Design & Analysis of Algorithms
COMP 482 / ELEC 420

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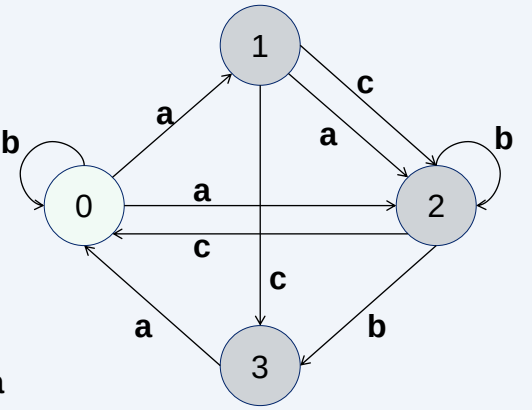
Dynamic Programming (and a little Greediness)

- Viterbi Algorithm
- Knapsack
- (See also the text examples.)

To do:
[CLRS] 15 #5

A Precursor Example to Viterbi Algorithm



Input: **acbba**

Algorithm for calculating set of paths?

3

Viterbi – Toy Example Problem

A Hidden Markov Model

Hidden states **H**,**L** *Observable outputs* **A**,**C**,**G**,**T**

Example from Didier Gonze's adaptation of Borodovsky & Ekisheva (2006), p. 80-81

4

Viterbi – Toy Example Problem

Observed output: **GGCACTGAA**

One example path, **LLHHHHLLL**, has probability

= p(**S**→**L**)×p(**G**|**L**) × p(**L**→**L**)×p(**G**|**L**) × p(**L**→**H**)×p(**C**|**H**) × p(**H**→**H**)×p(**A**|**H**) × …

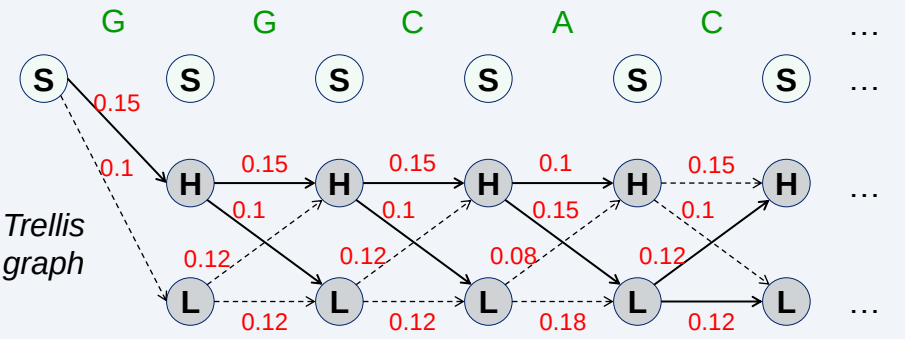
= **0.5**×**0.2** × **0.6**×**0.2** × **0.4**×**0.3** × **0.5**×**0.2** × …

Viterbi – Toy Example Problem

Observed output: **GGCACTGAA**

Algorithm for calculating **most likely** paths?

Viterbi – Two Traditional Views of Algorithm



	G	G	C	A	C	…
S	0.0	0.0	0.0	0.0	0.0	
H	0.15	0.0225	0.003375	0.0003375	0.00006075	
L	0.1	0.015	0.0025	0.00050625	0.00006075	

7

Using Logarithms for Numerical Precision

	Formula	Range	Goal
Replace	$x = a \cdot b \cdot \cdots$	[0,1]	Max
with	$\log x = \log a + \log b + \cdots$	[-∞,0]	Max
or	$-\log x = -\log a + -\log b + \cdots$	[∞,0]	Min

8

A Few Uses of Viterbi Algorithm

Bioinformatics

- Sequence alignment & prediction
- Inferring evolutionary relationships

NLP

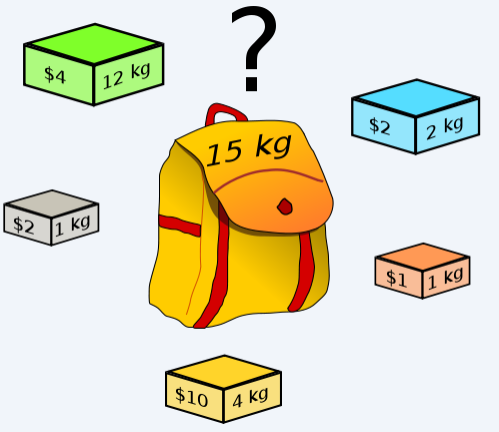
- speech, handwriting, text recognition

Convolutional coding

- Noisy communications
- Hard drives
- Often in implemented in hardware

9

Knapsack Example Problem



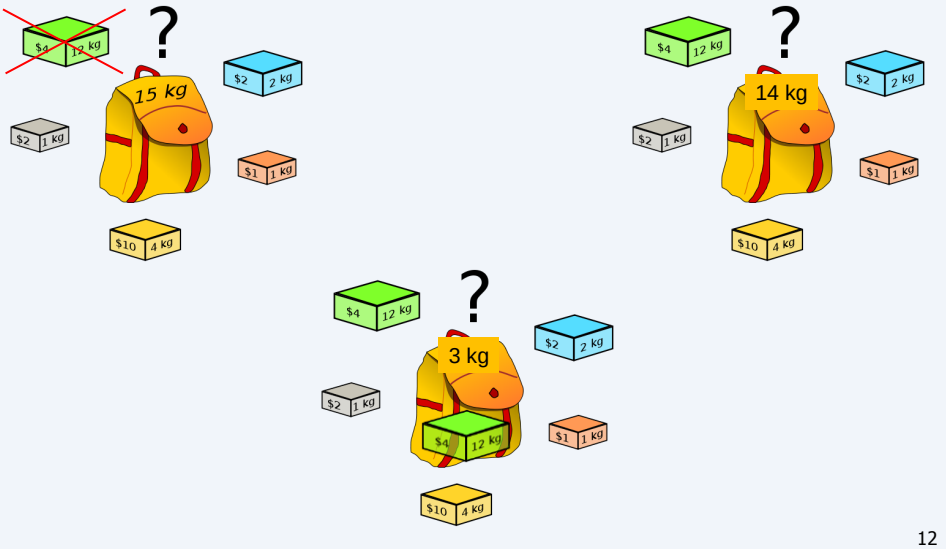
10

A Few Applications of Knapsack Problem

- Manufacturing
  - Cutting material for parts
- Financial decision making
  - Budgeting
  - Portfolio selection
  - Asset-back securitization
  - Combinatorial auctions
- Merkle-Hellman cryptosystem

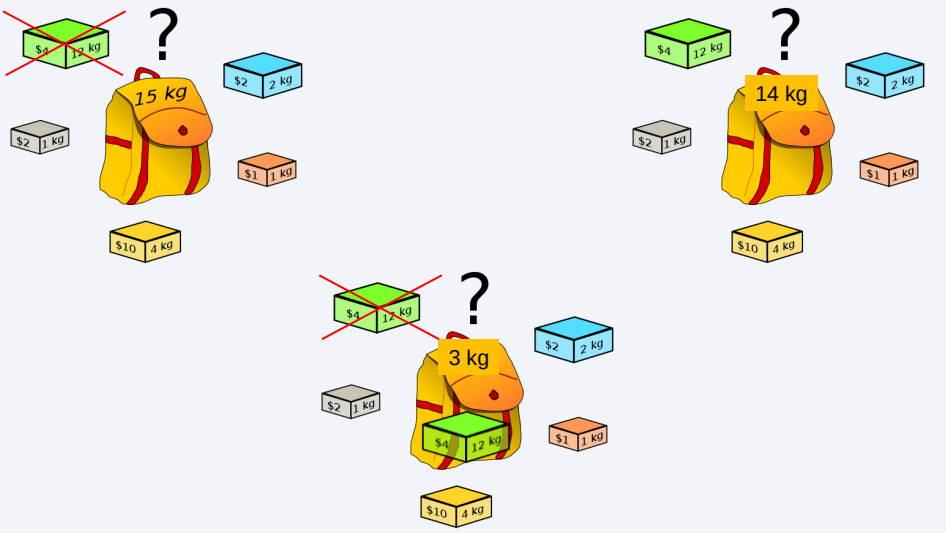
Knapsack with Repetition (aka Unbounded Knapsack)

Solve which subproblem(s)?



Knapsack without Repetition (aka 0-1 Knapsack)

Solve which subproblem(s)?



13

Analyze Knapsack Algorithms

But isn't Knapsack NP-Complete!?

Pseudo-polynomial

14

Knapsack Problems Reworded

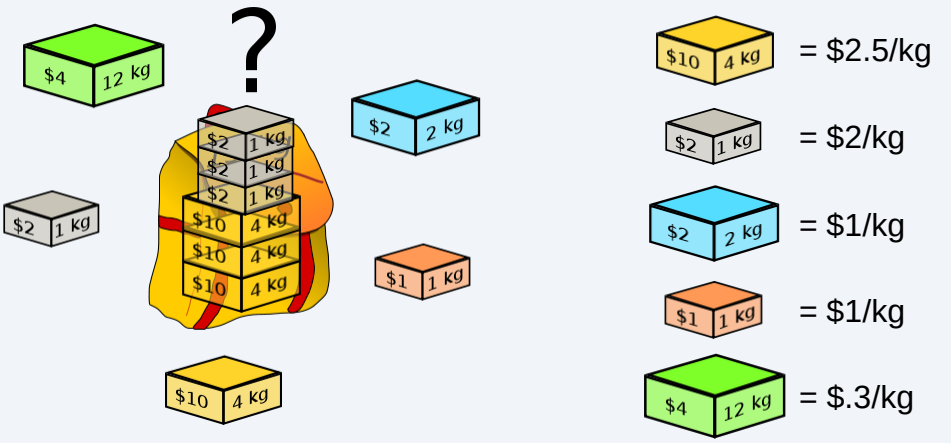
Maximize $\sum_{i=1}^{n} v_i x_i$

Subject to $\sum_{i=1}^{n} w_i x_i \leq W$

$x_i \in \mathbb{N}$ or $x_i \in \{0,1\}$

15

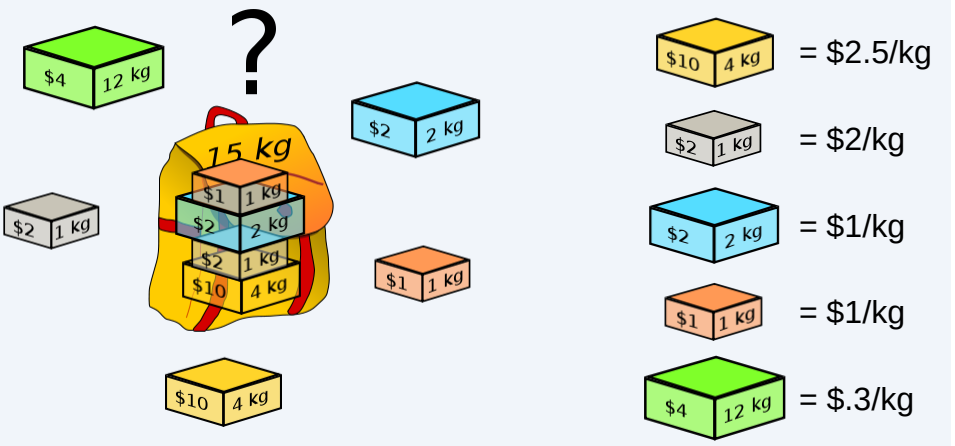
Knapsack with Repetition – Greedy



Later – Prove always ≥ half of optimal.

16

Knapsack without Repetition – Greedy



Construct example when this strategy is bad?

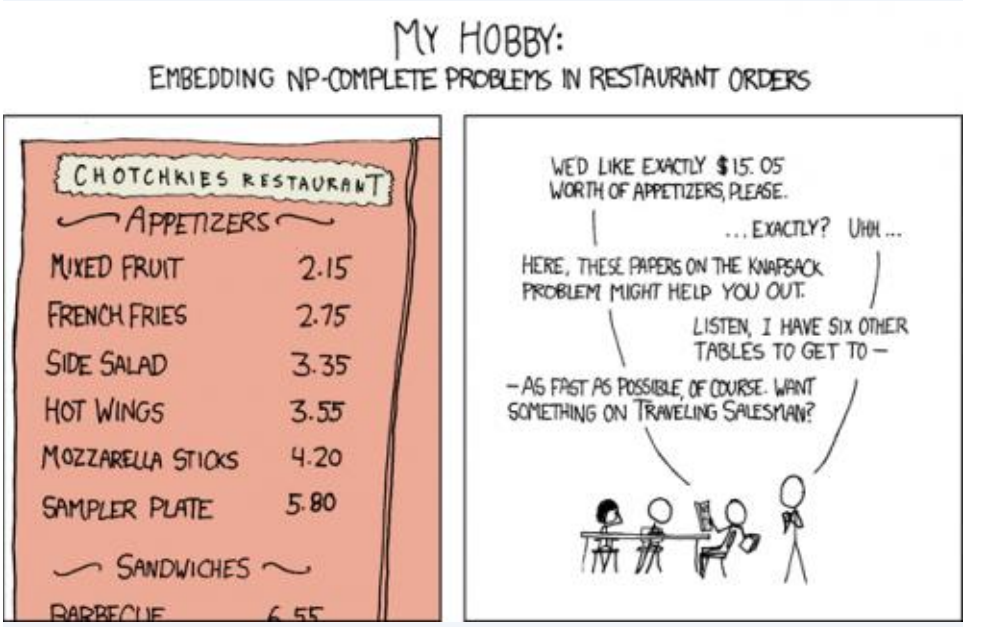
Later – Return to this problem.

17

Supplementary Resources

- Viterbi – Lots online, but most expects background in application area or in Hidden Markov Models.
- Viterbi algorithm
- Wikipedia Knapsack problem

18



Special case: Subset sum