



COMP 512
Rice University
Spring 2015

Lazy Code Motion

— *The Data-Flow Approach to Code Motion* —

J. Knoop, O. Ruthing, & B. Steffen, “Lazy Code Motion”, in Proceedings of the ACM SIGPLAN 92 Conference on Programming Language Design and Implementation, June 1992. [\[225\]](#)

K. Drechsler & M. Stadel, “A Variation of Knoop, Ruthing, and Steffen’s Lazy Code Motion,” SIGPLAN Notices, 28(5), May 1993. [\[134\]](#)

§ 10.3.1 of EaC2e

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[Citation numbers refer to entries in the EaC2e bibliography.](#)

Announcements



- Next week is break
- Midterm exam
 - ◆ Available today, in class
 - ◆ Due back on Tuesday 3/10/2015 at 5 **PM**
 - ◆ Three questions:
 - Matching question through today's lecture
 - Question on data-flow analysis
 - Question on the construction of **SSA** form
 - ◆ Two-hours, closed-notes, closed-literature, take-home exam
- You should be working on your labs
 - ◆ Three benchmarks available on **CLEAR**
 - ◆ More to come

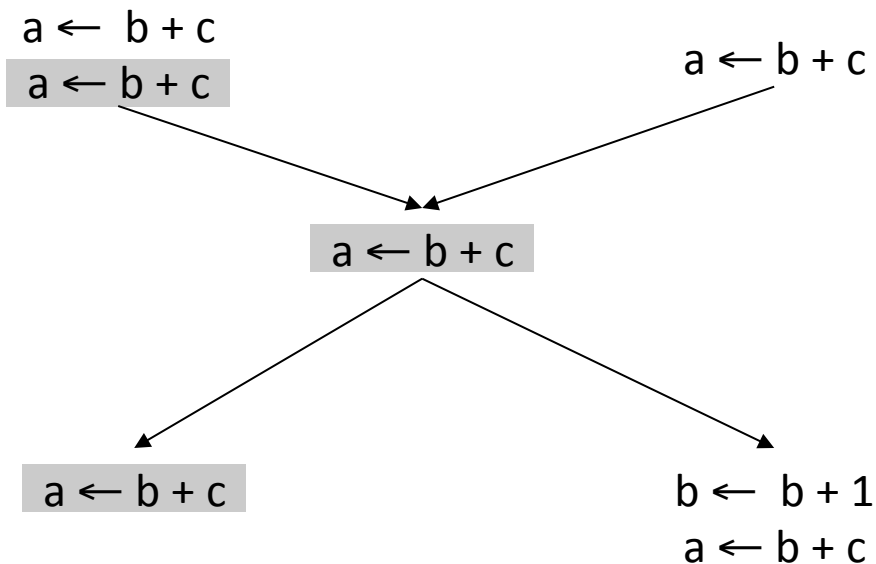


Redundant Expression

An expression is redundant at point p if, on every path to p

1. It is evaluated before reaching p , and
2. Non of its constituent values is redefined before p

Example



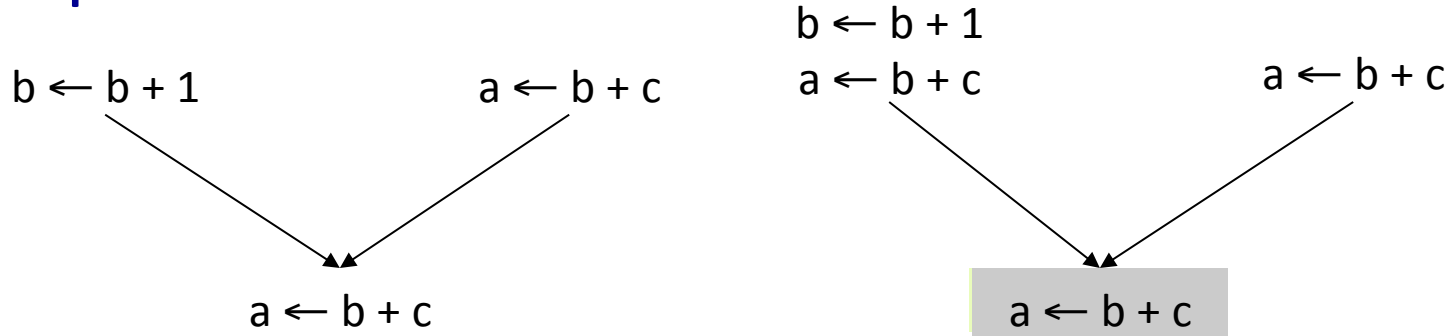
Some occurrences of $b+c$ are redundant



Partially Redundant Expression

An expression is partially redundant at p if it is redundant along some, but not all, paths reaching p

Example



Inserting a copy of “ $a \leftarrow b + c$ ” after the definition of b can make it redundant

fully redundant?



Loop Invariant Expression

Another Example



Loop invariant expressions are partially redundant

- Partial redundancy elimination performs code motion
- Major part of the work is figuring out where to insert operations



Lazy Code Motion

The Concept

- Solve data-flow problems that show opportunities & limits
 - ◆ Availability & anticipability, then placement
- Compute **INSERT** & **DELETE** sets from solutions
- Linear pass over the code to rewrite it (using **INSERT** & **DELETE**)

The History

- Partial redundancy elimination (**PRE**) [267] (*Morel & Renvoise, CACM, 1979*)
- Improvements by Drechsler & Stadel [133], Joshi & Dhamdhere [209], Chow [81], Knoop, Ruthing & Steffen [225], Dhamdhere [130], Sorkin [321], Hailperin [178], Kennedy, Lo, et al. [220] ...
- All versions of **PRE** optimize placement
 - ◆ Guarantee that no path is lengthened
- **LCM** was published by Knoop et al. in **PLDI 92**
- Drechsler & Stadel simplified the equations

PRE and its descendants are conservative

Dhamdhere applied these same ideas to strength reduction [127, 131] and hoisting [129]. Others have followed this path, as well [209, 220, 226].



Lazy Code Motion

The Intuitions

- Compute *available expressions*
- Compute *anticipable expressions*
- From **AVAIL** & **ANT**, we can compute an earliest placement for each expression
- Push expressions down the **CFG** until it changes behavior

LCM operates on expressions
It moves expression evaluations, not assignments

Assumptions

- Uses a lexical notion of identity (*not value identity*)
- ILOC-style code with unlimited name space
- Consistent, disciplined use of names
 - ◆ Identical expressions define the same name
 - ◆ No other expression defines that name

Avoids copies
Result name serves as proxy

LCM operates on code that is *not* in **SSA** form.
Lexical identity conflicts with **SSA**'s notion of unique names.

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Digression in Chapter 5 of EAC2e:
"The impact of naming"



The Name Space

- $r_i + r_j \rightarrow r_k$, always, with both $i < k$ and $j < k$ *(hash to find k)*
 - ◆ r_k is always set by $r_i + r_j$ or $r_j + r_i$, and by no other expression
- We can refer to $r_i + r_j$ by r_k *(bit-vector sets)*
- Variables must be set by copies
 - ◆ No consistent definition for a variable
 - ◆ Break the rule for this case, but require $r_{source} < r_{destination}$
 - ◆ To achieve this, assign register names to variables first

Without this name space

- **LCM** must insert copies to preserve redundant values
- **LCM** must compute its own map of expressions to unique ids

The restrictions on the name space in **LCM** goes all the way back to Morel & Renvoise [267]. It is mentioned as an assumption in the original paper.

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Local Information

(Computed for each block)

- **DEEXPR(b)** contains expressions defined in b that survive to the end of b

(downward exposed expressions)

$e \in \text{DEEXPR}(b) \Rightarrow$ evaluating e at the end of b produces the same value for e

- **UEEXPR(b)** contains expressions defined in b that have upward exposed arguments (both args)

(upward exposed expressions)

$e \in \text{UEEXPR}(b) \Rightarrow$ evaluating e at the start of b produces the same value for e

- **EXPRKILL(b)** contains those expressions that have one or more arguments defined (*killed*) in b

(killed expressions)

$e \notin \text{EXPRKILL}(b) \Rightarrow$ evaluating e produces the same result at the start and end of b

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Availability

$$\mathbf{AVAILIN}(n) = \bigcap_{m \in \text{preds}(n)} \mathbf{AVAILOUT}(m), \quad n \neq n_0$$

$$\mathbf{AVAILOUT}(m) = \mathbf{DEEXPR}(m) \cup (\mathbf{AVAILIN}(m) \cap \overline{\mathbf{EXPRKILL}(m)})$$

Initialize $\mathbf{AVAILIN}(n)$ to the set of all names, except at n_0

Set $\mathbf{AVAILIN}(n_0)$ to \emptyset

Interpreting AVAILOUT

- $e \in \mathbf{AVAILOUT}(b) \Leftrightarrow$ evaluating e at end of b produces the same value for e .
 $\mathbf{AVAILOUT}$ tells the compiler how far forward e can move
- This interpretation differs from the way we *talk* about $\mathbf{AVAILOUT}$ in global redundancy elimination; the equations, however, are unchanged.

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Anticipability is identical to VeryBusy expressions



Anticipability

$$\mathbf{ANTOUT}(n) = \bigcap_{m \in \text{succs}(n)} \mathbf{ANTIN}(m), \quad n \text{ not an exit block}$$

$$\mathbf{ANTIN}(m) = \mathbf{UEEXPR}(m) \cup (\mathbf{ANTOUT}(m) \cap \overline{\mathbf{EXPRKILL}(m)})$$

Initialize $\mathbf{ANTOUT}(n)$ to the set of all names, except at exit blocks

Set $\mathbf{ANTOUT}(n)$ to \emptyset , for each exit block n

Interpreting \mathbf{ANTOUT}

- $e \in \mathbf{ANTIN}(b) \Leftrightarrow$ evaluating e at start of b produces the same value for e .
 \mathbf{ANTIN} tells the compiler how far backward e can move
- This view shows that anticipability is, in some sense, the inverse of availability (& explains the new interpretation of \mathbf{AVAIL})



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The Intuitions

Available expressions

- $e \in \mathbf{AVAILOUT}(b) \Rightarrow$ evaluating e at exit of b gives same result
- $e \in \mathbf{AVAILIN}(b) \Rightarrow e$ is available from every predecessor of b
 \Rightarrow an evaluation at entry of b is redundant

Anticipable expressions

- $e \in \mathbf{ANTIN}(b) \Rightarrow$ evaluating e at entry of b gives same result
- $e \in \mathbf{ANTOUT}(b) \Rightarrow e$ is anticipable from every successor of b
 \Rightarrow evaluation at exit of b would a later evaluation redundant, on every path, so exit of b is a profitable place to insert e

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Earliest Placement On An Edge

$$\text{EARLIEST}(i,j) = \text{ANTIN}(j) \cap \overline{\text{AVAILOUT}(i)} \cap \overline{(\text{EXPRKILL}(i) \cup \text{ANTOUT}(i))}$$

$$\text{EARLIEST}(n_0,j) = \text{ANTIN}(j) \cap \overline{\text{AVAILOUT}(n_0)}$$

Can move e to head of j & it is not redundant from i and

Either killed in i or would not be busy at exit of i

\Rightarrow insert e on the edge

EARLIEST is a predicate

- Computed for edges rather than nodes

(*placement*)

- $e \in \text{EARLIEST}(i,j)$ if

- ◆ It can move to head of j ,

(**ANTIN(j)**)

- ◆ It is not available at the end of i and

(**AVAILOUT(i)**)

- ◆ either it cannot move to the head of i or another edge leaving i prevents its placement in i

(**ANTOUT(i)**)

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Later (than earliest) Placement

$$\mathbf{LATERIN}(j) = \bigcap_{i \in \text{pred}(j)} \mathbf{LATER}(i,j), \quad j \neq n_0$$

$$\mathbf{LATER}(i,j) = \mathbf{EARLIEST}(i,j) \cup (\mathbf{LATERIN}(i) \cap \overline{\mathbf{UEEXPR}(i)})$$

Initialize $\mathbf{LATERIN}(n_0)$ to \emptyset

$x \in \mathbf{LATERIN}(k) \Leftrightarrow$ every path that reaches k has $x \in \mathbf{EARLIEST}(i,j)$ for some edge (i,j) leading to x , and the path from the entry of j to k is x -clear & does not evaluate x

\Rightarrow the compiler can move x through k without losing any benefit

$x \in \mathbf{LATER}(i,j) \Leftrightarrow \langle i,j \rangle$ is its earliest placement, or it can be moved forward from i ($\mathbf{LATER}(i)$) and placement at entry to i does not anticipate a use in i (*moving it across the edge exposes that use*)

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Rewriting the code

$$\text{INSERT}(i,j) = \text{LATER}(i,j) \cap \overline{\text{LATERIN}(j)}$$

Can go on the edge but not in j
⇒ no later placement

$$\text{DELETE}(k) = \text{UEEXPR}(k) \cap \overline{\text{LATERIN}(k)}, k \neq n_0$$

Upward exposed (so we will cover it) & not an evaluation that might be used later

INSERT & **DELETE** are predicates

Compiler uses them to guide the rewrite step

- $x \in \text{INSERT}(i,j) \Rightarrow$ insert x at start of j , end of i , or new block
- $x \in \text{DELETE}(k) \Rightarrow$ delete first evaluation of x in k ¹

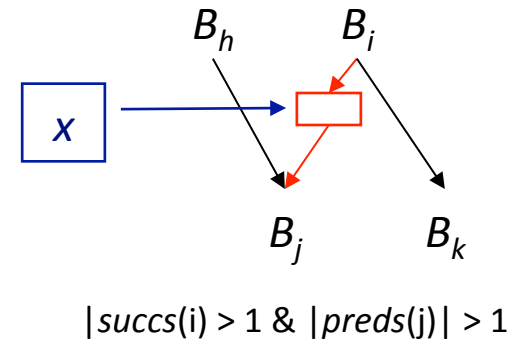
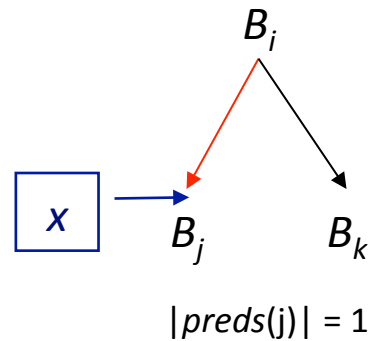
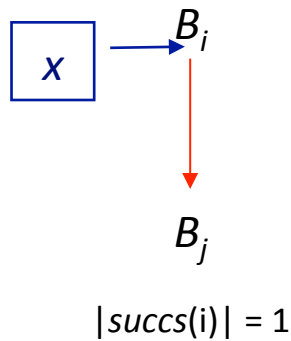
¹ If local redundancy elimination has already been performed, only one copy of x exists. Otherwise, remove all upward exposed copies of x .



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Edge placement

- $x \in \text{INSERT}(i,j)$



A “critical” edge

Three cases

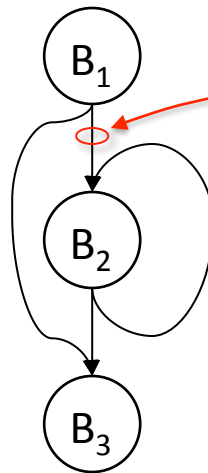
- $|succs(i)| = 1 \Rightarrow$ insert x at end of i
- $|succs(i)| > 1$, but $|preds(j)| = 1 \Rightarrow$ insert x at start of j
- $|succs(i)| > 1$, & $|preds(j)| > 1 \Rightarrow$ create new block in $\langle i,j \rangle$ for x



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Example

B_1 : $r_1 \leftarrow 1$
 $r_2 \leftarrow r_0 + @m$
 if $r_1 < r_2 \rightarrow B_2, B_3$
 B_2 : ...
 $r_{20} \leftarrow r_{17} * r_{18}$
 ...
 $r_4 \leftarrow r_1 + 1$
 $r_1 \leftarrow r_4$
 if $r_1 < r_2 \rightarrow B_2, B_3$
 B_3 : ...



	B1	B2
DEEXPR	r1,r2	r1,r4,r20
UEEXPR	r1,r2	r4,r20
NotKilled	r17,r18,r20	r2,r17,r18,r20

	B1	B2
AVAILIN	r17,r18	r1,r2,r17,r18
AVAILOUT	r1,r2,r17,r18	r1,r2,r4,r17,r18,r20
ANTIN	{}	r20
ANTOUT	{}	{}

	1,2	1,3	2,2	2,3
EARLIEST	r20	{}	{}	{}

Critical edge rule will create landing pad when needed, as on edge (B₁,B₂)

Example is too small to show off **LATER**

INSERT(1,2) = { r₂₀ }

DELETE(2) = { r₂₀ }

See the papers for more detailed examples.

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Improving the Results

Simpson attacked the problem of **LCM**'s reliance on lexical identity

- Performed global value numbering, then rewrote the name space to encode value identity into lexical identity
- In essence, his technique joined the code placement aspects of **LCM** with the value-based equivalence detection of global value numbering

Briggs rearranged expressions to expose more lexical identities

- Used algebraic reassociation to rewrite expressions into a canonical form
 - ◆ Associativity & commutativity, + distribution in some limited forms
- Preconditioning the code with reassociation exposed more opportunities