COMP 512
Rice University
Spring 2015

Example Transformations on SSA Form

Dead, Clean, and Constant Propagation

Citation numbers refer to entries in the EaC2e bibliography.

Dead Code Elimination

Three distinct problems

- Useless operations
  - ♦ Any operation whose value is not used in some visible way
  - ♦ Use the **SSA**-based mark/sweep algorithm (**DEAD**)
- Useless control flow
  - ♦ Branches to branches, empty blocks
  - ♦ Simple **CFG**-based algorithm (**CLEAN**)
- Unreachable blocks
  - ♦ No path from n_0 to $b \Rightarrow b$ cannot execute
  - ♦ Simple graph reachability problem

Using SSA – Dead code elimination

```
Mark
   for each op i
      clear i's mark
      if i is critical then
         mark i
         add i to WorkList

   while (Worklist ≠ Ø)
      remove i from WorkList
          (i has form "x←y op z")
      if def(y) is not marked then
         mark def(y)
         add def(y) to WorkList
      if def(z) is not marked then
         mark def(z)
         add def(z) to WorkList

      for each b ∈ RDF(block(i))
         mark the block-ending
            branch in b
         add it to WorkList
```

```
Sweep
   for each op i
      if i is not marked then

         if i is a branch then
            rewrite with a jump to i's
            nearest useful post-dominator

         if i is not a jump then
            delete i
```

Notes:

- Eliminates some branches
- Reconnects dead branches to the remaining live code
- Find useful post-dominator by walking post-dom tree
  - Entry & exit nodes are always "useful"

Eliminating Useless Control Flow

The Problem

- After optimization, the **CFG** can contain empty blocks
- "Empty" blocks still end with either a <u>branch</u> or a <u>jump</u>
- Produces jump to jump, which wastes time & space
- Need to simplify the **CFG** & eliminate these

We must distinguish between branch & jump
- Branch is conditional
- Jump is absolute

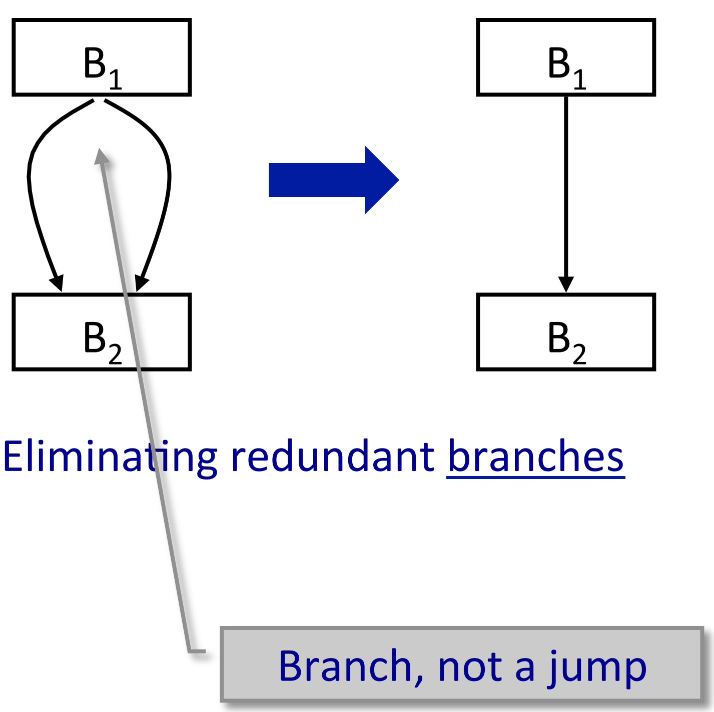
The **CLEAN** Algorithm

- Use four distinct transformations
- Apply them in a carefully selected order
- Iterate until done

Devised by Rob Shillingsburg (1992), documented by John Lu (1994)

Eliminating Useless Control Flow

Transformations in CLEAN



Both sides of branch target B_i

- Neither block must be empty
- Replace it with a jump to B_i
- Simple rewrite of last op in B_1

How does this happen?

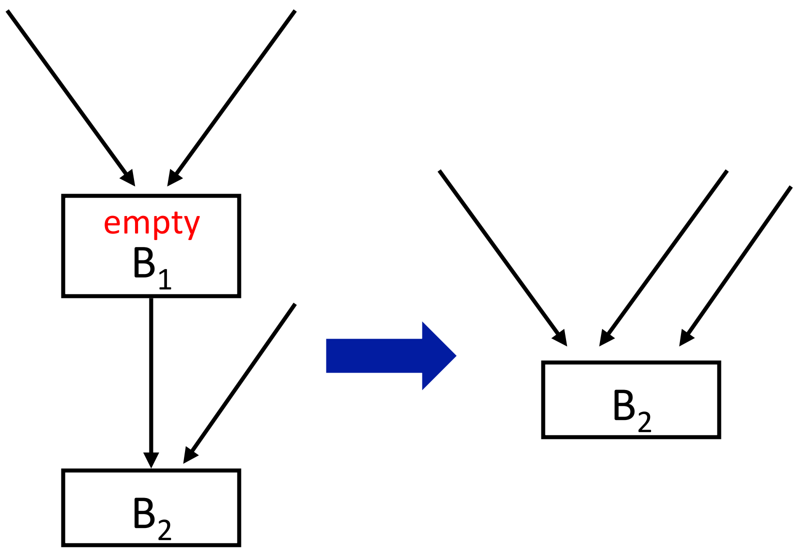
- Rewriting other branches

How do we find it?

- Check each branch

Eliminating Useless Control Flow

Transformations in CLEAN



Eliminating empty blocks

Merging an empty block

- Empty B_1 ends in a jump
- Coalesce B_1 with B_2
- Move B_1's incoming edges
- Eliminates extraneous jump
- Faster, smaller code

How does this happen?

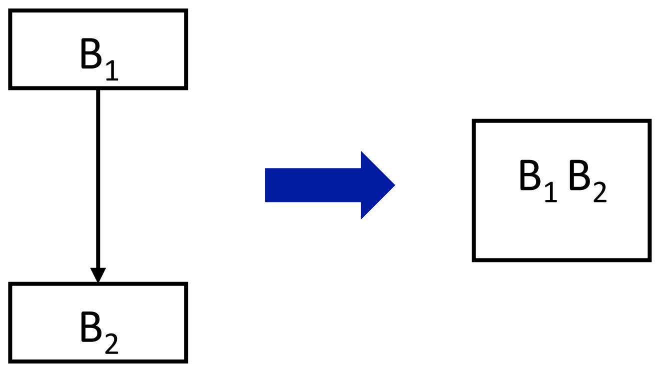
- Eliminate operations in B_1

How do we find it?

- Test for empty block

Eliminating Useless Control Flow

Transformations in CLEAN



Combining non-empty blocks

> B_1 and B_2 should be a single basic block
>
> If one executes, both execute, in linear order.

Coalescing blocks

- Neither block must be empty
- B_1 ends with a jump
- B_2 has 1 predecessor
- Combine the two blocks
- Eliminates a jump

How does this happen?

- Simplifying edges out of B_1

How do we find it?

- Check target of jump |*preds*|

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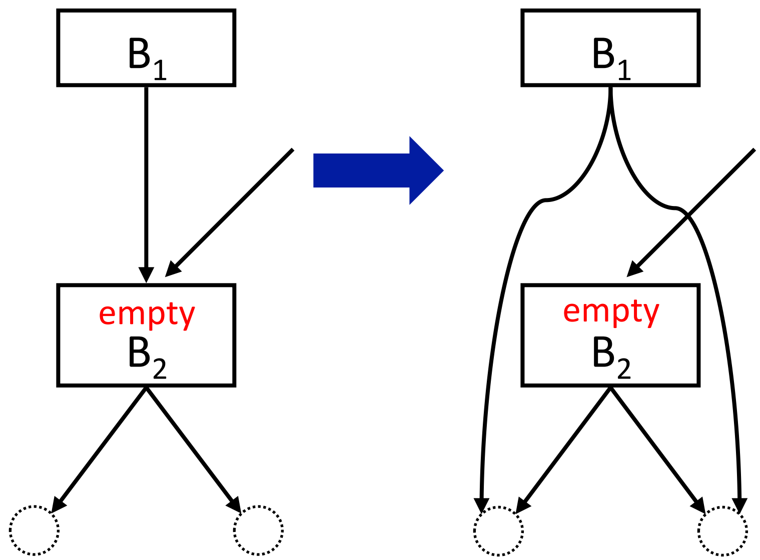
Eliminating Useless Control Flow

Transformations in CLEAN



Hoisting branches from
empty blocks

Jump to a branch

- B_1 ends with jump, B_2 is empty
- Eliminates pointless jump
- Copy branch into end of B_1
- Might make B_2 unreachable

How does this happen?

- Eliminating operations in B_2

How do we find this?

- Jump to empty block

Eliminating Useless Control Flow

Putting the transformations together

- Process the blocks in postorder
  - ♦ Clean up B_i's successors before B_i
  - ♦ Simplifies implementation & understanding
- At each node, apply transformations in a fixed order
  - ♦ Eliminate redundant branch
  - ♦ Eliminate empty block
  - ♦ Merge block with successor
  - ♦ Hoist branch from empty successor
- May need to iterate
  - ♦ Postorder ⇒ unprocessed successors along back edges
  - ♦ Can bound iterations, but a deriving tight bound is hard
  - ♦ Must recompute postorder between iterations

Handles jump

← Elim
← Elim
— Elim
— Adds an edge

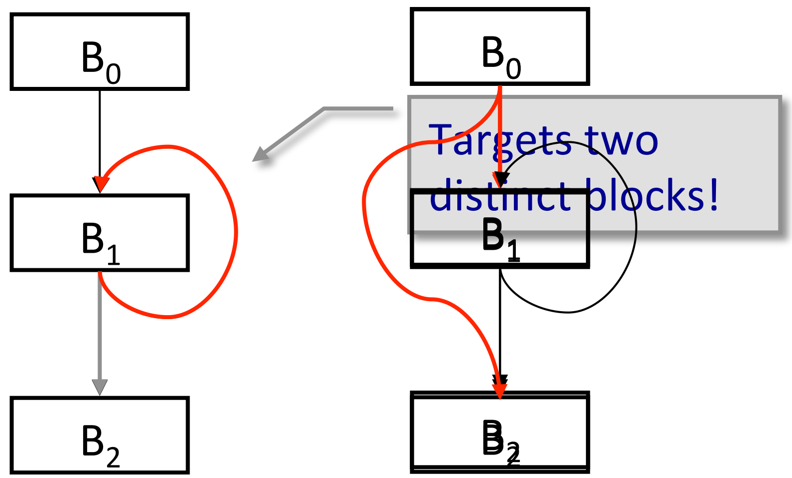
Creates a jump that should be checked in the same pass

Montonicity is not obvious

Eliminating Useless Control Flow

What about an empty loop?

- By itself, **CLEAN** cannot eliminate the loop
- Loop body branches to itself
  - ♦ Branch is not redundant *
  - ♦ Doesn't end with a jump
  - ♦ Hoisting does not help *
- Key is to eliminate self-loop
  - ♦ Add a new transformation? *
  - ♦ Then, B_1 merges with B_2 *



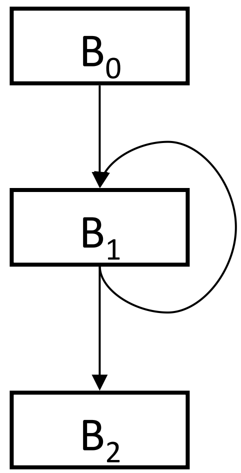
New transformation must recognize that B_1 is empty. Presumably, it has code to test exit condition & (probably) increment an induction variable.

This requires looking at code inside B_1 and doing some sophisticated pattern matching. This seems awfully complicated.

Eliminating Useless Control Flow

What about an empty loop?

- How to eliminate <B_1,B_1> ?
  - ♦ Pattern matching ?
  - ♦ Useless code elimination ?

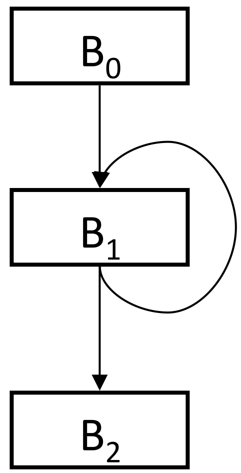


Eliminating Useless Control Flow

What about an empty loop?

- How to eliminate <B_1,B_1> ?
  - ♦ Pattern matching ?
  - ♦ Useless code elimination ?

- **What does DEAD do to B_1?**
  - ♦ Remember, it is empty
  - ♦ Contains only the branch
  - ♦ B_1 has only one exit
  - ♦ So, $B_1 \notin$ **RDF**(B_2)
  - ♦ B_1's branch is useless
  - ♦ **DEAD** rewrites it as a jump to B2



Using SSA – Dead code elimination

Added to the algorithm by Shillingsburg

```
Mark
    for each op i
        clear i's mark
        if i is critical then
            mark i
            add i to WorkList

    while (Worklist ≠ Ø)
        remove i from WorkList
            (i has form "x←y op z")
        if def(y) is not marked then
            mark def(y)
            add def(y) to WorkList
        if def(z) is not marked then
            mark def(z)
            add def(z) to WorkList

        for each b ∈ RDF(block(i))
            mark the block-ending
                branch in b
            add it to WorkList
```

```
Sweep
    for each op i
        if i is not marked then
            if i is a branch then
                rewrite with a jump to i's
                nearest useful post-dominator
            if i is not a jump then
                delete i
```

Notes:

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- Find useful post-dominator by walking post-dom tree
  - Entry & exit nodes are always "useful"

Eliminating Useless Control Flow

What about an empty loop?

- How to eliminate <B_1,B_1> ?
  - ♦ Pattern matching ?
  - ♦ Useless code elimination ?

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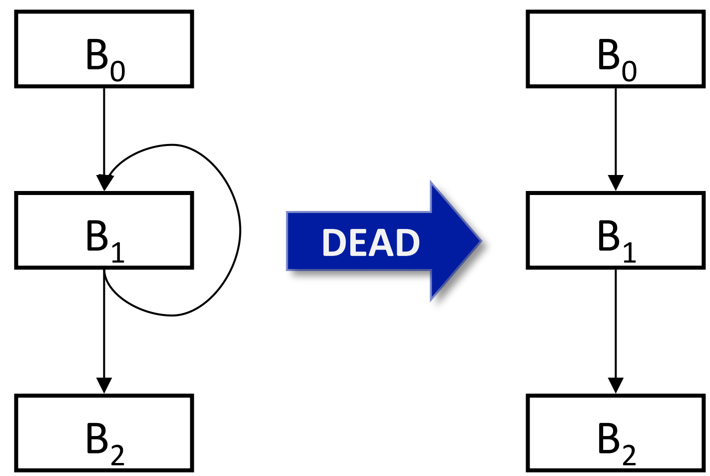

DEAD converts the empty loop to a form where **CLEAN** handles it !

Eliminating Useless Control Flow

The Algorithm

CleanPass()
  for each block *i*, in postorder
    if *i* ends in a branch then
      if both targets are identical then
        rewrite with a jump
    if *i* ends in a jump to *j* then
      if *i* is empty then
        merge *i* with *j*
      else if *j* has only one predecessor
        merge *i* with *j*
      else if *j* is empty & *j* has a branch then
        rewrite *i*'s jump with *j*'s branch

Clean()
  until CFG stops changing
    compute postorder
    CleanPass()

Summary

- Simple, structural algorithm
- Limited transformation set
- Cooperates with **DEAD**
- In practice, its quite fast

How many calls to CleanPass are needed before **CLEAN** halts?

- Clearly a fixed point algorithm
- Answer is not obvious

Eliminating Useless Control Flow

Putting the transformations together

- Process the blocks in postorder
  - ♦ Clean up B_i's successors before B_i
  - ♦ Simplifies implementation & understanding
- At each node, apply transformations in a fixed order
  - ♦ Eliminate redundant branch ← Eliminates an edge
  - ♦ Eliminate empty block ← Eliminates a node
  - ♦ Merge block with successor ← Eliminates node & edge
  - ♦ Hoist branch from empty successor ← Adds an edge

Montonicity is not obvious

- May need to iterate
  - ♦ Postorder ⇒ unprocessed successors along back edges
  - ♦ Can bound iterations, but a deriving tight bound is hard
  - ♦ Must recompute postorder between iterations

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Eliminating Unreachable Code

The Problem

- Block with no entering edge
- Situation created by other optimizations

The Cure

- Compute reachability & delete unreachable code
- Simple mark/sweep algorithm on **CFG**
- Mark during computation of postorder, reverse postorder ...
- In **MSCP**, importing **ILOC** did this (every time)

Dead Code Elimination

Summary

- Useless Computations ⇒ **DEAD**
- Useless Control-flow ⇒ **CLEAN**
- Unreachable Blocks ⇒ Simple housekeeping

Other Transformations that eliminate dead code

- Constant propagation can eliminate some branch targets
- Algebraic identities & redundancy elimination make some operations useless or outright remove them (depends on implementation style)

Use of SSA Form

- **DEAD** used **SSA** form as a convenient way to get **DEF-USE** chains
- **CLEAN** operated on the **CFG** without much regard to contents of a block

Constant Propagation

We have seen two formulations of constant propagation

- Classical formulation as a global data-flow problem
  - ♦ Annotate each node in the CFG (each block) with a CONSTANTS set
  - ♦ Complicated transfer functions to model effect of single op
  - ♦ Compose transfer function of individual ops to get function for entire block
    - → ***Resembles a symbolic interpretation***
  - ♦ Verdict: conceptually complex and (potentially) slow
- Sparse formulation over the graph formed by **DEF-USE** chains
  - ♦ Each value is treated separately — propagated along chain and used in a meet operation with the values of other defs that reach the same use
  - ♦ Algorithm is conceptually simple
  - ♦ Argument for termination and speed are based on lattice height, not some transfer function and the **CFG** structure (e.g., d(G) + 3 passes a la Kam-Ullman)
  - ♦ Verdict: conceptually simpler and (arguably) faster

Constant Propagation

Safety

- Proves that name always has known value *at point p*
- Specializes code around that value
  - ♦ Moves some computations to compile time (⇒ *code motion*)
  - ♦ Exposes some unreachable blocks (⇒ *dead code*)

Opportunity

- Value ≠ ⊥ signifies an opportunity

Profitability

- Compile-time evaluation is cheaper than run-time evaluation
- Branch removal may lead to block coalescing (**CLEAN**)
  - ♦ If not, it still avoids the test & makes branch predictable

Constant Propagation over DEF-USE Chains

Worklist ← Ø

For i ← 1 to number of operations
  if in_1 of operation *i* is a constant c_i
    then Value(in_1,*i*) ← c_i
    else Value(in_1,*i*) ← T
  if in_2 of operation *i* is a constant c_j
    then Value(in_2,*i*) ← c_j
    else Value(in_2,*i*) ← T
  if (Value(in_1,*i*) is a constant &
    Value(in_2,*i*) is a constant)
    then Value(*out,i*) ← evaluate op *i*
      Worklist ← Worklist ∪ {*i* }
    else Value(*out,i*) ← T

Initialization Step

while (Worklist ≠ Ø)
  remove a definition *i* from WorkList
  for each *j* ∈ USES(*out,i*)
    set *x* so that out of i is in_x of j
    Value(in_x,*j*) ← Value(in_x,*j*)
        ∧ Value(*out,,i*)
    if (Value(in_1,*j*) is a constant &
      Value(in_2,*j*) is a constant)
      then Value(*out,j*) ← evaluate op *j*
        Worklist ← Worklist ∪ {*j* }
      else if (Value(in_1,*j*) is ⊥ or
          Value(in_2,*j*) is ⊥)
      then Value(*out,j*) ← ⊥
        Worklist ← Worklist ∪ {*j* }

Propagation Step

Last time we saw constant propagation, we had this algorithm over **DEF-USE** chains …

Constant Propagation over DEF-USE Chains

Worklist ← Ø

for i ← 1 to number of operations
    if in_1 of operation i is a constant c_i
        then Value(in_1,i) ← c_i
        else Value(in_1,i) ← T
    if in_2 of operation i is a constant c_j
        then Value(in_2,i) ← c_j
        else Value(in_2,i) ← T
    if (Value(in_1,i) is a constant &
        Value(in_2,i) is a constant)
        then Value(out,i) ← evaluate op i
            Worklist ← Worklist ∪ {i }
        else Value(out,i) ← T

Initialization Step

while (Worklist ≠ Ø)
    remove a definition i from WorkList
    for each j ∈ **USES**(out,i)
        let x be operand where j occurs
        Value(in_x,j) ← Value(in_x,j)
            ∧ Value(out,i)
        if (Value(in_1,j) is a constant &
            Value(in_2,j) is a constant)
            then Value(out,j) ← evaluate op j
                Worklist ← Worklist ∪ {j }
            else if (Value(in_1,j) is ⊥ or
                Value(in_2,j) is ⊥)
            then Value(out,j) ← ⊥
                Worklist ← Worklist ∪ {j }

Propagation Step

The same algorithm, formulated over SSA interpreted as a graph.

Using SSA — Sparse Constant Propagation [W&Z, 347]

∀ expression, e
  Value(e) ← { **TOP** if its value is unknown; c_i if its value is known; **BOT** if its value is known to vary }
WorkList ← Ø

∀ SSA edge s = <u,v>
  if Value(u) ≠ **TOP** then
    add s to WorkList

while (WorkList ≠ Ø)
  remove s = <u,v> from WorkList
  let o be the operation that uses v
  if Value(o) ≠ **BOT** then
    t ← result of evaluating o
    if t ≠ Value(o) then
      ∀ SSA edge <o,x>
        add <o,x> to WorkList

i.e., o is "a←b op v" or "a ←v op b"

Evaluating a Ø-function:

Ø(x_1,x_2,x_3, … x_n) is
  Value(x_1) ∧Value(x_2) ∧ Value(x_3)
  ∧ … ∧ Value(x_n)

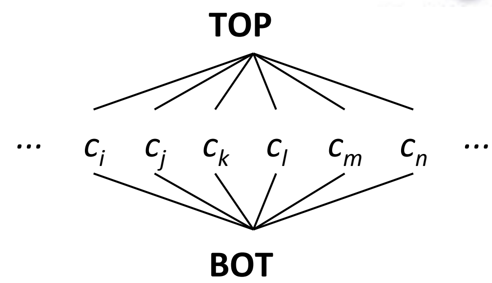
where

TOP ∧ x = x	∀ x
c_i ∧ c_j = c_i	if c_i = c_j
c_i ∧ c_j = BOT	if c_i ≠ c_j
BOT ∧ x = **BOT**	∀ x

Same result, fewer ∧ operations
Performs ∧ only at Ø nodes

Using SSA — Sparse Constant Propagation

How long does this algorithm take to halt?

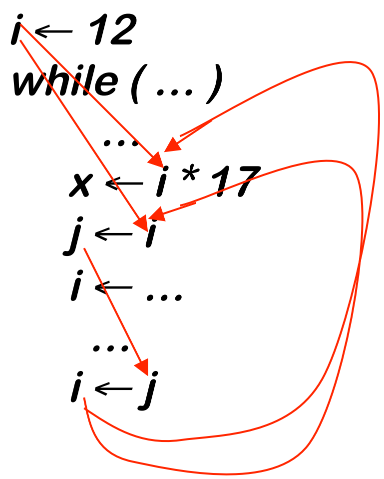


- Initialization is two passes
  - ♦ |ops| + 2 x |ops| edges
- In propagation, Value(x) can take on 3 values
  - ♦ **TOP**, c_i, **BOT**
  - ♦ Each use can be on WorkList twice
  - ♦ 2 x |args| = 4 x |ops| evaluations, WorkList pushes & pops

This algorithm is much simpler than the **DEF-USE** version

Constant Propagation over DEF-USE Chains

Optimism versus Pessimism



Clear that *i* is always 12 at def of *x*

Optimism

- This version of the algorithm is an *optimistic* formulation
- Initializes values to $\top$
- Prior version used $\bot$ (*pessimism*)

M.N. Wegman & F.K. Zadeck, "Constant Propagation With Conditional Branches", **ACM TOPLAS**, 13(2), April 1991, pages 181–210.

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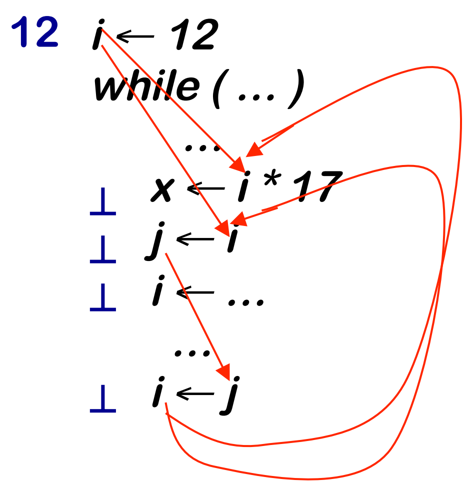
Constant Propagation over DEF-USE Chains

Optimism versus Pessimism



Pessimistic initializations

Leads to
$i \leftarrow 12 \wedge \bot = \bot$

Optimism

- This version of the algorithm is an *optimistic* formulation
- Initializes values to $\top$
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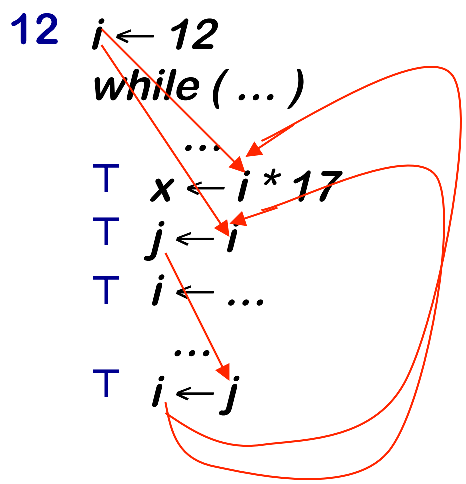
Constant Propagation over DEF-USE Chains

Optimism versus Pessimism



Optimistic initializations

Leads to
$i \leftarrow 12 \wedge \top = 12$

Optimism

- This version of the algorithm is an *optimistic* formulation
- Initializes values to $\top$
- Prior version used $\bot$ (*pessimism*)

In general

- Optimism helps inside loops
- Determined by the initial value

M.N. Wegman & F.K. Zadeck, "Constant Propagation With Conditional Branches", **ACM TOPLAS**, 13(2), April 1991, pages 181–210.

Sparse Constant Propagation

What happens when SCP propagates a value into a branch?

- **TOP** ⇒ we gain no knowledge
- **BOT** ⇒ either path can execute
- **TRUE** or **FALSE** ⇒ only one path can execute

} But, the algorithm does not use this knowledge ...

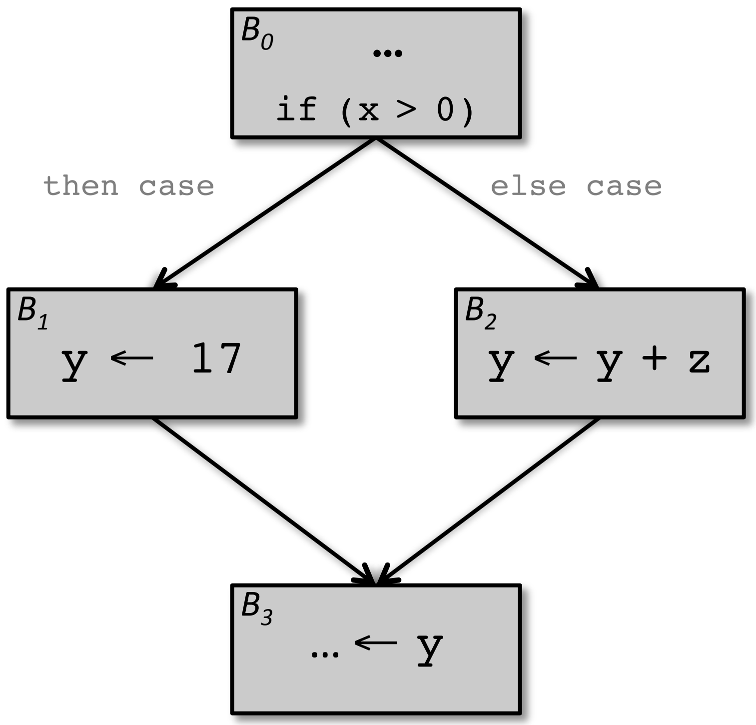
Using this observation, we can add an element of refining feasible paths to the algorithm that will take it beyond the standard limits of **DFA**

→ Until a block can execute, treat it as unreachable

→ Optimistic initializations allow analysis to proceed with unevaluated blocks

Result is an analysis that can use *limited symbolic evaluation* to combine constant propagation with unreachable code elimination

Sparse Conditional Constant Propagation



Classic DFA assumes that all paths can be taken at runtime, including (B_0, B_2, B_3)

Can use constant-valued control predicates to refine the CFG

- If compiler knows the value of `x`, it can eliminate either the then or the else case
  - ♦ $(x > 0) \Rightarrow$ y is 17 in B_3
  - ♦ $(x > 0) \Rightarrow B_2$ is unreachable
- This approach combines constant propagation with **CFG** reachability analysis to produce better results in each
- Example of Click's notion of *"combining optimizations"*
  - ♦ Predated & motivated Click

Aside on Combining Optimizations

Sometimes, combining two optimizations can produce solutions that cannot be obtained by solving them independently.

- Requires bilateral interactions between optimizations
  - ♦ C. Click and K.D. Cooper, "Combining Analyses, Combining Optimizations", TOPLAS 17(2), March 1995 [86]

Sparse Conditional Constant Propagation is an example

- Combines constant propagation and unreachable code elimination
- Achieves results that no combination of the two can reach independently
- In the paper, they also suggest combining inline substitution
  - ♦ While that idea is nice, it does not achieve the kind of same synergy
  - ♦ Inlining followed by SCCP would achieve the same results

Interdependence versus
a phase ordering problem

Sparse Constant Propagation

To work simplification of conditionals into the algorithm, requires several modifications:

- Use two worklists:
  - ♦ SSAWorkList
    - → Holds edges in the **SSA** graph
    - → **SSA** worklist propagates changing values
  - ♦ CFGWorkList
    - → Holds edges in the control-flow graph
    - → **CFG** worklist propagates information on reachability
- Do not evaluate operations until block is reachable
- When algorithm marks a block as reachable, must evaluate all operations in the block and propagate their effects forward

The statement of this algorithm in EaC1e is mangled. It is fixed in EaC2e.

Sparse Conditional Constant Propagation

```
SSAWorkList ← Ø
CFGWorkList ← n0
∀ block b
    clear b's mark
    ∀ operation o in b
        Value(o) ← TOP
```

Initialization Step

To evaluate a branch

```
if arg is BOT then
    put both targets on CFGWorklist
else if arg is TRUE then
    put TRUE target on CFGWorkList
else if arg is FALSE then
    put FALSE target on CFGWorkList
```

To evaluate a jump

```
place its target on CFGWorkList
```

```
while ((CFGWorkList ∪ SSAWorkList) ≠ Ø)
    while(CFGWorkList ≠ Ø)
        remove b from CFGWorkList
        mark b
        evaluate each Ø-function in b
        evaluate each op o in b, in order
            ∀ SSA edge <o,x>
                if block(x) is marked
                    add <o,x> to SSAWorklist
    while(SSAWorkList ≠ Ø)
        remove s = <u,v> from WorkList
        let o be the operation that contains v
        t ← result of evaluating o
        if t ≠ Value(o) then
            Value(o) ← t
            ∀ SSA edge <o,x>
                if block(x) is marked, then
                    add <o,x> to SSAWorkList
```

Propagation Step

Sparse Conditional Constant Propagation

There are some subtle points

- Branch conditions should not be **TOP** when evaluated
  - ♦ Indicates an upwards-exposed use (*no initial value*)
  - ♦ Hard to envision compiler producing such code
- Initialize Value attribute for each operation to **TOP**
  - ♦ Block processing will fill in the non-top initial values
  - ♦ Unreachable paths contribute **TOP** to Ø-functions
- Code shows **CFG** edges first, then SSA edges
  - ♦ Can intermix them in arbitrary order (*correctness*)
  - ♦ Taking **CFG** edges first may help with speed (*minor effect*)

Sparse Conditional Constant Propagation

More subtle points

- **TOP** * **BOT** → **TOP**
  - ♦ If **TOP** becomes 0, then 0 * **BOT** → 0
  - ♦ This prevents non-monotonic behavior for the result value
  - ♦ Uses of the result value might go irretrievably to **BOT**
  - ♦ Similar effects with any operation that has a "zero"

- Some values reveal simplifications, rather than constants
  - ♦ **BOT** * c_i → **BOT**, but might turn into shifts & adds (c_i = 2, **BOT** ≥ 0)
    - → ***Multiply to shift removes commutativity*** (*reassociation*)
  - ♦ **BOT****2 → **BOT** * **BOT** (*vs. series or call to library*)

- cbr **TRUE** → L_1,L_2 becomes br → L_1
  - ♦ Method discovers this; it must rewrite the code, too!

Sparse Conditional Constant

Unreachable Code

17	$i \leftarrow 17$	
	if ($i > 0$) then	
10	$j_1 \leftarrow 10$	
	else	
20	$j_2 \leftarrow 20$	
$\perp$	$j_3 \leftarrow \emptyset(j_1, j_2)$	
$\perp$	$k \leftarrow j_3 * 17$	

Assume that all paths execute

Optimism

- Initialization to **TOP** is still important
- Unreachable code keeps **TOP**
- $\wedge$ with **TOP** has desired result

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Sparse Conditional Constant

Unreachable Code

17	$i \leftarrow 17$
	if ($i > 0$) then
TOP	$\quad j_1 \leftarrow 10$
	else
TOP	$\quad j_2 \leftarrow 20$
TOP	$j_3 \leftarrow Ø(j_1, j_2)$
	$k \leftarrow j_3 * 17$

Initial values in SCC

Optimism

- Initialization to **TOP** is still important
- Unreachable code keeps **TOP**
- $\wedge$ with **TOP** has desired result

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Sparse Conditional Constant

Unreachable Code

17	$i \leftarrow 17$
17	if ($i > 0$) then
10	$j_1 \leftarrow 10$
	~~else~~
TOP	~~$j_2 \leftarrow 20$~~
10	$j_3 \leftarrow \varnothing(j_1, j_2)$
170	$k \leftarrow j_3 * 17$

After propagation

Optimism

- Initialization to **TOP** is still important
- Unreachable code keeps **TOP**
- $\wedge$ with **TOP** has desired result

Sparse Conditional Constant

Unreachable Code

17	$i \leftarrow 17$
17	if ($i > 0$) then
10	$\quad j_1 \leftarrow 10$
	~~else~~
TOP	$\quad$ ~~$j_2 \leftarrow 20$~~
10	$j_3 \leftarrow Ø(j_1, j_2)$
170	$k \leftarrow j_3 * 17$

Optimism

- Initialization to **TOP** is still important
- Unreachable code keeps **TOP**
- $\wedge$ with **TOP** has desired result

Cannot get this result with separate transformations

- DEAD cannot test ($i > 0$)
- DEAD marks j_2 as useful

In general, combining two optimizations can lead to answers that cannot be produced by any combination of running them separately.

This algorithm is one example of that general principle.

Combining allocation & scheduling is another ...

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Sparse Conditional Constant Propagation

And one more thing …

- Wegman and Zadeck proposed integrating inline substitution into **SCCP**
- They were aware of the difficulty of the decision problem for inlining
  - ♦ The "einey, meiney, miney, moe" problem

They proposed a simple solution:

*Inline during **SCCP** when known constants propagate into a call site*

- Constant-valued parameters & globals are one important source of improvement with inline substituion (see Ball [31])
- Compiler might inline for analysis and undo transformation if it did not find significant opportunities for simplification — constant folding, loop invariant code motion, redundancy expression

I know of no experimental evaluation of this idea.