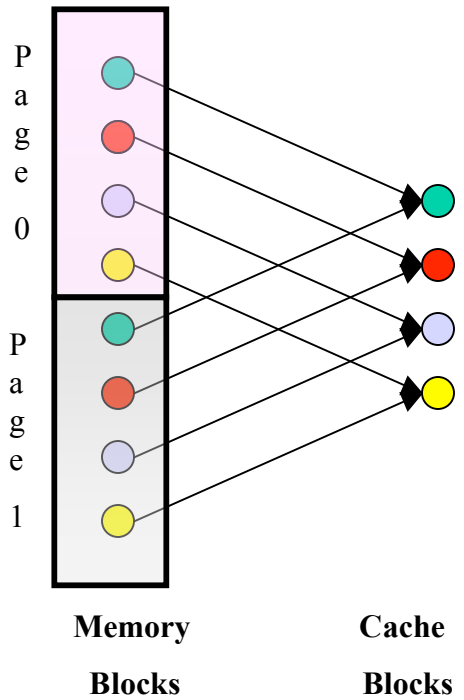
Reduce **Conflict Misses** improving **Miss ratio**

No Conflict Misses in a Fully Associative Cache

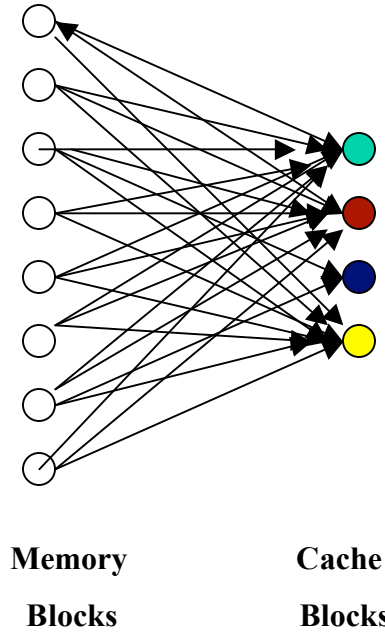
Set Associative Cache

Compromise between **miss rate** and **complexity (power, speed)**

Direct Mapped and Fully Associative Cache Organizations



Direct-Mapped Cache mapping



Fully Associative mapping

- All cache blocks have different colors
- Memory blocks in each page cycle through the same colors in order
- A memory block can be placed only in a cache block of matching color

- A memory block can be placed in any cache block

Set-Associative Cache: Motivation

Direct Mapped Cache:

Only one cache location to store any memory block

Single collision: cache forces eviction even if other cache blocks unused

Improve miss ratio by providing choice of locations for each memory block

Example: Cache size = M words

Therefore memory words with addresses M apart will map to the same cache block in a DM cache

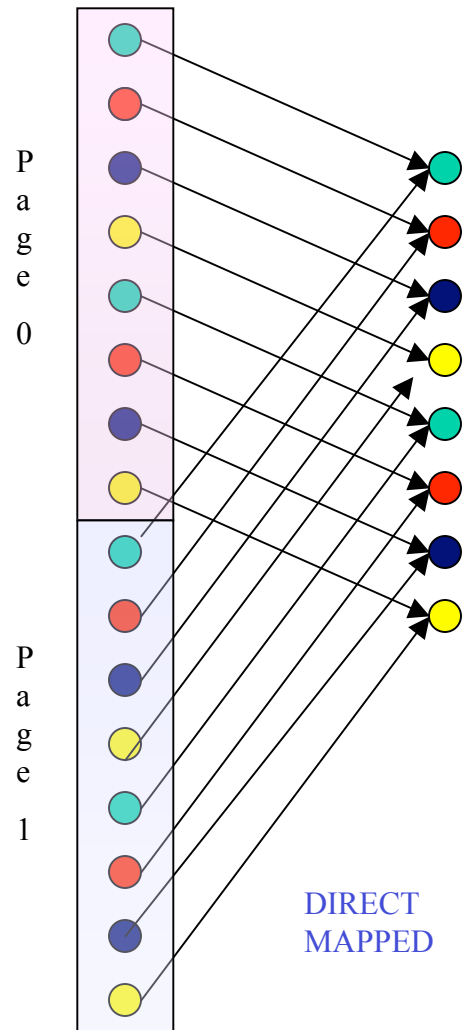
```
while (!done) {  
  for (i = M; i < limit; i = i+M)  
    a[i] += (a[i-M] + a[i+M]) / 2;  
}
```

$a[i] += (a[i-M] + a[i+M])$ all map to same cache index: $(i \bmod M)$

Every memory access in every iteration could be a cache miss

Reduce Conflict Misses using set associative cache

Mapping between Memory Blocks and Cache Blocks

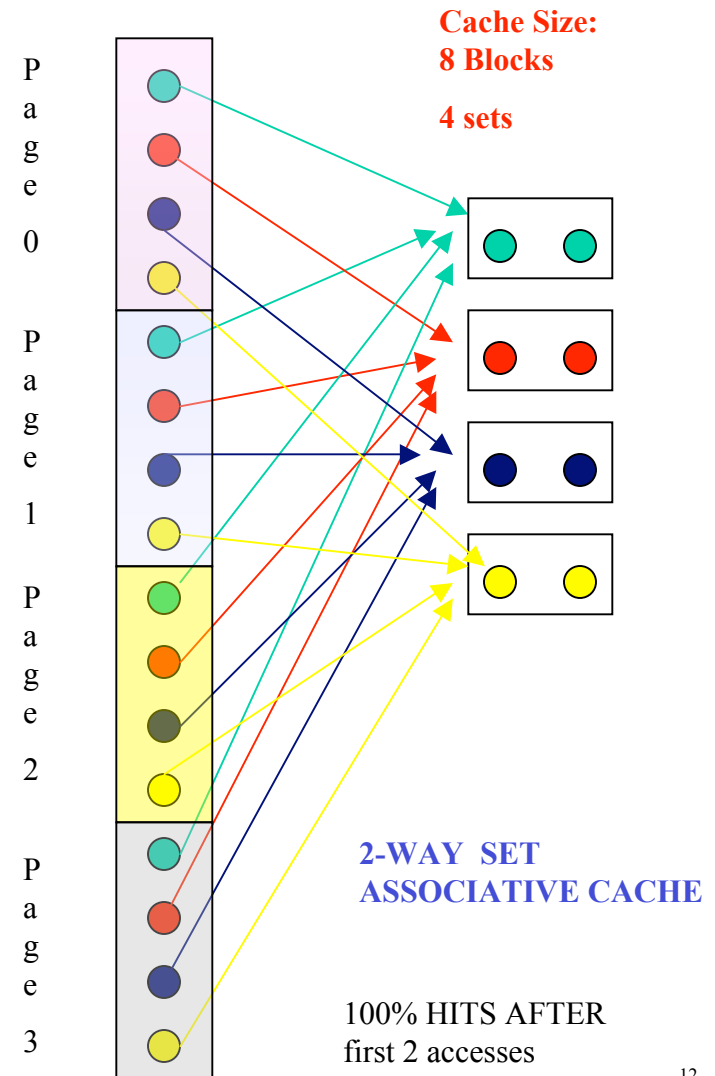


DIRECT MAPPED

Cache Size:
8 Blocks

EXAMPLE: 0,8,0,8,0,8,.....

100% MISS

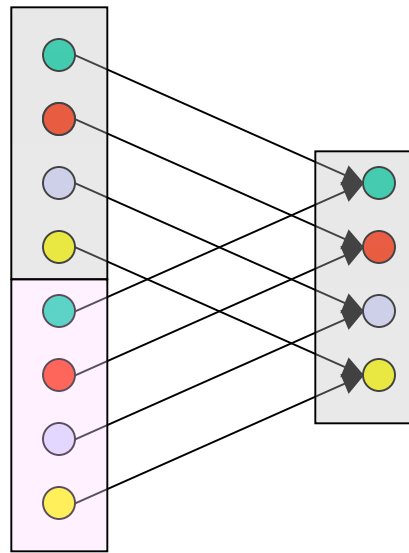


Cache Size:
8 Blocks
4 sets

2-WAY SET ASSOCIATIVE CACHE

100% HITS AFTER first 2 accesses

Mapping between Memory Blocks and Cache Blocks

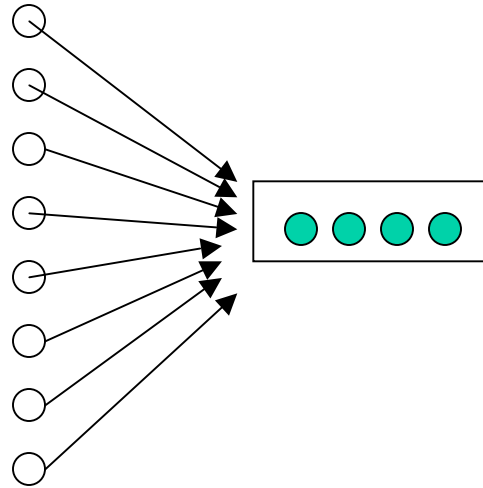


**Memory
Blocks**

**Cache
Blocks**

Direct-Mapped Cache mapping

- All cache blocks have different colors
- Memory blocks in any page cycle through the same colors in order
- A memory block can be placed only in a cache block of matching color

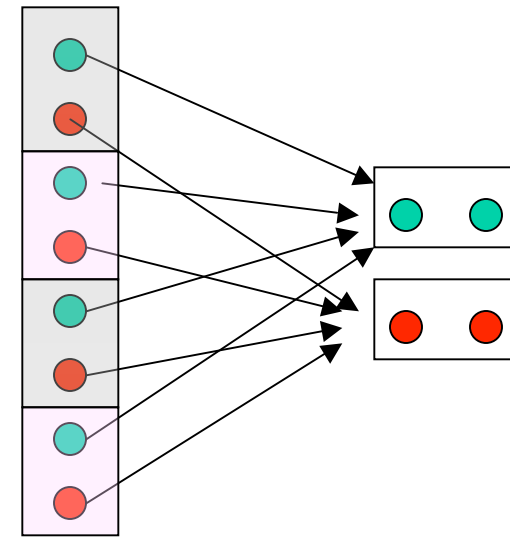


**Memory
Blocks**

**Cache
Blocks**

Fully Associative mapping

- Cache blocks grouped in *sets*
- Page size equals number of sets
- All sets of the cache have different colors
- All blocks within a set have the same color
- Number of blocks in set defines “way” of the cache
- A memory block can be placed only in set of matching color



**Memory
Blocks**

**Cache
Blocks**

2-way Set Associative mapping

Set-Associative Cache

K-way Set Associative Cache:

Cache size: $M = 2^m$ blocks

Cache divided into sets of size $K = 2^k$ blocks each (K-way set associative)

Cache consists of $S = 2^s = 2^{m-k}$ sets

Page Size = S blocks

A block in a page is mapped to exactly one set

Memory block with address A mapped to the **unique set**: $(A \bmod S)$

Memory block may be stored in **any** cache block **in the set**

With each cache block store a **tag** of $(n - s)$ MSBs of memory address A

Example:

Cache size: $M = 32$ blocks,

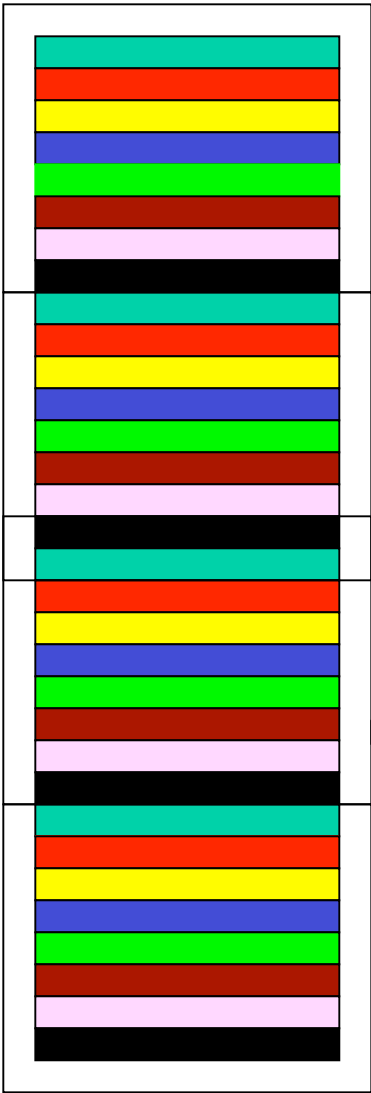
Cache “way”: $K = 4$

Number of sets: $S = M/K = 8$

Consider address trace 0, 32, 64, 96, 128,

In Direct mapped cache ($K=1$) all blocks mapped to cache block 0

In this example ($K=4$) all blocks mapped to set 0; but 4 cache blocks available in each set



Example:

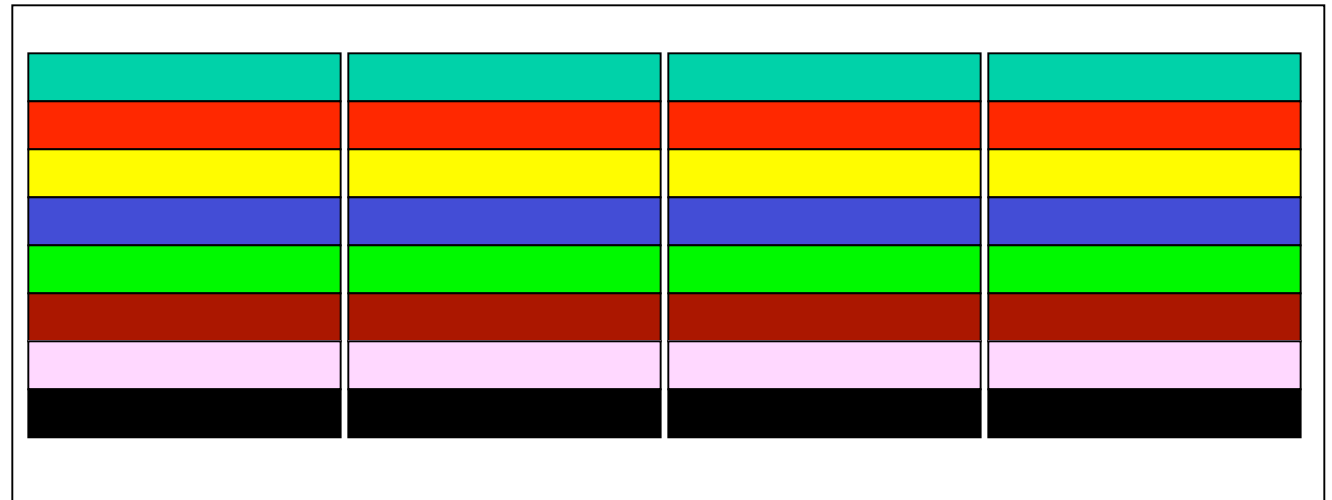
Cache size: $M = 32$ blocks

Cache “way”: $K = 4$

Number of sets $S = M/K = 8$

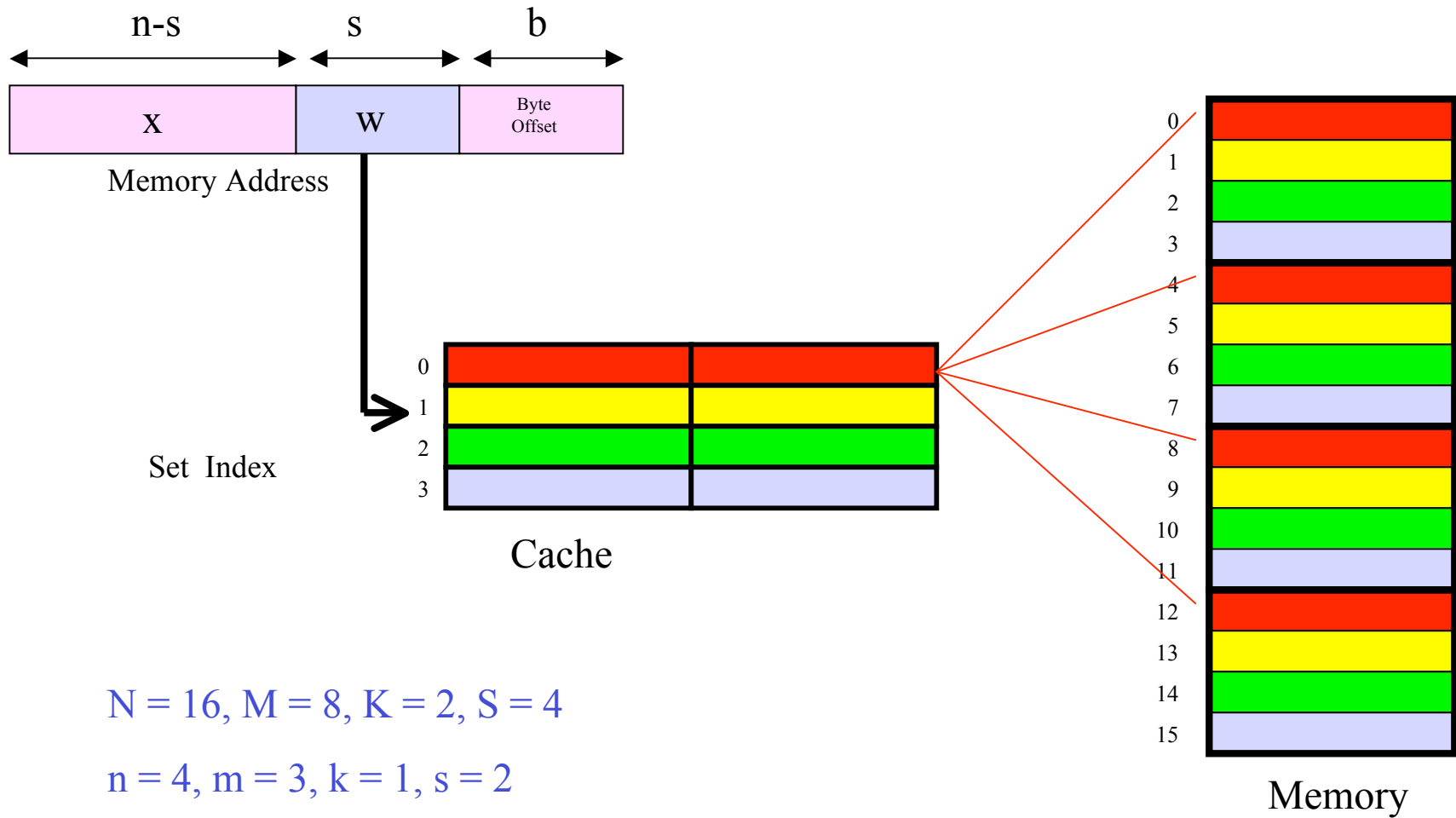
Set Index

- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7



Cache

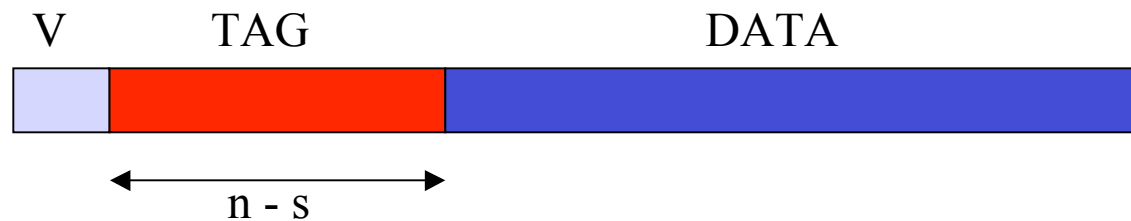
K-way Set-Associative Cache (K = 2)



Set-Associative Cache Organization

To identify which of the 2^{n-s} possible memory blocks is actually stored in a given cache block, each cache block is given a TAG of $n-s$ bits.

Cache Entry:

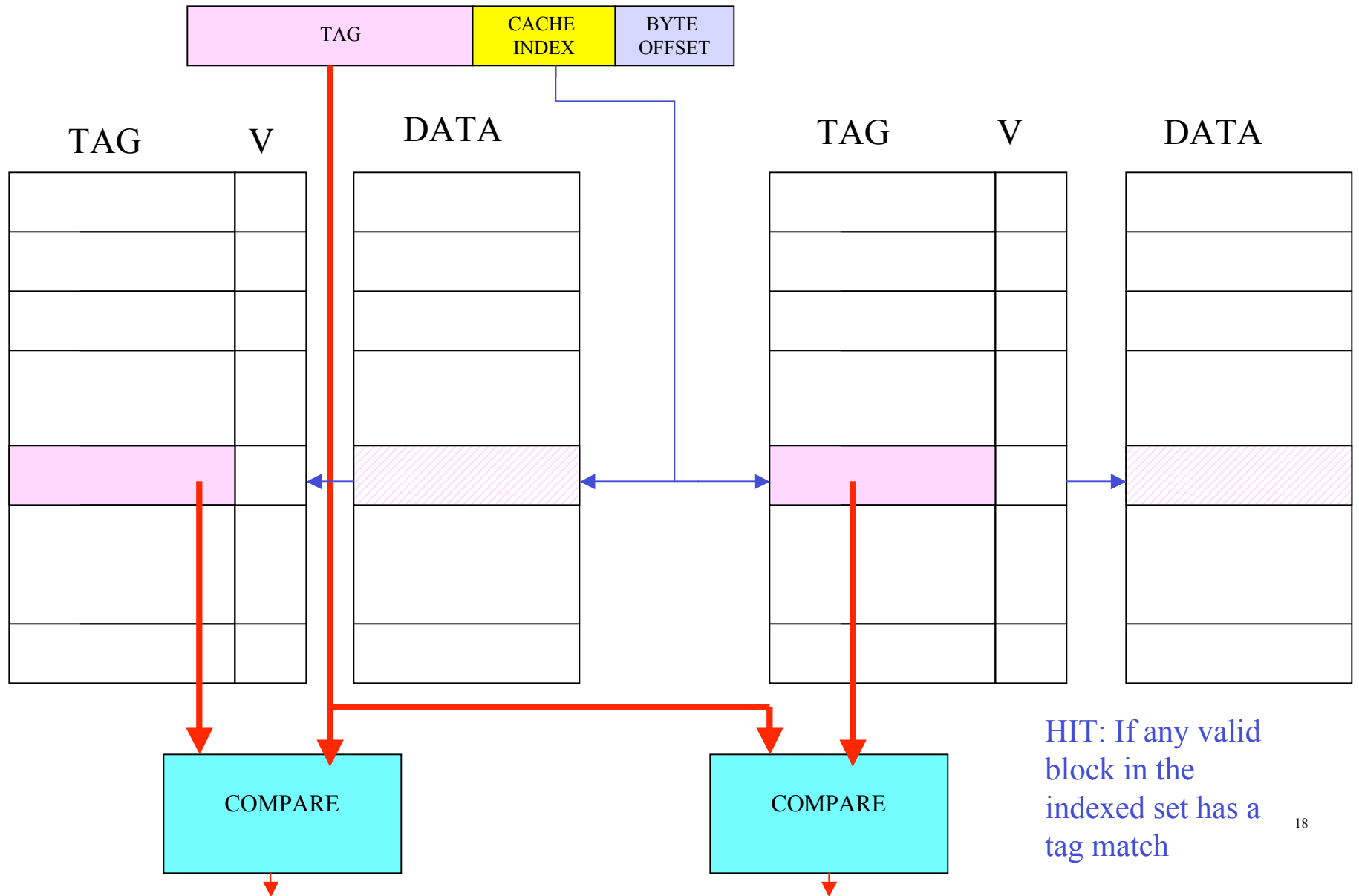


V (Valid) bit: Indicates that the cache entry contains valid data

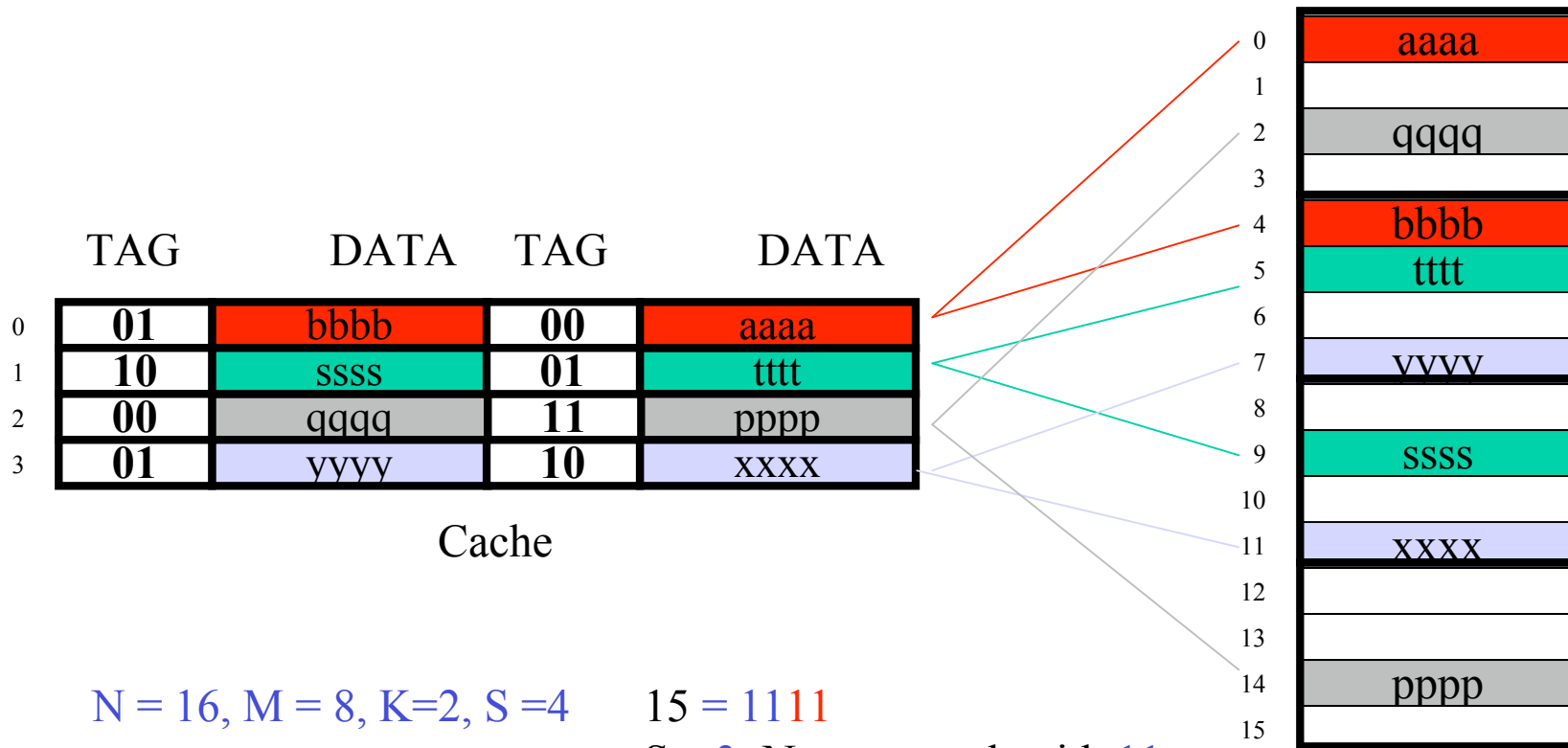
TAG : identifies which of the 2^{n-s} memory blocks stored in cache block

DATA : Copy of the memory block stored in this cache block

2-way Set Associative Cache



Set-Associative Cache Organization



$N = 16, M = 8, K=2, S = 4$

$n = 4, m = 3, k=1, s=2$

$15 = 1111$

Set 3: No tag match with 11

$7 = 0111$

Set 3: Tag match with 01

Set-Associative Cache: Operation

Assume write through (so all blocks are clean)

Memory Read Protocol: n-bit memory block address $A = [x]_{n-s} [w]_s$

Compute cache *set index* $w = A \bmod S$

Read *all K blocks* in set $\text{cache}[w]$

Simultaneously check tags against x

if **cache hit**

 Read **DATA** field of matching block into processor

else /* **cache miss** : no block in set matches */

Stall processor till block brought into cache

Choose a victim block in set $\text{cache}[w]$ to **evict** from the cache

Load main memory block at address A into **DATA** field of **victim**

Update TAG field of cache block to x and **V** to **TRUE**

Restart processor from start of cycle

Cache Hit if **there is a block** in set $\text{cache}[w]$ such that its **V** bit is set and its **TAG** field matches x

Require **K comparators** to compare tags **simultaneously**

Set-Associative Cache: Example

	TAG	DATA	TAG	DATA
0	00	AAAA		
1				
2				
3				

	TAG	DATA	TAG	DATA
0	00	AAAA		
1				
2	01	BBBB		
3				

	TAG	DATA	TAG	DATA
0	00	AAAA	01	CCCC
1				
2	01	BBBB		
3				

Cache

Address Trace: 0, 6, 4, 0, 8

0000: Set 00 Tag: 00 AAAA

0110: Set 10 Tag: 01 BBBB

0100: Set 00 Tag: 01 CCCC

0000: Set 00 Tag: 00 AAAA

Hit!

1000: Set 00 Tag: 10 DDDD

Replacement needed!

Set-Associative Cache Replacement

Replacement Strategy:

Which of the **K** blocks in the selected **set** is **replaced**?

Random: One of the **K** blocks in the set chosen at **random** and replaced

LRU (Least Recently Used) Policy: Replace the block that has not been referenced for the longest time -- block whose last reference most in the past

Difficult to implement efficiently in hardware

Approximations to LRU often used

In example: **0** referenced **more lately** than **4**: **replace 4**

	TAG	DATA	TAG	DATA
0	00	AAAA	10	DDDD
1				
2	01			
3				

Cache

Set-Associative Cache: Write Allocate with Write-Through

Write Allocate and Write-Through Protocol: write `data` to address $A = [x]_{n-s} [w]_s$

Compute cache *set* index $w = A \bmod S$

Search for *match* among blocks in set `cache[w]`

if **cache hit**

 Write `data` into `DATA` field of matching block

 Store `data` into memory address `A`

else /* **cache miss** */

 Stall processor

 Select **victim** to **replace** from set `cache[w]`

 Load cache entry of **victim** with memory block at `A`

 Update fields `TAG` to `x` and `V` to **TRUE**

 Restart cache access

Set-Associative Cache: Write Allocate with Write Back

Write Allocate and Write-Back Protocol : write data to address $A = [x]_{n-s} [w]_s$

If cache hit update data field of cache block

If cache miss

select a block to replace writing it to main memory if dirty

update cache block with new data and V, D, TAG fields

Compute cache set index $w = A \bmod S$

if cache hit

Write data into DATA field of matching block

Update D field to TRUE

else /* cache miss */

Stall processor

Choose a victim block in set $cache[w]$ to replace from the cache

if victim block is dirty

Store DATA field of victim into memory at address $[tag][w]$

Load memory block at A into victim entry of cache

Update TAG to x, V = TRUE, D fields to FALSE

Restart cache access

Set-Associative Cache: Reads in a Write Back Cache

Write-Back Protocol : **read** address $A = [x]_{n-s} [w]_s$

If **cache hit** read data field of cache block

If **cache miss**

select a block to **replace** writing it to **memory** if **dirty**

read in new block from **memory** and install in **cache**

Compute cache index set $w = A \bmod S$

if **cache hit**

Read $\text{cache}[w].\text{DATA}$ into processor

else /* **cache miss** */

Stall processor

Choose a victim block in set $\text{cache}[w]$ to **replace** from the cache

if **victim** block is **dirty**

Store **DATA** field of **victim** into **memory** at **address** $[\text{tag}][w]$

Load block at memory address A into **DATA** field of selected block

Update fields of selected block: **TAG** to x , **V** to **TRUE**, **D** to **FALSE**

Restart processor