COMP 512
Rice University
Spring 2013

THE PARTITIONING ALGORITHM FOR DETECTING CONGRUENT EXPRESSIONS

YET ANOTHER WAY TO ACHIEVE REDUNDANCY ELIMINATION

Background

This algorithm discovers *congruent expressions*, which should have the same value at runtime. Sets of congruent expressions can serve as the basis for a powerful form of redundancy elimination.

Critical points:

- The Partitioning Method
  - ♦ Using a well-known algorithm for a new purpose
  - ♦ Classic example of an optimistic algorithm
  - ♦ Performs analysis not transformation
- Compiler must still rewrite the code
  - ♦ Paper suggests a method based on dominance
  - ♦ Better options are available

We want to cover this technique, in part, as a prelude to the lecture on algebraic reassociation of expressions.

PARTITIONING METHOD

Key idea

Operations *x* and *y* are congruent *iff* they have the same operator & their operands are congruent

Congruence is related to redundancy, but not the same

- ♦ *Redundancy implicitly includes a notion that* x *and* y *share an execution path*
- ♦ *Congruence defers that issue for later consideration*

Using the idea

- Start with SSA form (*we need the name space*)
- Partition the set of all expressions into congruence classes
  - ♦ Find largest possible sets
- Each class needs one (*or more*) representative computation
- Other uses can be replaced with the representer

Alpern, Wegman, & Zadeck, POPL 1988

PARTITIONING METHOD

The algorithm

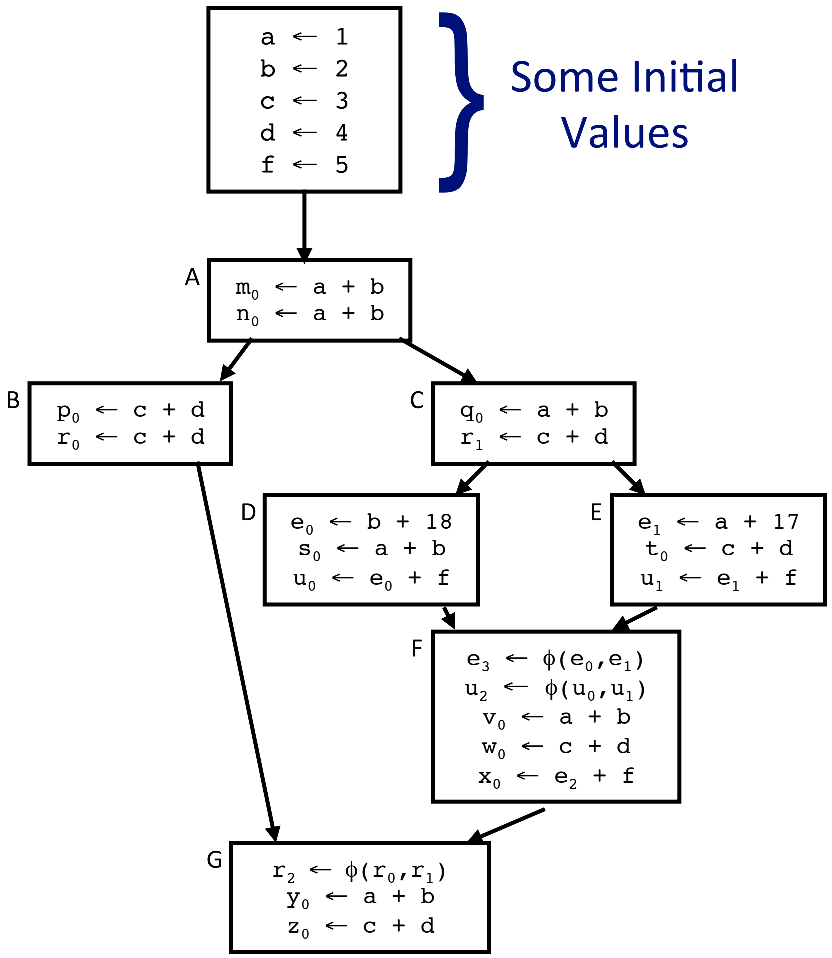
```
1. Partition operations into initial classes by opcode
           *Treat constant values as unique opcodes*
2. *Worklist* ← all classes
3. While *Worklist* is not empty
   A. Remove a class $c$ from *Worklist*
   B. For each class $s$ that uses some $x$ defined in $c$
      i. Split $s$ into $s_1$ and $s_2$ around uses of $c$
      ii. Remove $s$ from *Worklist* & add the smaller of $s_1$ and $s_2$
4. Pick a representer for each class & rewrite the code to replace
   other uses of the class with representer
```

Based on Hopcroft's algorithm for DFA minimization

- ♦ *A widely-used, often re-invented algorithm*

There is a reason that COMP 412 is a prerequisite for this course!

Partitioning Method



$a \leftarrow 1$
$b \leftarrow 2$
$c \leftarrow 3$
$d \leftarrow 4$
$f \leftarrow 5$

Some Initial Values

A
$m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$

B
$p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$

C
$q_0 \leftarrow a + b$
$r_1 \leftarrow c + d$

D
$e_0 \leftarrow b + 18$
$s_0 \leftarrow a + b$
$u_0 \leftarrow e_0 + f$

E
$e_1 \leftarrow a + 17$
$t_0 \leftarrow c + d$
$u_1 \leftarrow e_1 + f$

F
$e_3 \leftarrow \phi(e_0,e_1)$
$u_2 \leftarrow \phi(u_0,u_1)$
$v_0 \leftarrow a + b$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$

G
$r_2 \leftarrow \phi(r_0,r_1)$
$y_0 \leftarrow a + b$
$z_0 \leftarrow c + d$

All Operations

$m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$
$p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$
$q_0 \leftarrow a + b$
$r_1 \leftarrow c + d$
$e_0 \leftarrow b + 18$
$s_0 \leftarrow a + b$
$u_0 \leftarrow e_0 + f$
$e_1 \leftarrow a + 17$
$t_0 \leftarrow c + d$
$u_1 \leftarrow e_1 + f$
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$u_2 \leftarrow \phi(u_0,u_1)$
$v_0 \leftarrow a + b$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$
$r_2 \leftarrow \phi(r_0,r_1)$
$y_0 \leftarrow a + b$
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Partitioning Method

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$u_0 \leftarrow e_0 + f$
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$v_0 \leftarrow a + b$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$
$r_2 \leftarrow \phi(r_0,r_1)$
$y_0 \leftarrow a + b$
$z_0 \leftarrow c + d$
$a \leftarrow 1$
$b \leftarrow 2$
$c \leftarrow 3$
$d \leftarrow 4$
$f \leftarrow 5$

Compute → Initial Partition

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

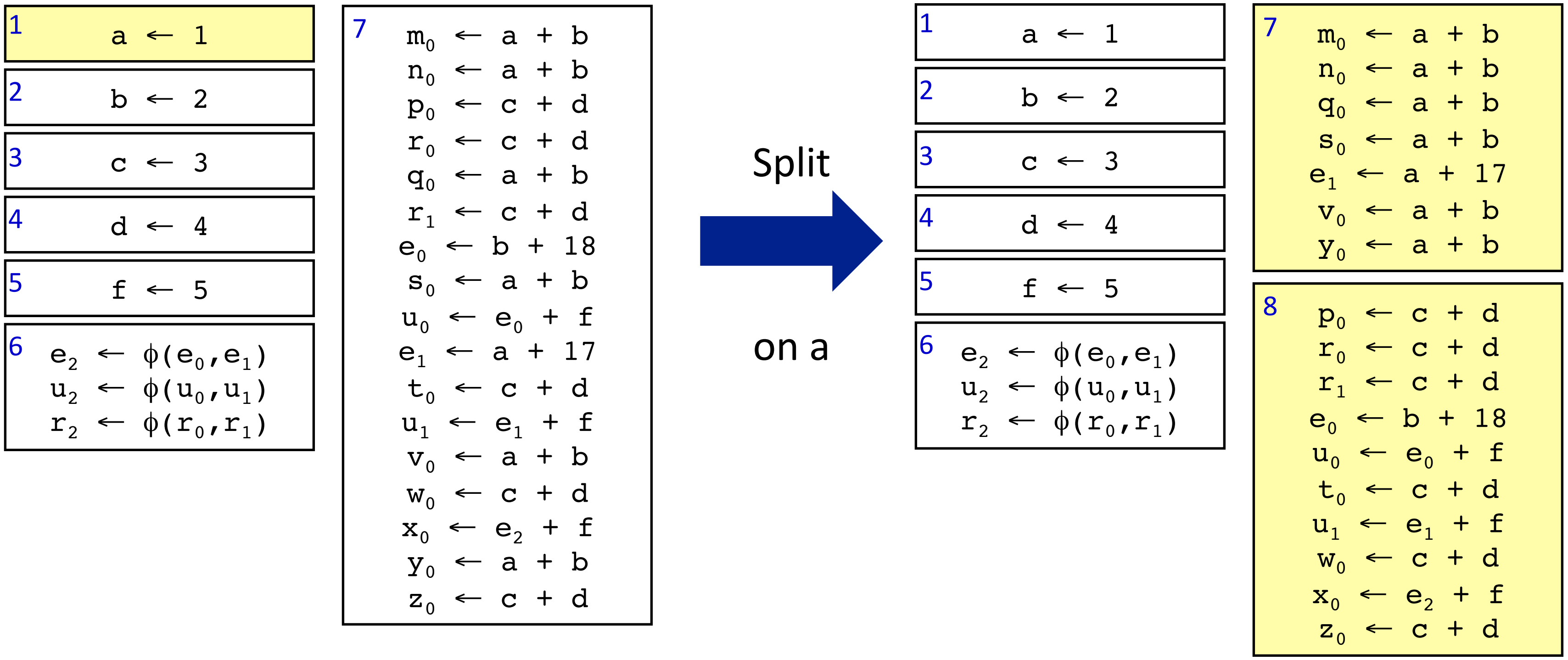
5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$
$u_2 \leftarrow \phi(u_0,u_1)$
$r_2 \leftarrow \phi(r_0,r_1)$

7 $m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$
$p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$
$q_0 \leftarrow a + b$
$r_1 \leftarrow c + d$
$e_0 \leftarrow b + 18$
$s_0 \leftarrow a + b$
$u_0 \leftarrow e_0 + f$
$e_1 \leftarrow a + 17$
$t_0 \leftarrow c + d$
$u_1 \leftarrow e_1 + f$
$v_0 \leftarrow a + b$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$
$y_0 \leftarrow a + b$
$z_0 \leftarrow c + d$

Worklist
1, 2, 3, 4, 5, 6, 7

Partitioning Method



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$t_0 \leftarrow c + d$
$u_1 \leftarrow e_1 + f$
$v_0 \leftarrow a + b$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$
$y_0 \leftarrow a + b$
$z_0 \leftarrow c + d$

Split

on a

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6
$e_2 \leftarrow \phi(e_0,e_1)$
$u_2 \leftarrow \phi(u_0,u_1)$
$r_2 \leftarrow \phi(r_0,r_1)$

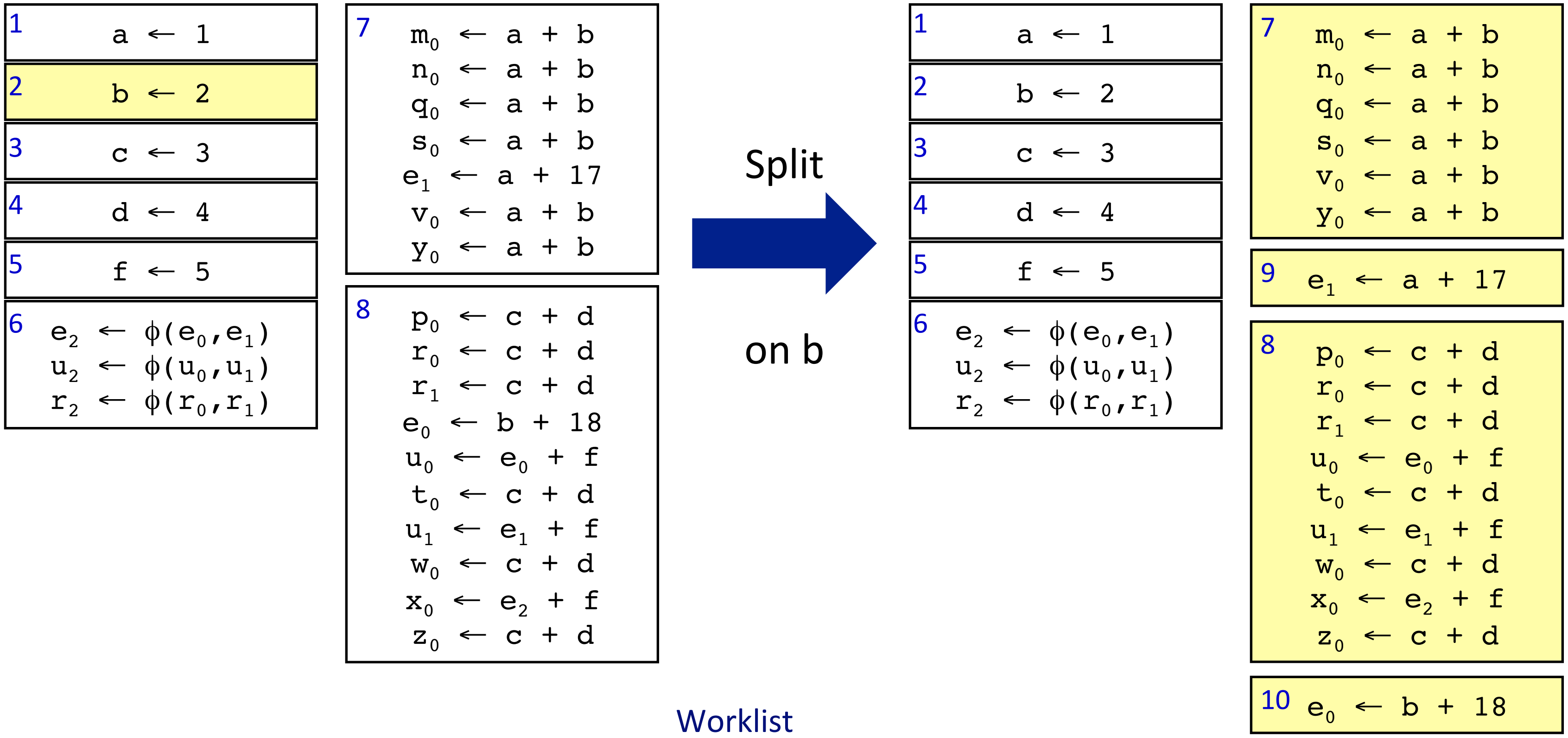
7
$m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$
$q_0 \leftarrow a + b$
$s_0 \leftarrow a + b$
$e_1 \leftarrow a + 17$
$v_0 \leftarrow a + b$
$y_0 \leftarrow a + b$

8
$p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$e_0 \leftarrow b + 18$
$u_0 \leftarrow e_0 + f$
$t_0 \leftarrow c + d$
$u_1 \leftarrow e_1 + f$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$
$z_0 \leftarrow c + d$

Worklist:
2, 3, 4, 5, 6, 7

-1, -7, +7

Partitioning Method



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5 $f \leftarrow 5$

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$n_0 \leftarrow a + b$
$q_0 \leftarrow a + b$
$s_0 \leftarrow a + b$
$e_1 \leftarrow a + 17$
$v_0 \leftarrow a + b$
$y_0 \leftarrow a + b$

8 $p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$e_0 \leftarrow b + 18$
$u_0 \leftarrow e_0 + f$
$t_0 \leftarrow c + d$
$u_1 \leftarrow e_1 + f$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$
$z_0 \leftarrow c + d$

Split

on b

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$
$u_2 \leftarrow \phi(u_0,u_1)$
$r_2 \leftarrow \phi(r_0,r_1)$

7 $m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$
$q_0 \leftarrow a + b$
$s_0 \leftarrow a + b$
$v_0 \leftarrow a + b$
$y_0 \leftarrow a + b$

9 $e_1 \leftarrow a + 17$

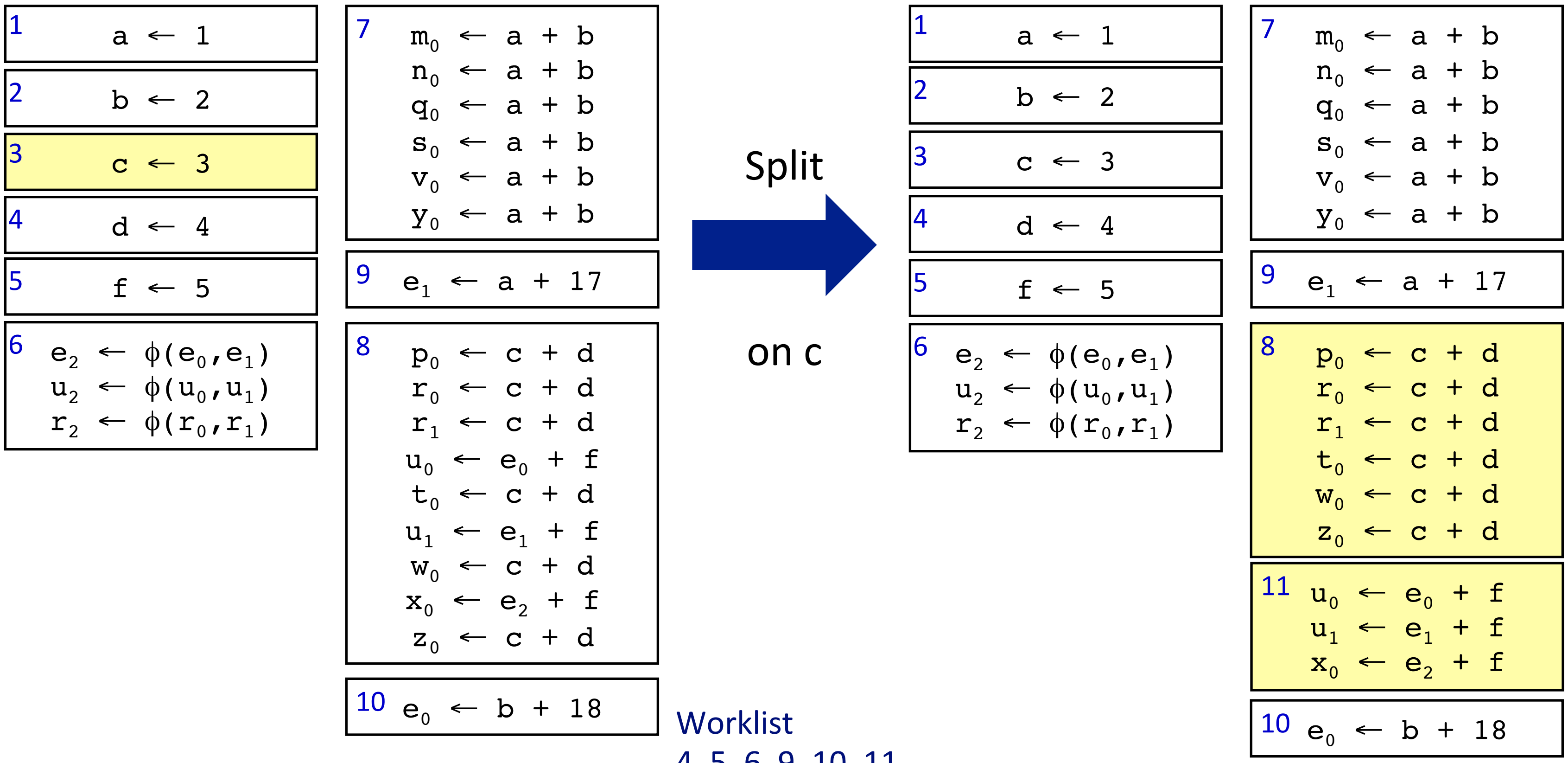
8 $p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$u_0 \leftarrow e_0 + f$
$t_0 \leftarrow c + d$
$u_1 \leftarrow e_1 + f$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$
$z_0 \leftarrow c + d$

10 $e_0 \leftarrow b + 18$

Worklist
3, 4, 5, 6, 9, 10

-2, -7, +9, -8, +10

Partitioning Method



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$v_0 \leftarrow a + b$
$y_0 \leftarrow a + b$

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$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$u_0 \leftarrow e_0 + f$
$t_0 \leftarrow c + d$
$u_1 \leftarrow e_1 + f$
$w_0 \leftarrow c + d$
$x_0 \leftarrow e_2 + f$
$z_0 \leftarrow c + d$

10 $e_0 \leftarrow b + 18$

Split

on c

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$
$u_2 \leftarrow \phi(u_0,u_1)$
$r_2 \leftarrow \phi(r_0,r_1)$

7 $m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$
$q_0 \leftarrow a + b$
$s_0 \leftarrow a + b$
$v_0 \leftarrow a + b$
$y_0 \leftarrow a + b$

9 $e_1 \leftarrow a + 17$

8 $p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$t_0 \leftarrow c + d$
$w_0 \leftarrow c + d$
$z_0 \leftarrow c + d$

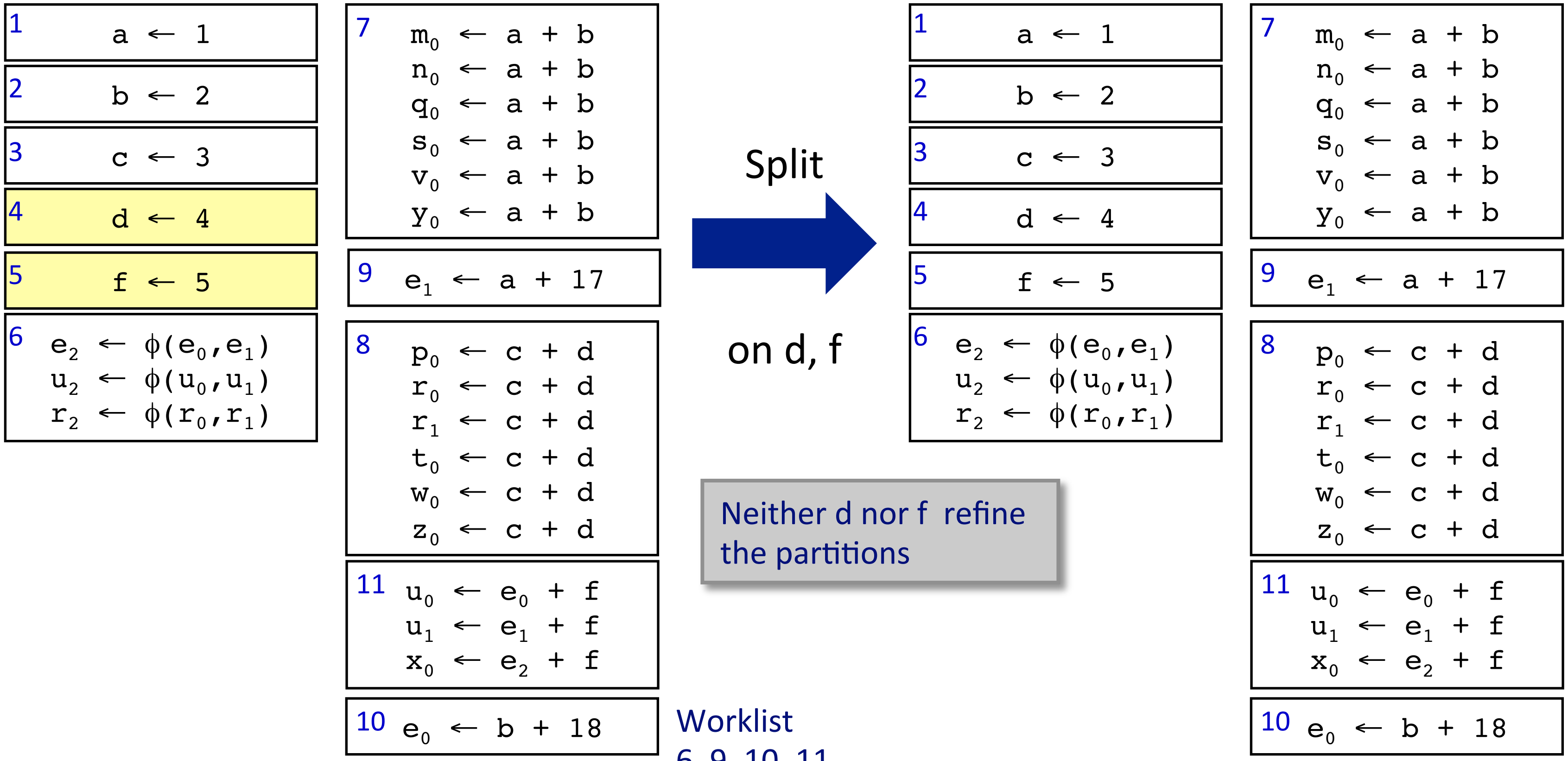
11 $u_0 \leftarrow e_0 + f$
$u_1 \leftarrow e_1 + f$
$x_0 \leftarrow e_2 + f$

10 $e_0 \leftarrow b + 18$

Worklist
4, 5, 6, 9, 10, 11

-3, -8, +11

Partitioning Method



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9 $e_1 \leftarrow a + 17$

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$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$t_0 \leftarrow c + d$
$w_0 \leftarrow c + d$
$z_0 \leftarrow c + d$

11 $u_0 \leftarrow e_0 + f$
$u_1 \leftarrow e_1 + f$
$x_0 \leftarrow e_2 + f$

10 $e_0 \leftarrow b + 18$

Split

on d, f

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$
$u_2 \leftarrow \phi(u_0,u_1)$
$r_2 \leftarrow \phi(r_0,r_1)$

7 $m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$
$q_0 \leftarrow a + b$
$s_0 \leftarrow a + b$
$v_0 \leftarrow a + b$
$y_0 \leftarrow a + b$

9 $e_1 \leftarrow a + 17$

8 $p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$t_0 \leftarrow c + d$
$w_0 \leftarrow c + d$
$z_0 \leftarrow c + d$

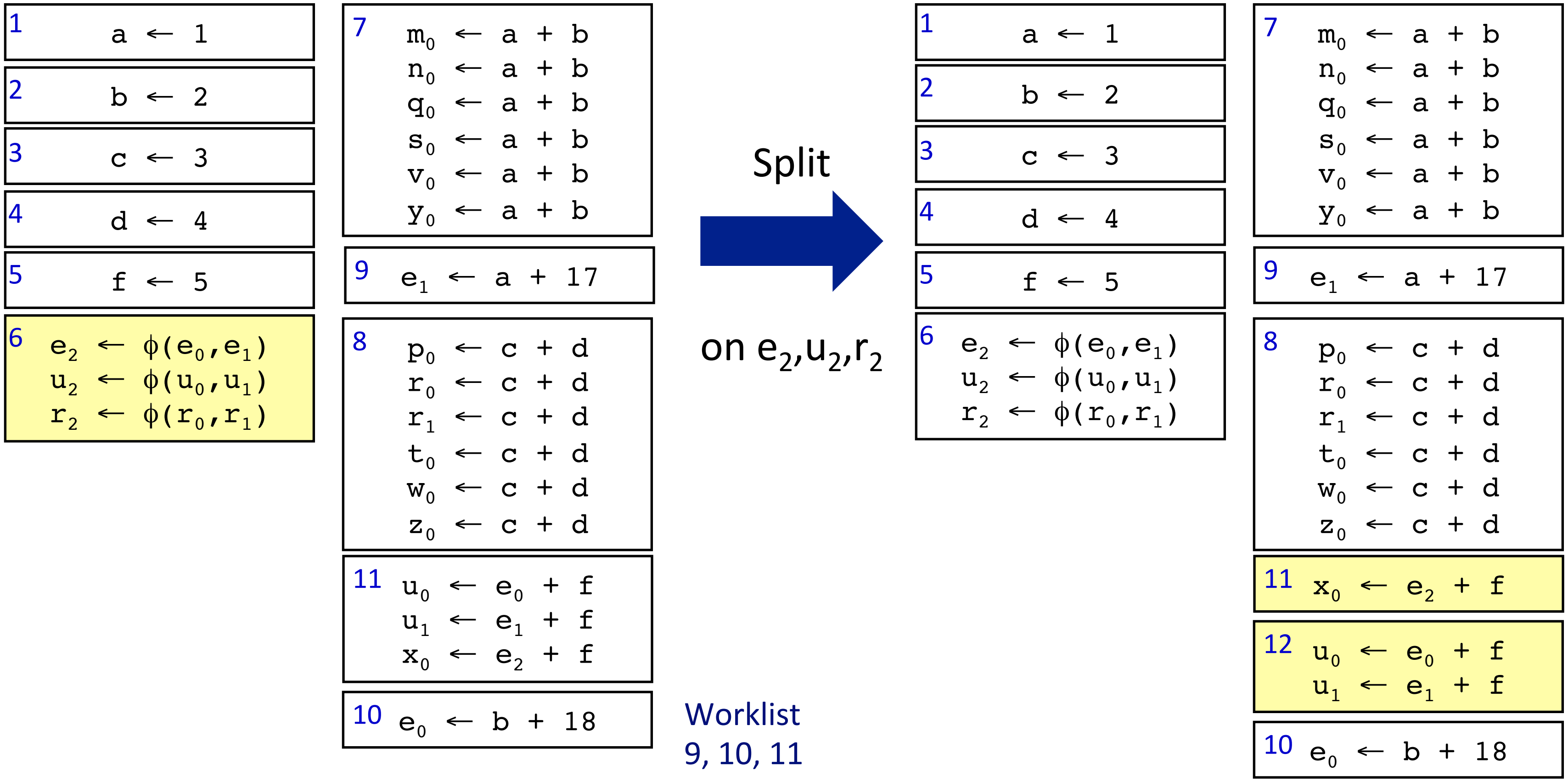
11 $u_0 \leftarrow e_0 + f$
$u_1 \leftarrow e_1 + f$
$x_0 \leftarrow e_2 + f$

10 $e_0 \leftarrow b + 18$

Neither d nor f refine the partitions

Worklist
6, 9, 10, 11

Partitioning Method



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$s_0 \leftarrow a + b$
$v_0 \leftarrow a + b$
$y_0 \leftarrow a + b$

9 $e_1 \leftarrow a + 17$

8 $p_0 \leftarrow c + d$
$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$t_0 \leftarrow c + d$
$w_0 \leftarrow c + d$
$z_0 \leftarrow c + d$

11 $u_0 \leftarrow e_0 + f$
$u_1 \leftarrow e_1 + f$
$x_0 \leftarrow e_2 + f$

10 $e_0 \leftarrow b + 18$

Split

on e_2,u_2,r_2

Worklist
9, 10, 11

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

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$r_2 \leftarrow \phi(r_0,r_1)$

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$v_0 \leftarrow a + b$
$y_0 \leftarrow a + b$

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$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$t_0 \leftarrow c + d$
$w_0 \leftarrow c + d$
$z_0 \leftarrow c + d$

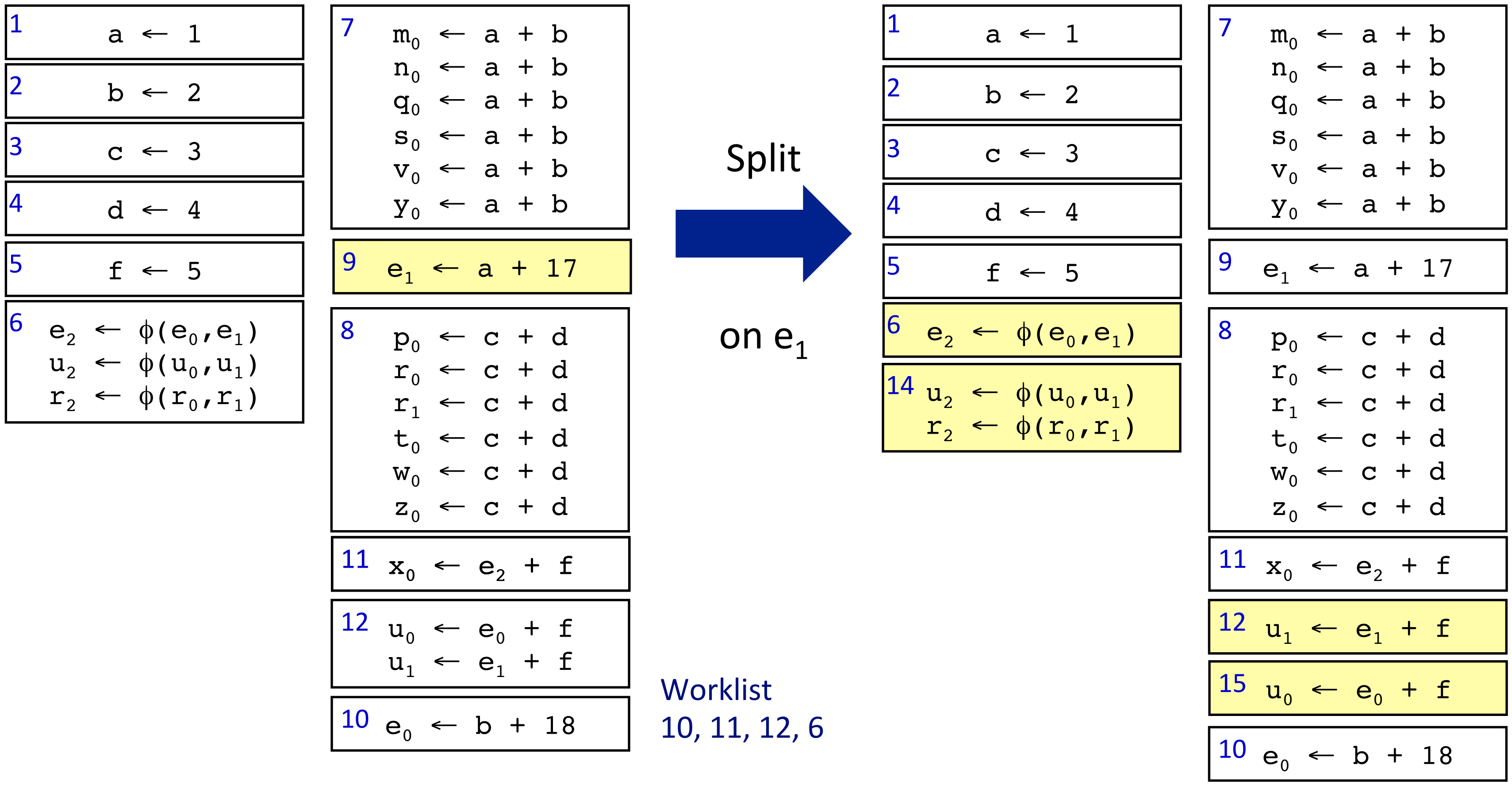
11 $x_0 \leftarrow e_2 + f$

12 $u_0 \leftarrow e_0 + f$
$u_1 \leftarrow e_1 + f$

10 $e_0 \leftarrow b + 18$

-6, -11, + 11

PARTITIONING METHOD



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$z_0 \leftarrow c + d$

11 $x_0 \leftarrow e_2 + f$

12 $u_0 \leftarrow e_0 + f$
$u_1 \leftarrow e_1 + f$

10 $e_0 \leftarrow b + 18$

Split

on e_1

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$

14 $u_2 \leftarrow \phi(u_0,u_1)$
$r_2 \leftarrow \phi(r_0,r_1)$

7 $m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$
$q_0 \leftarrow a + b$
$s_0 \leftarrow a + b$
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$y_0 \leftarrow a + b$

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$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$t_0 \leftarrow c + d$
$w_0 \leftarrow c + d$
$z_0 \leftarrow c + d$

11 $x_0 \leftarrow e_2 + f$

12 $u_1 \leftarrow e_1 + f$

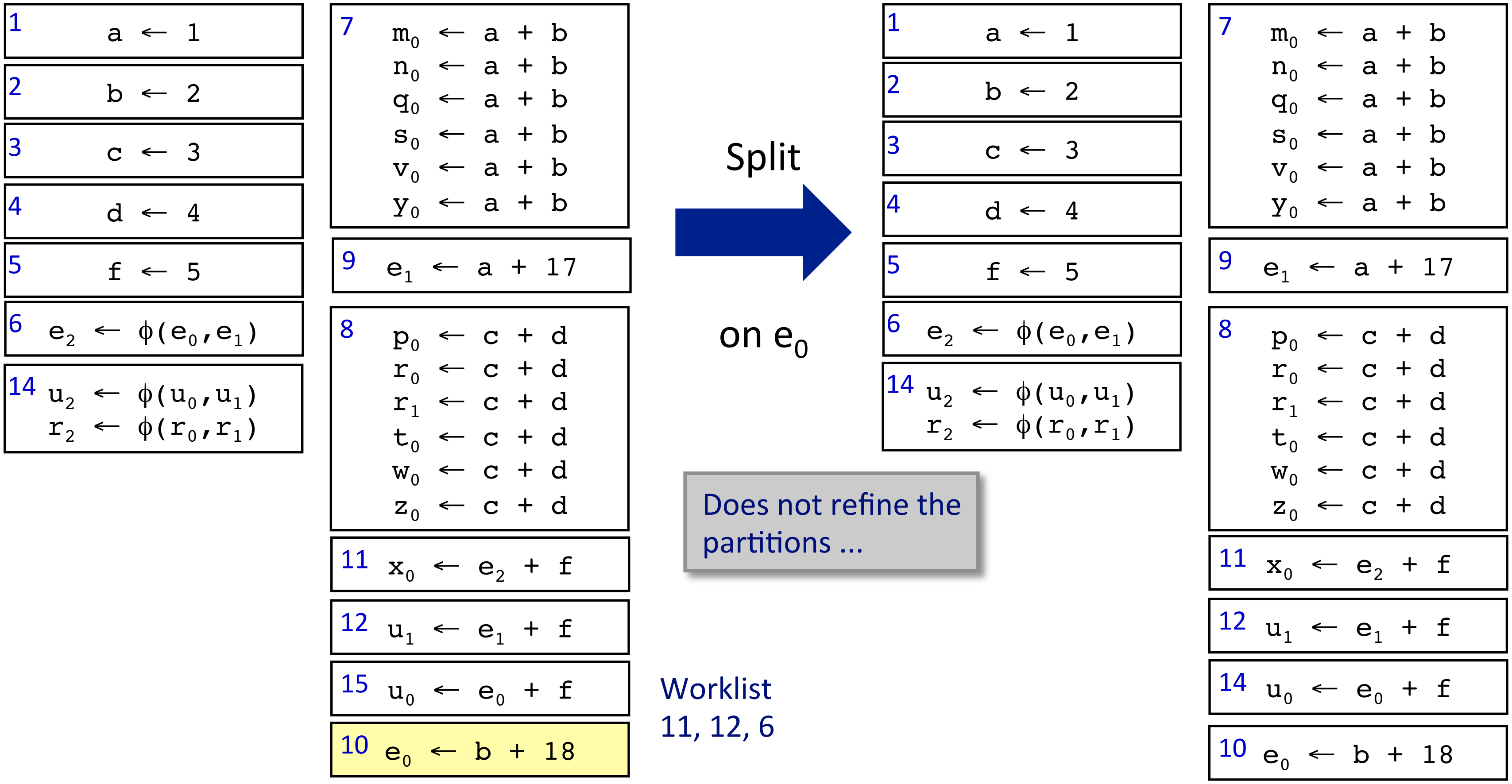
15 $u_0 \leftarrow e_0 + f$

10 $e_0 \leftarrow b + 18$

Worklist
10, 11, 12, 6

-9, -6, +6, -12, +12

PARTITIONING METHOD



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11 $x_0 \leftarrow e_2 + f$

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15 $u_0 \leftarrow e_0 + f$

10 $e_0 \leftarrow b + 18$

Split

on e_0

1 $a \leftarrow 1$

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3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

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$t_0 \leftarrow c + d$
$w_0 \leftarrow c + d$
$z_0 \leftarrow c + d$

11 $x_0 \leftarrow e_2 + f$

12 $u_1 \leftarrow e_1 + f$

14 $u_0 \leftarrow e_0 + f$

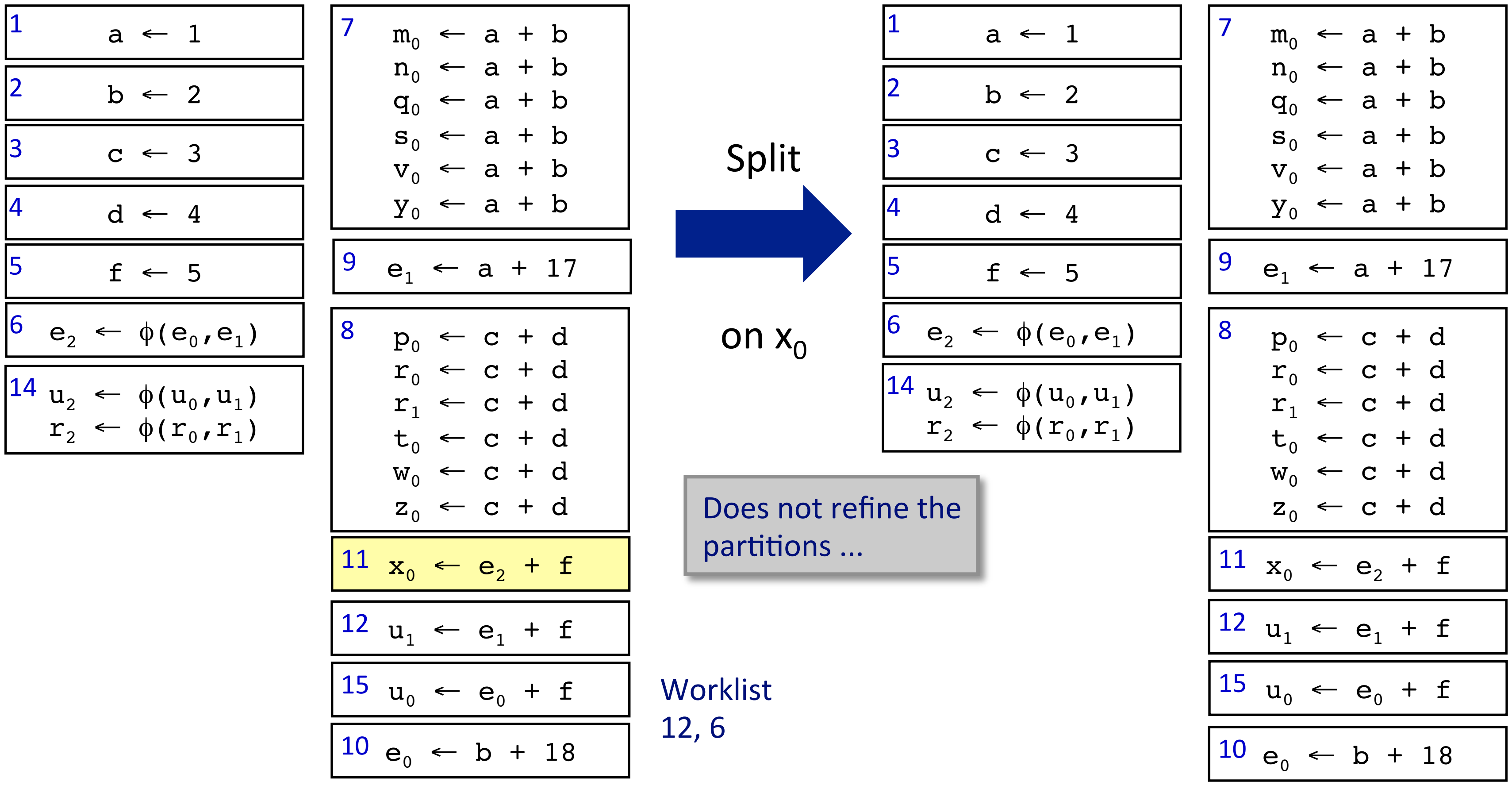
10 $e_0 \leftarrow b + 18$

Does not refine the partitions ...

Worklist
11, 12, 6

-10

Partitioning Method



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11 $x_0 \leftarrow e_2 + f$

12 $u_1 \leftarrow e_1 + f$

15 $u_0 \leftarrow e_0 + f$

10 $e_0 \leftarrow b + 18$

Split

on x_0

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$

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$z_0 \leftarrow c + d$

11 $x_0 \leftarrow e_2 + f$

12 $u_1 \leftarrow e_1 + f$

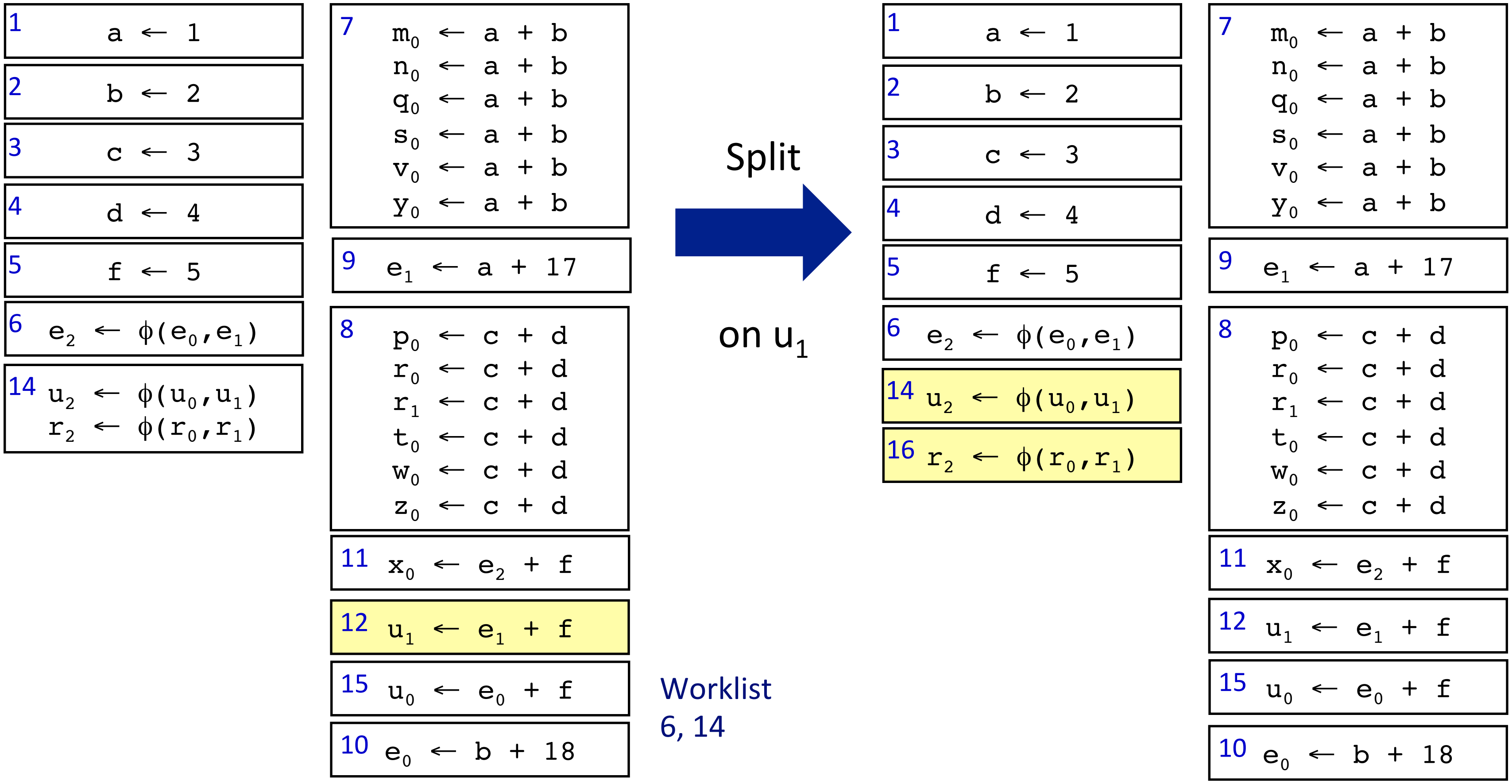
15 $u_0 \leftarrow e_0 + f$

10 $e_0 \leftarrow b + 18$

Does not refine the partitions ...

Worklist
12, 6

PARTITIONING METHOD



1 $a \leftarrow 1$

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3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$

14 $u_2 \leftarrow \phi(u_0,u_1)$
$r_2 \leftarrow \phi(r_0,r_1)$

7 $m_0 \leftarrow a + b$
$n_0 \leftarrow a + b$
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Split

on u_1

Worklist
6, 14

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$

14 $u_2 \leftarrow \phi(u_0,u_1)$

16 $r_2 \leftarrow \phi(r_0,r_1)$

7 $m_0 \leftarrow a + b$
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11 $x_0 \leftarrow e_2 + f$

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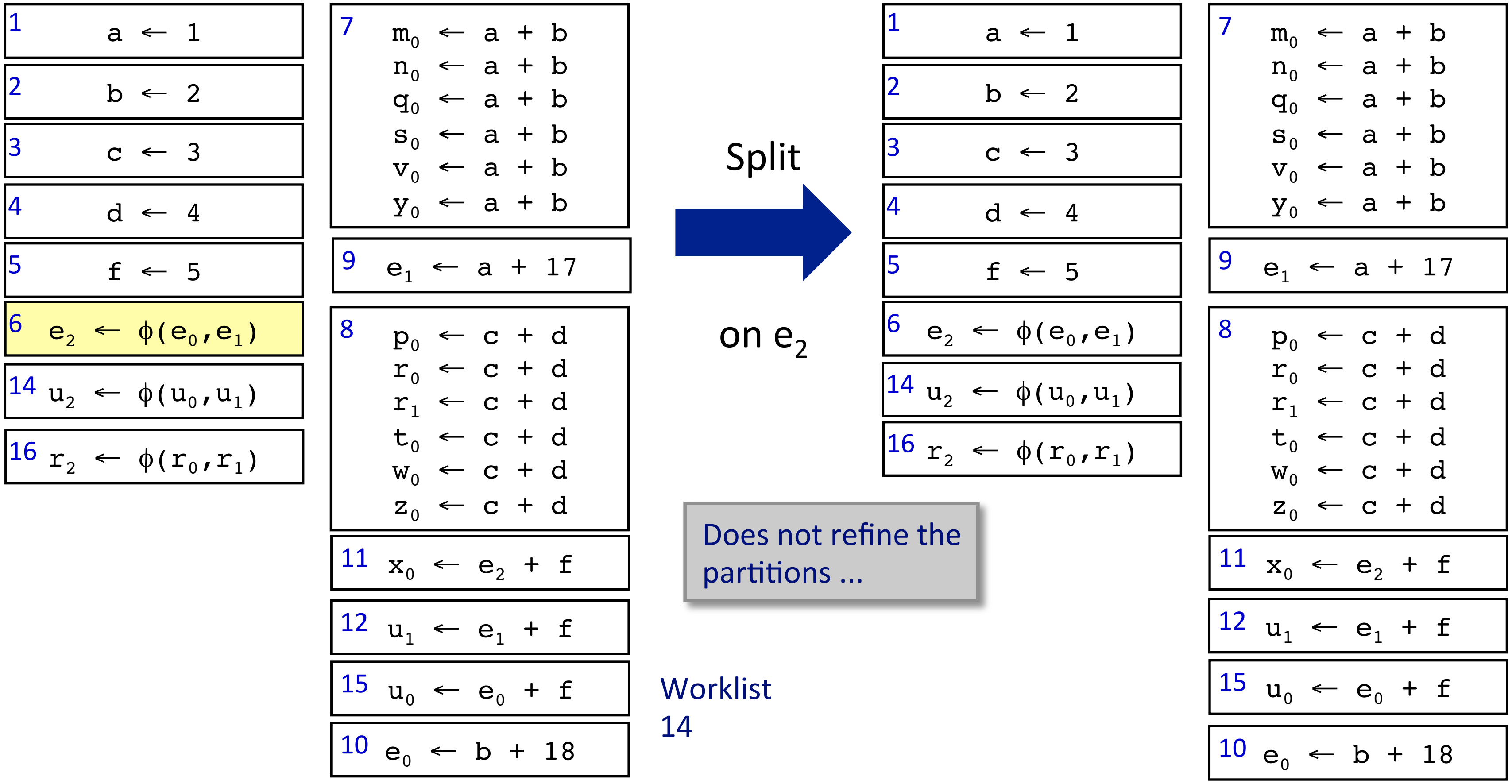
10 $e_0 \leftarrow b + 18$

-12, -14, + 14

PARTITIONING METHOD



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Split

on e_2

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4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$

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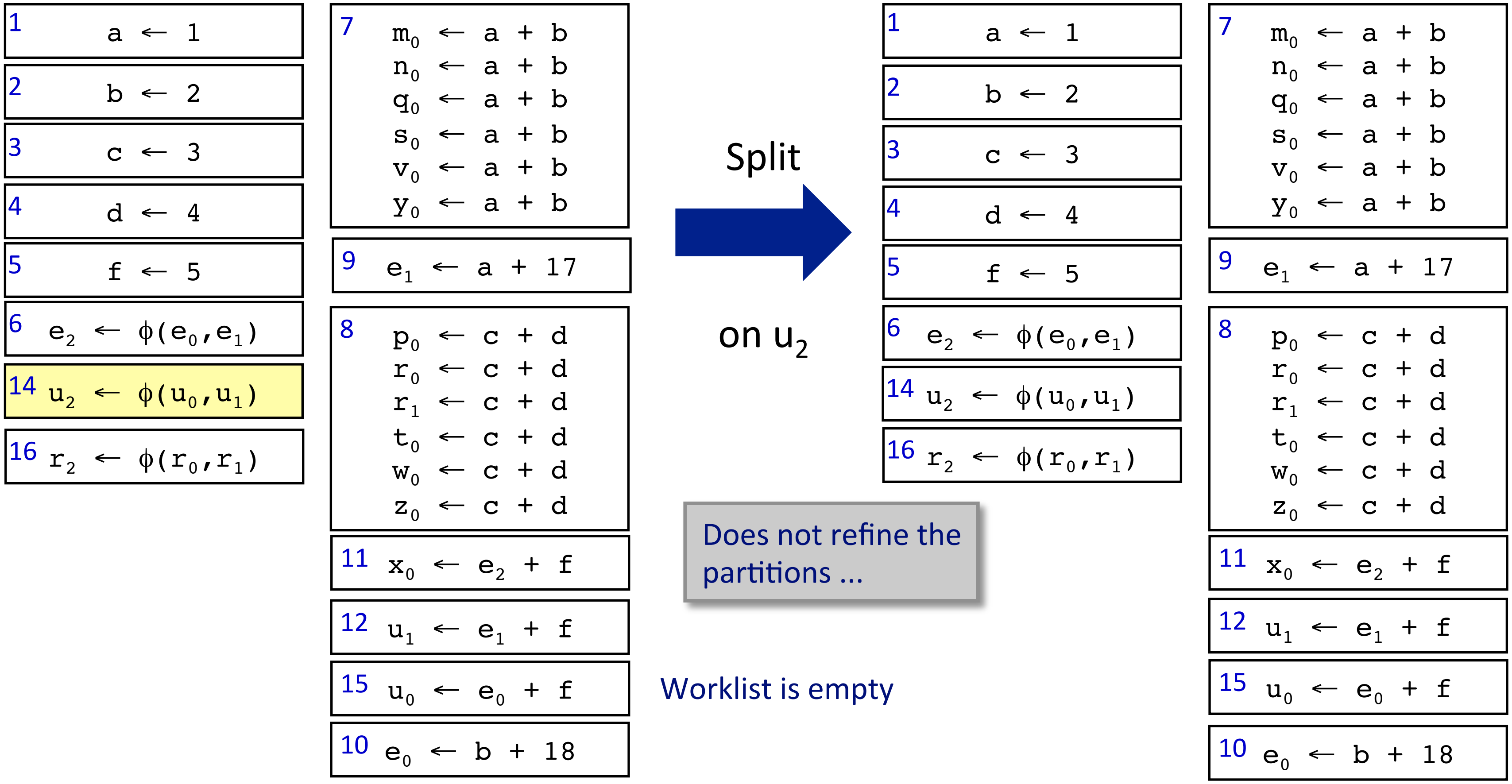
Does not refine the partitions ...

Worklist
14

-6

PARTITIONING METHOD



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Split

on u_2

Does not refine the partitions ...

Worklist is empty

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0,e_1)$

14 $u_2 \leftarrow \phi(u_0,u_1)$

16 $r_2 \leftarrow \phi(r_0,r_1)$

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$r_0 \leftarrow c + d$
$r_1 \leftarrow c + d$
$t_0 \leftarrow c + d$
$w_0 \leftarrow c + d$
$z_0 \leftarrow c + d$

11 $x_0 \leftarrow e_2 + f$

12 $u_1 \leftarrow e_1 + f$

15 $u_0 \leftarrow e_0 + f$

10 $e_0 \leftarrow b + 18$

-14

Partitioning Method

Now what?

1 $a \leftarrow 1$

2 $b \leftarrow 2$

3 $c \leftarrow 3$

4 $d \leftarrow 4$

5 $f \leftarrow 5$

6 $e_2 \leftarrow \phi(e_0, e_1)$

14 $u_2 \leftarrow \phi(u_0, u_1)$

16 $r_2 \leftarrow \phi(r_0, r_1)$

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We have proved a set of theorems

- Must rewrite code to use them
- Knowledge alone won't make the code run faster!

Use the partition to rebuild the code

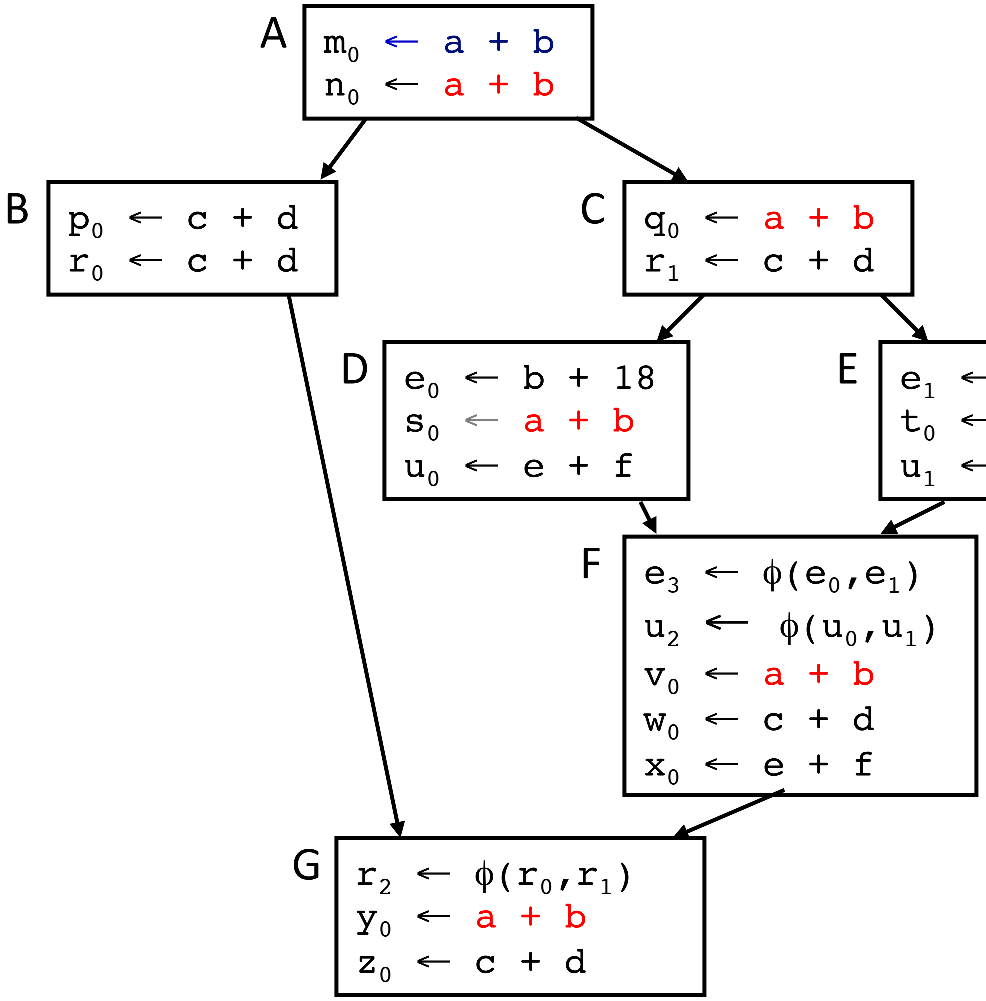
- Sets with > 1 member ⇒ reuse
- Sets with 1 member ⇒ no reuse

Let's look at the code

- Opportunities are `a+b` & `c+d`
- Can we make this work?

Partitioning Method

These slides omits the initialization block so that it can fit into the available space.



A

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B

$p_0 \leftarrow c + d$

$r_0 \leftarrow c + d$

C

$q_0 \leftarrow a + b$

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D

$e_0 \leftarrow b + 18$

$s_0 \leftarrow a + b$

$u_0 \leftarrow e + f$

E

$e_1 \leftarrow a + 17$

$t_0 \leftarrow c + d$

$u_1 \leftarrow e + f$

F

$e_3 \leftarrow \phi(e_0,e_1)$

$u_2 \leftarrow \phi(u_0,u_1)$

$v_0 \leftarrow a + b$

$w_0 \leftarrow c + d$

$x_0 \leftarrow e + f$

G

$r_2 \leftarrow \phi(r_0,r_1)$

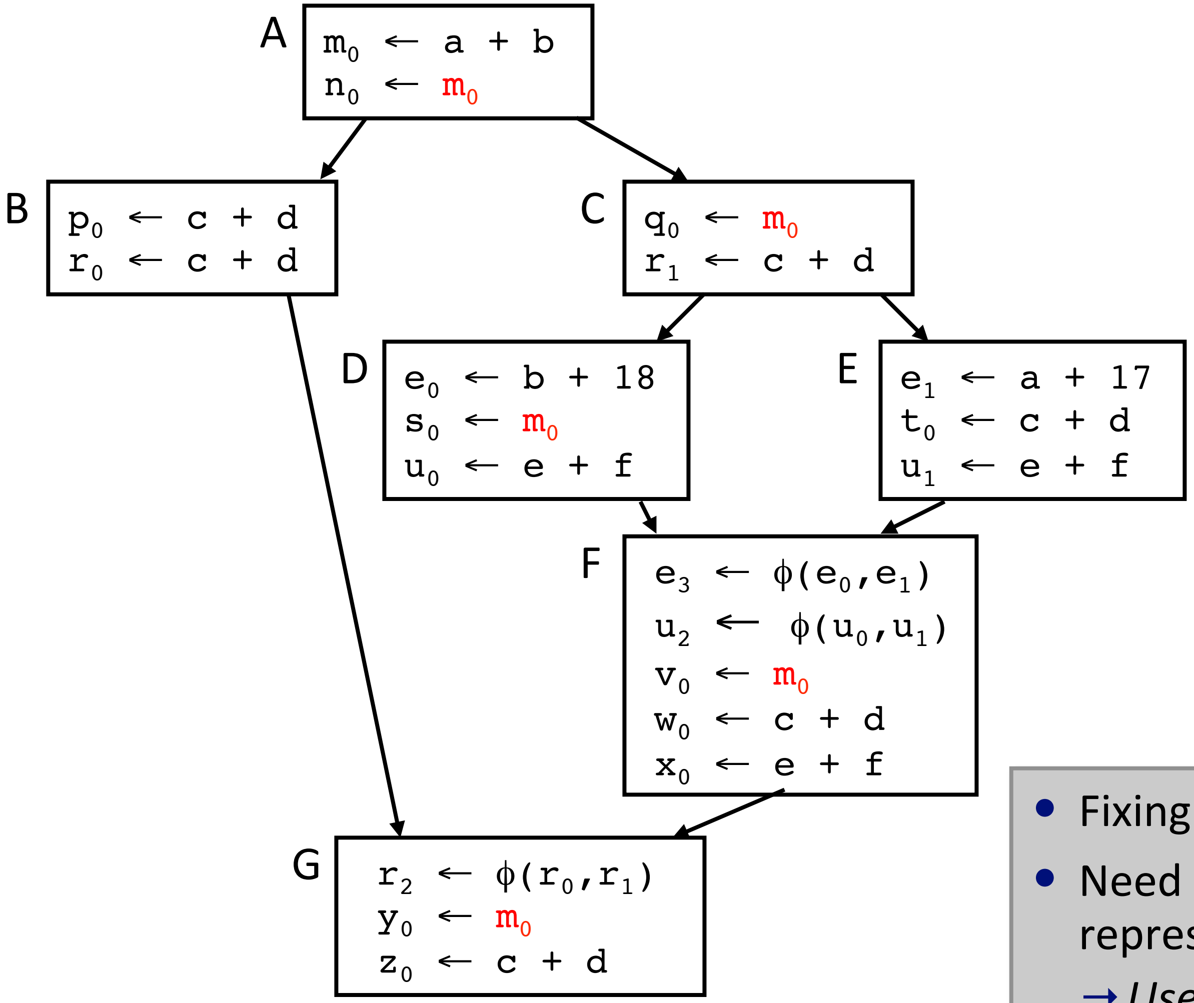
$y_0 \leftarrow a + b$

$z_0 \leftarrow c + d$

- Fixing a+b is easy
- Need to choose the right representer
  - → *Use the instance of* a+b *in A*
  - → *It makes the others redundant*

Partitioning Method

These slides omits the initialization block so that it can fit into the available space.



A

$m_0 \leftarrow a + b$

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  - → *Use the instance of* a+b *in A*
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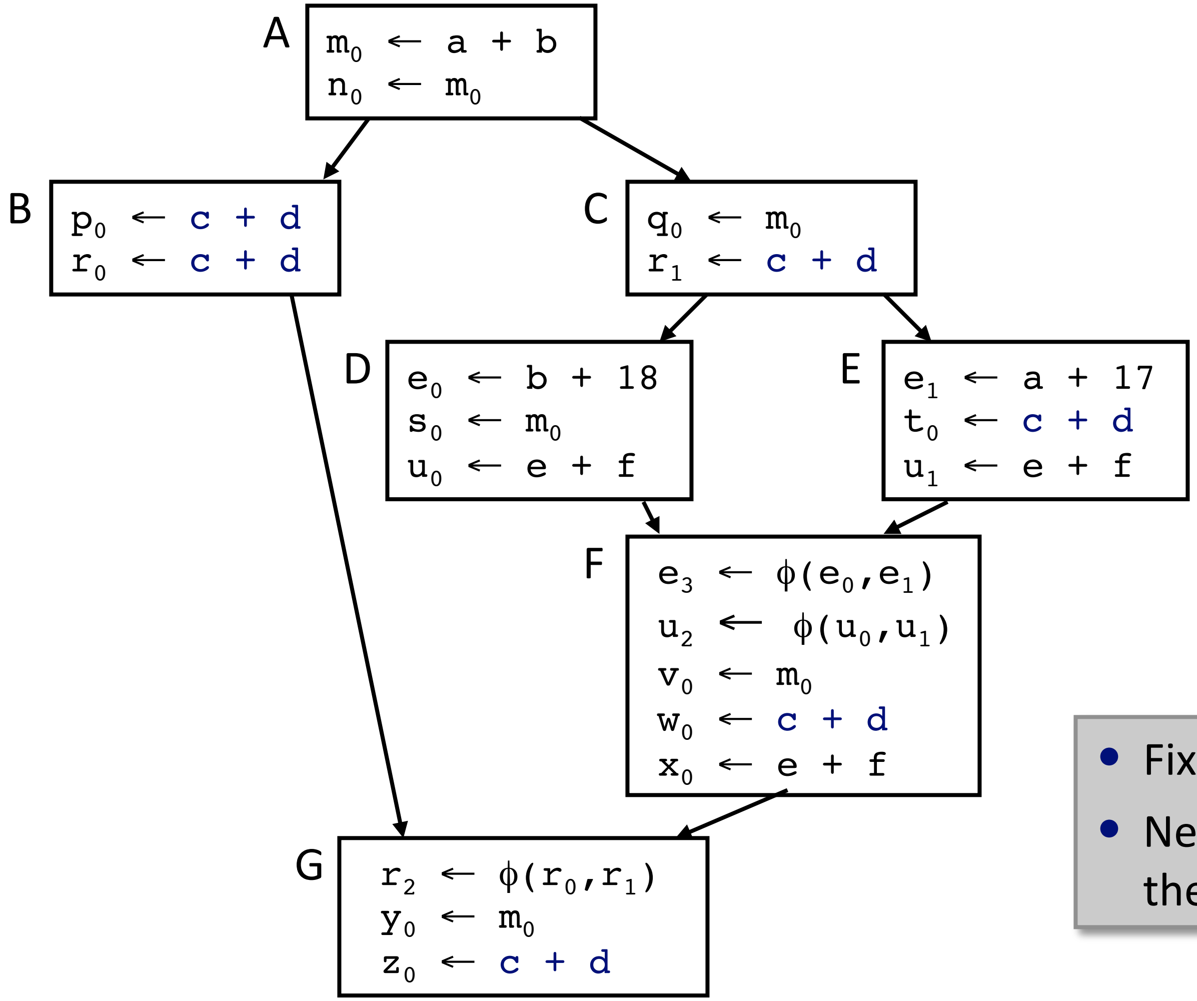
PARTITIONING METHOD



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G

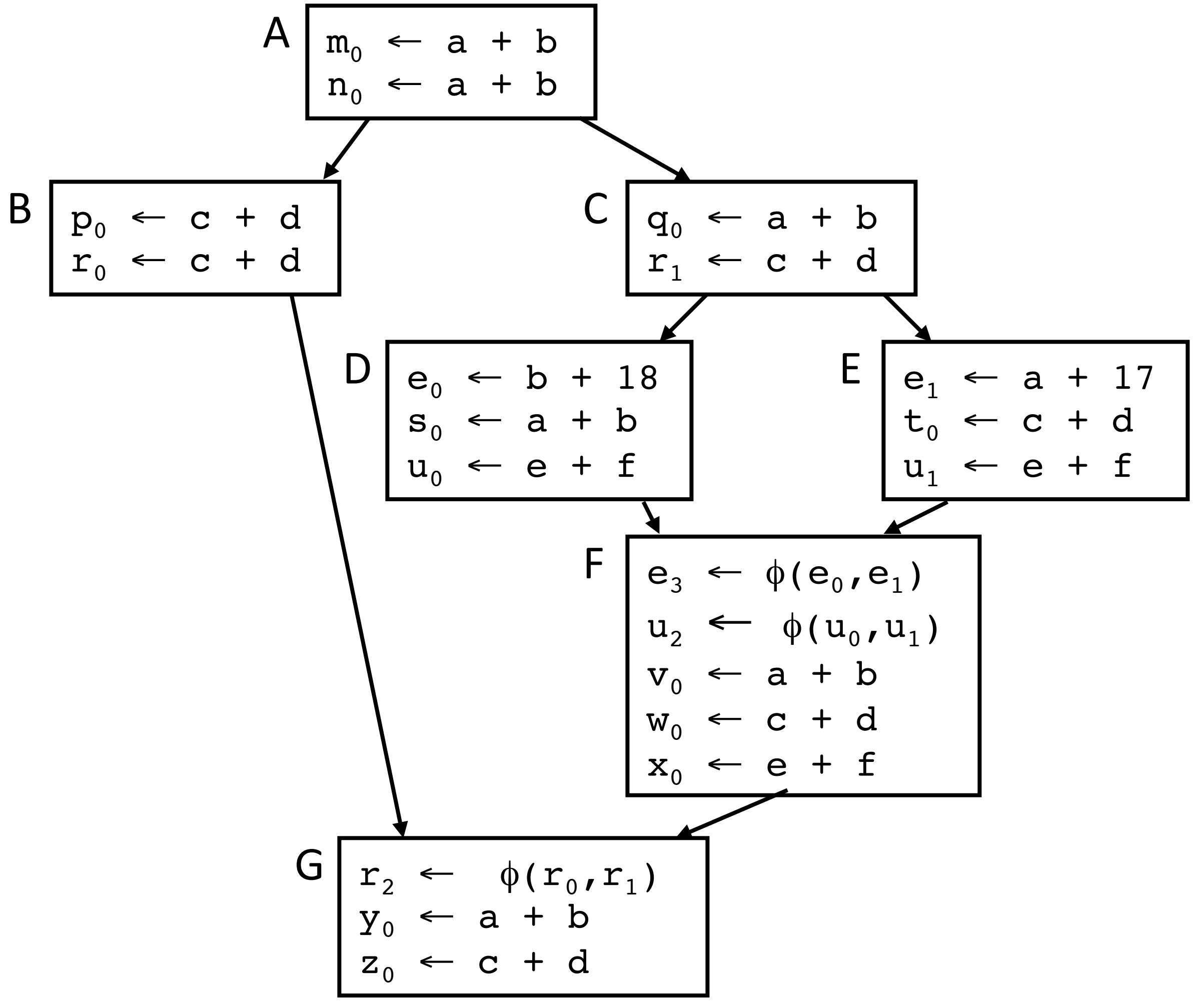
$r_2 \leftarrow \phi(r_0,r_1)$

$y_0 \leftarrow m_0$

$z_0 \leftarrow c + d$

- Fixing c+d is harder
- Need some rationale for the replacement scheme

Partitioning Method



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G

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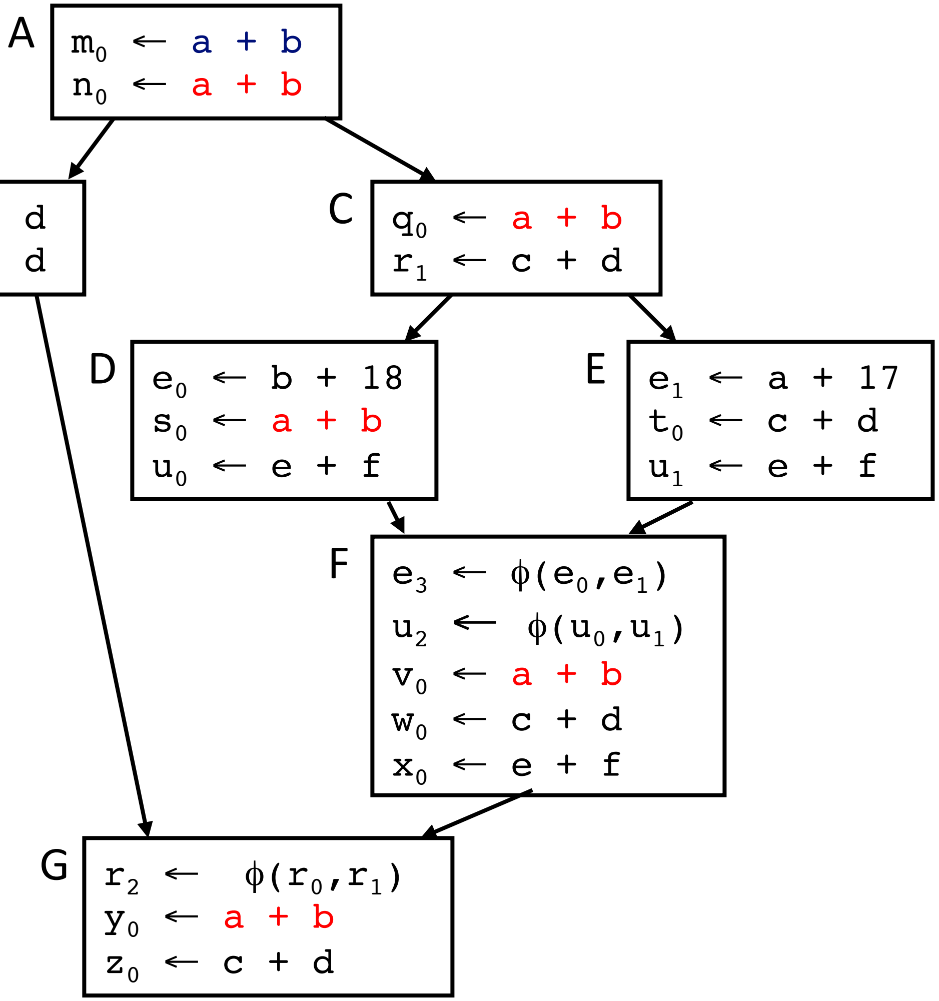
AWZ suggest removing any computation that is dominated by another computation in the same partition

PARTITIONING METHOD



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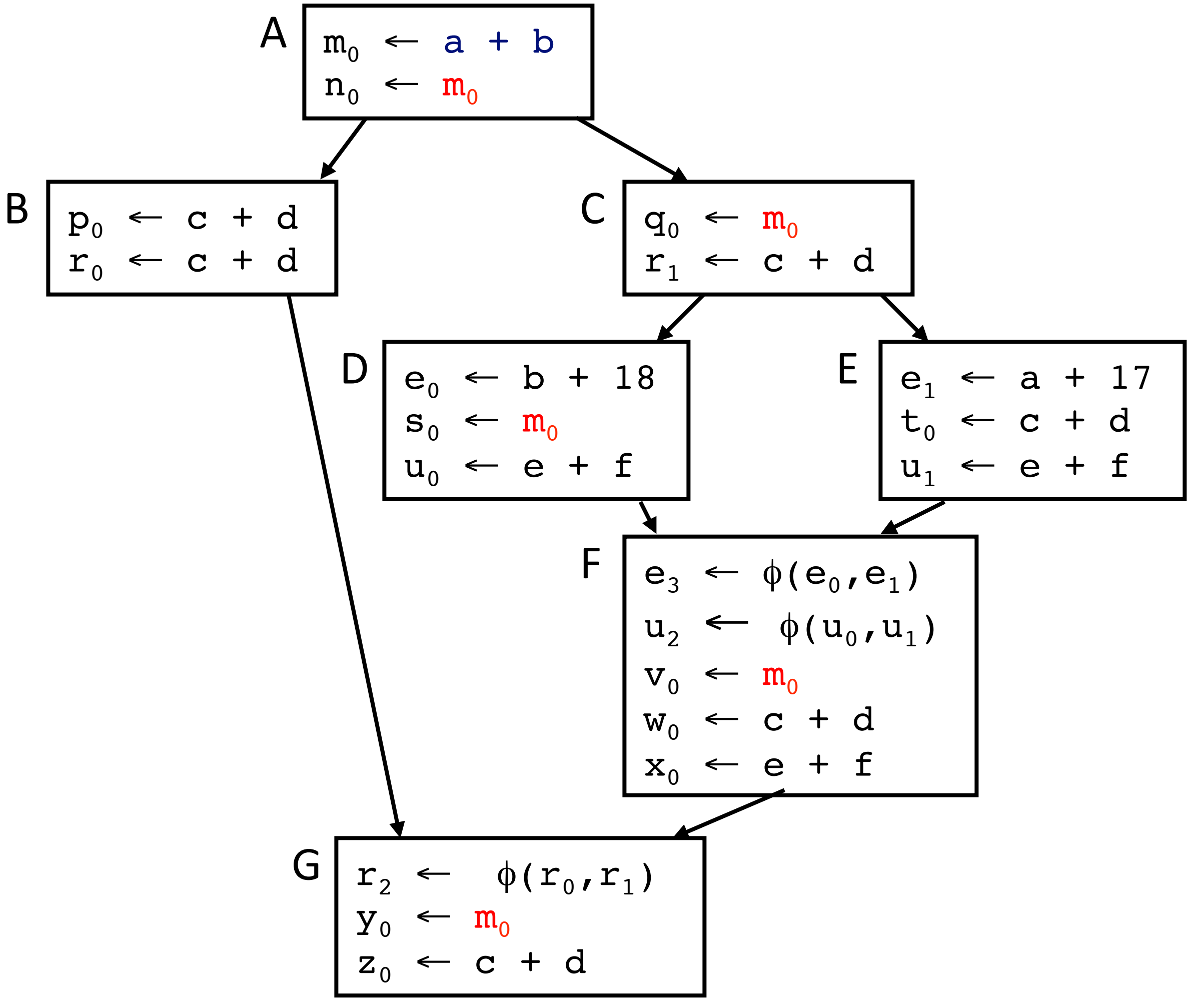
AWZ suggest removing any computation that is dominated by another computation in the same partition

- For a+b, the def of m_0 dominates all other instances

PARTITIONING METHOD



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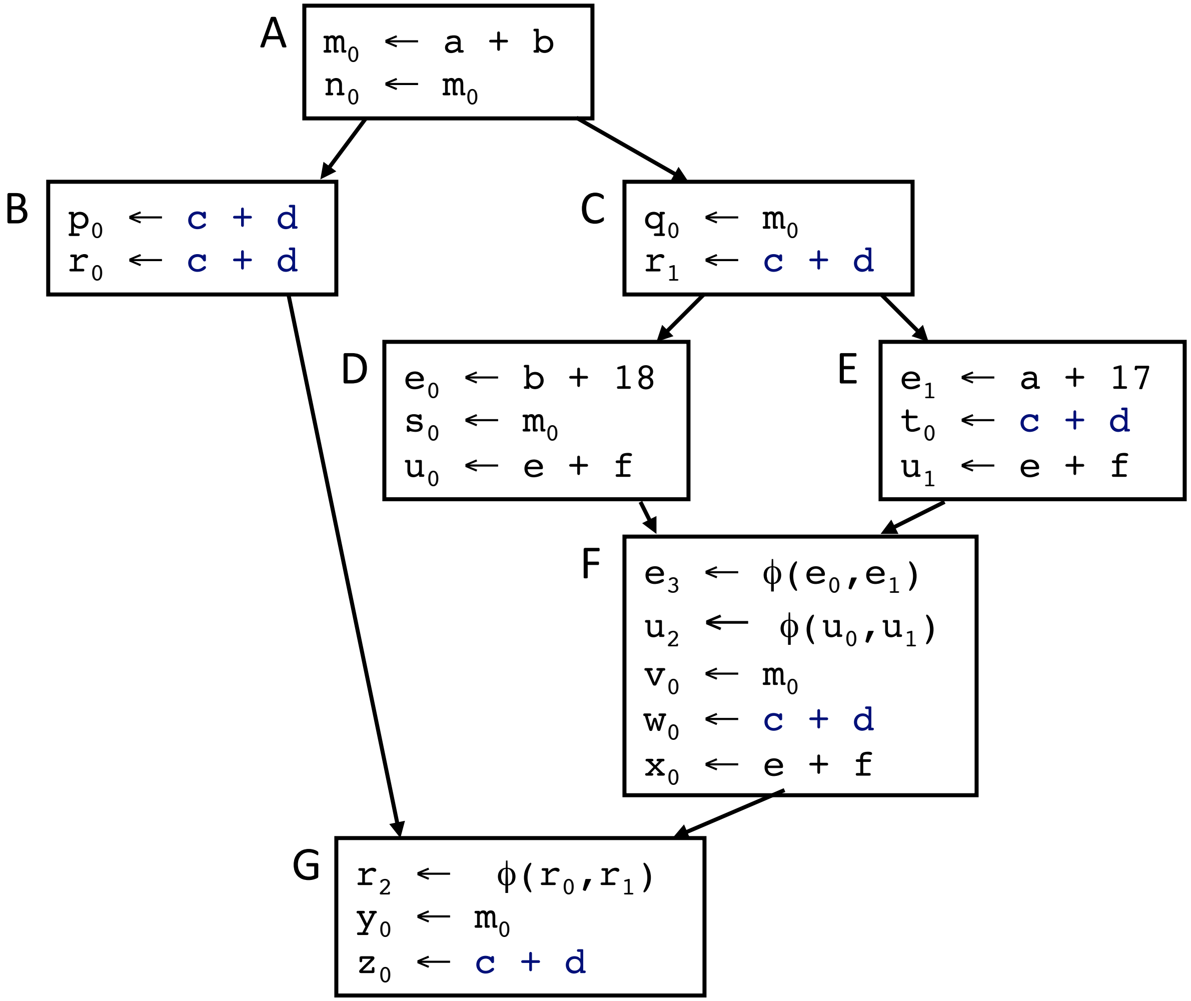
Partitioning Method



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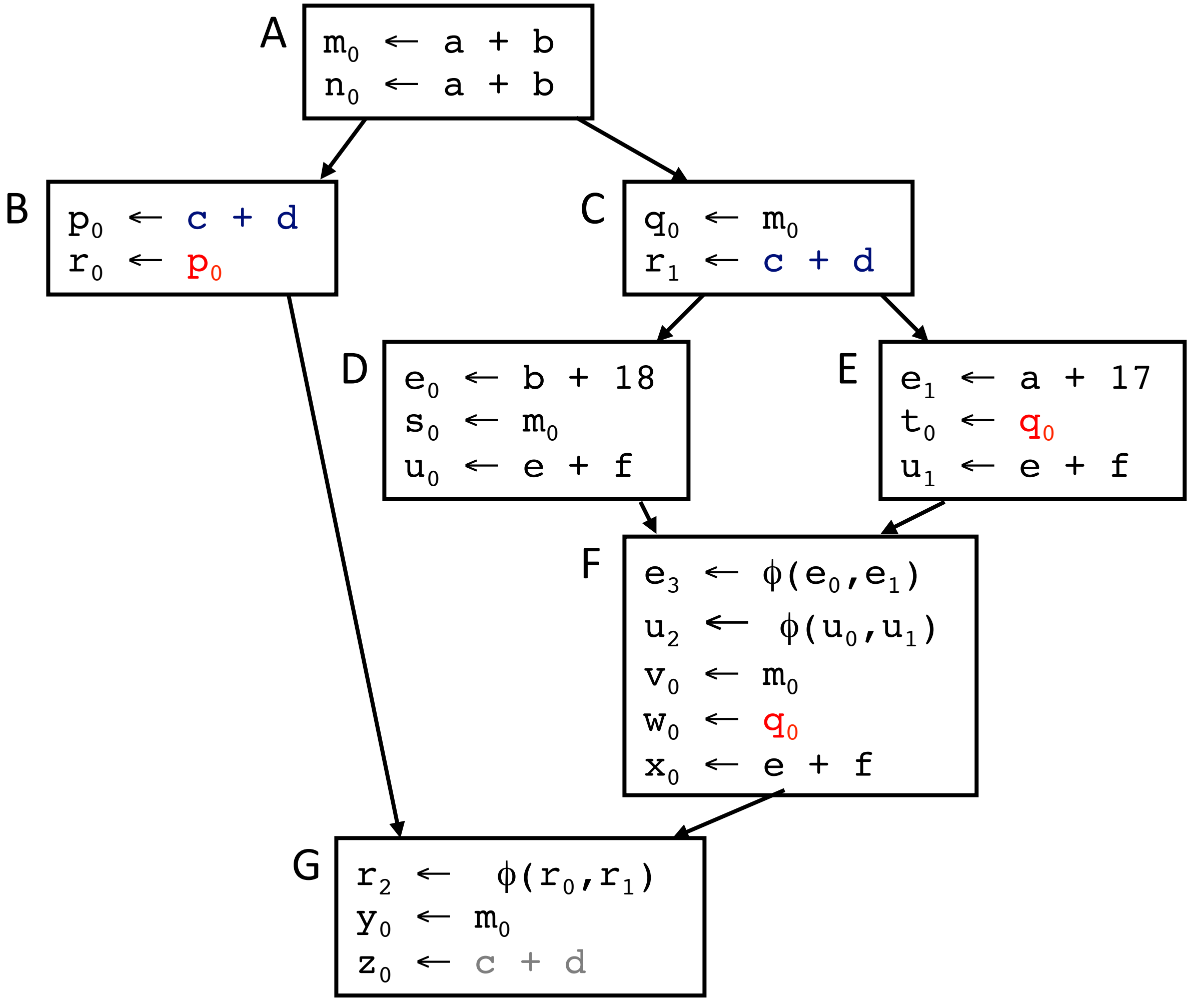
AWZ suggest removing any computation that is dominated by another computation in the same partition

- For a+b, the def of m_0 dominates all other instances
- For c+d no single instance will do

*

PARTITIONING METHOD



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AWZ suggest removing any computation that is dominated by another computation in the same partition

- For a+b, the def of m_0 dominates all other instances
- For c+d no single instance will do
- Some cases cannot be caught this way

Simpson suggested another approach.

PARTITIONING METHOD

Simpson's suggestion

- Encode congruences into the name space
  - *This idea is critical to and derives from the Briggs-Cooper approach to algebraic reassociation for LCM/PRE*
  - Use LCM to remove redundancies

Simplification

- In SSA name space, there are no KILLS
- Knocks terms out of some equations
- Makes initial information cheaper to compute
- Larger name space, but O(n) rather than O(n^2) initialization

Consistently out-performs dominator-based replacement

Partitioning Method

Is This Algorithm Optimistic Or Pessimistic?

- When we talked about optimism in data-flow analysis, we said
  - ♦ *Initialize to top ⇒ optimistic*
  - ♦ *Initialize to bottom ⇒ pessimistic*
- This algorithm does neither

AWZ builds a set of congruence relations

- Assumes maximal size sets
- Knocks out infeasible elements
  - ♦ Over-estimates set of congruences
  - ♦ Finds largest set of self-consistent congruences

} inherently optimistic

⇒ Must let it run to completion, or sets might contain lies

Click points that set construction can be either optimistic or pessimistic. An optimistic construction starts with the universe and eliminates elements. A pessimistic construction starts with the empty set and adds elements.

PARTITIONING METHOD

Strengths

- Global algorithm for finding redundancies
  - ♦ *Handles arbitrary control flow by ignoring it*
  - ♦ *Finds largest congruence classes consistent with definition*
- Relies on well understood (& efficient) algorithm

Weaknesses

- Cannot put `2*a` and `a+a` in same class (Neither can lexical methods)
- Not obvious how to handle identities or constants (Click)
- Pays no systematic attention to placement (LCM)
- Slower than DVNT (1(LVN):4(DVNT):10(AWZ))