Comp487/587 - Exercise 4

- This exercise is due at the **beginning** of class.
- Questions labeled with **(COMP 587)** are required only for students in COMP 587. All other questions are required both for students in COMP 487 and students in COMP 587.
- Collaboration is allowed and encouraged, but every student must write the solutions in his/her own words. Indicate clearly the name(s) of people you collaborated with.
- Note on the submission the total number of hours spent on this assignment.

Problem 1: PSPACE and Log-Space Reductions (25 points)

Part 1 [10 points]: Prove that the following language is PSPACE-complete:

$$\mathit{AcceptTM} = \{(\lceil M \rceil, x, 1^n) \mid M \text{ is a DTM that accpets } x \text{ in space } n\}$$

Part 2 [15 points]: Prove that $\leq_{\log}$ is transitive. That is, for every triple of languages A, B, C, if $A \leq_{\log} B$ and $B \leq_{\log} C$ then $A \leq_{\log} C$.

Problem 2: Using the Proof of Savitch's Theorem (25 Points)

A directed graph $G = (V, E)$ is called a layer graph if there is a partition of V into k disjoint sets $V_1, \cdots V_k$ such that all the edges are from one layer to the next. (i.e $E \subseteq \bigcup_{1 \leq i < k}(V_i \times V_{i+1})$). Prove that the following language is in $DSPACE(\log^2(n))$:

$$\mathit{EvenCon} = \{(G = (V_1, V_2, \cdots, V_k, E), s, t) \mid \text{there are an even number of distinct paths from } s \text{ to } t \text{ in } G,\ s \in V_1, t \in V_k, \text{ and } G \text{ is a layer graph under the partition } V_1, \cdots, V_k\}$$

(Note that two paths are distinct if they do not contain exactly the same vertices in the same order).

Problem 3: 2SAT (25 points)

Part 1 [10 points]: Prove that $\overline{2SAT} \in NL$.

Part 2 [10 points]: Prove that $\overline{2SAT}$ is NL-hard.

- Hint: Show a reduction from *st-Con* that takes as input a graph with n vertices and m edges and constructs a formula with n variables and $m + 2$ clauses.

Part 3 [5 points]: Prove that $2SAT$ is NL-complete.

Problem 4: Linear-Exponential Time (25 points)

Consider $LinEXP = \bigcup_{c \in \mathbb{N}} DTIME(2^{cn})$ and $LinNEXP = \bigcup_{c \in \mathbb{N}} NTIME(2^{cn})$. Given a language L, we define $L_{pad} = \{(x, 1^{2^{|x|}}) \mid x \in L\}$ (i.e., the language where we pad every word $x \in L$ with $2^{|x|}$ 1's).

Part 1 [10 points]: Prove that if $L \in LinNEXP$ then $L_{pad} \in NP$.

Part 2 [10 points]: Prove that if $L_{pad} \in P$ then $L \in LinEXP$.

Part 3 [5 points]: Prove that if $P = NP$ then $LinEXP = LinNEXP$.

(COMP 587) Problem 5: Bipartite Perfect Matching (20 points)

We say that $G = (V, E)$ has a perfect matching if there is a set of edges $E' \subseteq E$ such that every vertex $v \in V$ has is contained in exactly one edge from E'. Consider the following language:

$$BipPerfMatch = \{G \mid G \text{ is an undirected bipartite graph with a perfect matching}\}$$

Prove that $st\text{-}Con \leq_{\log} BipPerfMatch$.