Comp487/587 - Exercise 5

- This exercise is due at the **beginning** of class.
- Questions labeled with **(COMP 587)** are required only for students in COMP 587. All other questions are required both for students in COMP 487 and students in COMP 587.
- Collaboration is allowed and encouraged, but every student must write the solutions in his/her own words. Indicate clearly the name(s) of people you collaborated with.
- Note on the submission the total number of hours spent on this assignment.

Problem 1: Log-Space NP Verification (20 points)

Consider the certificate definition of NL (Def. 4.17 in the online version of your textbook). Prove that, if the verifier's tape head is allowed to move back and forth on the certificate, the complexity class being defined changes from NL to NP. (Note that the verifier still uses at most $O(\log |x|)$ space.)

- Hint: Recall from class that 3SAT is also NP-complete under log-space reductions.

Problem 2: TQBF (20 points)

Part 1 [5 points]: Prove that TQBF restricted to formulas where the part following the quantifiers is in CNF is still PSPACE-complete.

Part 2 [15 points]: Prove that TQBF is complete for PSPACE under log-space reductions.

Problem 3: Space Hierarchy Theorems (20 points)

Let $s_1(n)$ and $s_2(n)$ be space-constructible functions such that $\log(n) \leq s_1(n), s_2(n)$.

Part 1 [5 points]: Prove that, if $\lim_{n\to\infty} \frac{(s_1(n))^2}{s_2(n)} = 0$, then $NSPACE(s_1(n)) \subsetneq NSPACE(s_2(n))$.

Part 2 [15 points]: Prove that, if $\lim_{n\to\infty} \frac{s_1(n)}{s_2(n)} = 0$, then $NSPACE(s_1(n)) \subsetneq NSPACE(s_2(n))$.

- Hint: Rely on the Immerman-Szelepscenyi Theorem, and recall the proof from class for DSPACE.

Problem 4: Intersecting NP and Co-NP Languages (20 points COMP 487 / 30 points COMP 587)

Consider the class of languages $\Delta_P = \{L \subseteq \Sigma^* \mid \text{there exist } L_1 \in \text{NP and } L_2 \in \text{Co-NP such that } L = L_1 \cap L_2\}$. Note that Δ_P is **not** equal to NP $\cap$ Co-NP.

Part 1 [6 points]: Prove that NP $\cup$ Co-NP $\subseteq \Delta_P$.

Part 2 [6 points]: Prove that $\Delta_P \subseteq \Sigma_2^P$.

Part 3 [8 points]: Prove that the following language is Δ_P-complete with respect to polynomial-time reductions:

$$SAT\text{-}noSAT = \{(\varphi_1, \varphi_2) \mid \varphi_1 \in 3SAT, \varphi_2 \notin 3SAT\}.$$

(COMP 587) Part 4 [10 points]: Prove that $3SAT \in$ Co-NP if and only if $\Delta_P = NP$.

Problem 5: Oracles (20 points COMP 487 / 30 points COMP 587)

Definition 1. A language A is called *shrinking* if there is a polynomial-time DTM M, with access to A as an oracle, that on every input x decides if $x \in A$ using at most 2 oracle queries to A, where each query must be of length strictly smaller than $|x|$.

Part 1 [20 points]: Prove that TQBF is shrinking.

(COMP 587) Part 2 [10 points]: Prove that, if a language A is shrinking, then $A \in PSPACE$.