COMP 512
Rice University
Spring 2015

Algebraic Reassociation of Expressions

— With Application To Lazy Code Motion —

P. Briggs & K.D. Cooper, "Effective Partial Redundancy Elimination," *Proceedings of the ACM SIGPLAN 1994 Conference on Programming Language Design and Implementation*, June 1994.

Citation numbers refer to entries in the EaC2e bibliography.

The Problem

Compiler front end generates expressions in arbitrary order

- Some orders (or shapes) may cost less to evaluate
- Sometimes “better” is a local property
- Sometimes “better” is a non-local property

Compiler should reorder expressions to fit their context

Old Problem

- Recognized in 1961 by Floyd
- Scarborough & Kolsky did it manually in Fortran H Enhanced
- PL.8 and HP compilers claimed to solve it, without publishing

Need an efficient & effective way to rearrange expressions

RESTRICTIONS

In the last example, the symbolic coding generated is at least comparable to the results of hand coding. Other examples, however, could disclose the limitations of the algorithm. Its inability to apply the associative laws may result in unnecessary mode conversions and storage of partial results in computing sums or products of quantities of unlike modes. In justification, it may be said that floating-point arithmetic is only approximately associative. Its inability to recognize equivalent subexpressions containing subscripted variables is a more serious drawback, and more nearly intrinsic to the algorithm. Finally, no provision has been made to recognize integral constant exponents. Most existing compilers waste time extravagantly by using exp $(2 \times \ln(x))$ to compute $x \uparrow 2$. It is possible to rewrite such expressions to be evaluated by a small number of multiplications. For example, $y \uparrow 9$ may be written

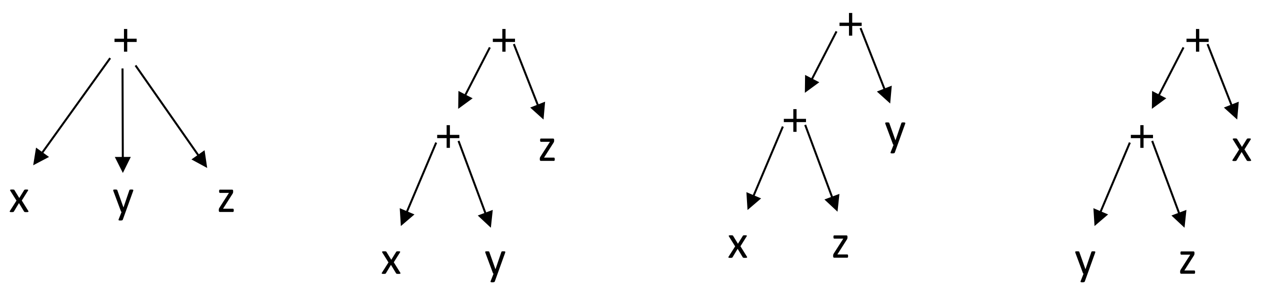
$$((((y \times y) \times (y \times y)) \times ((y \times y) \times (y \times y))) \times (((y)))).$$

Floyd, 1961

Opportunities

Common Subexpression Elimination & Constant Propagation

x + y + z	x + y → t1	x + z → t1	y + z → t1
	t1+ z → t2	t1+ y → t2	t1+ z → t2



- Best shape for **CP** (probably) moves constants together
  - ♦ Which operands are constant? x & y, x & z, or x & y
- Best shape for **CSE** is context dependent
  - ♦ Which expressions appear elsewhere? x + y, x + z, or x + y?

Local issue

Non-local issue

Assume that x, y, & z are integers & that addition is commutative.

Opportunities

Code Motion

- In a loop nest, want to move loop-invariant code into the outermost loop where it does not vary

$a \leftarrow \ldots ;\; b \leftarrow \ldots$
do i …
  $c \leftarrow \ldots ;\; d \leftarrow \ldots ;$
  do j …
    … $a+b$ …
    … $d+b+c$ …
    … $a+c+b+d$ …

might become

$a \leftarrow \ldots ;\; b \leftarrow \ldots$
$t_1 \leftarrow a+b$
do i …
  $c \leftarrow \ldots ;\; d \leftarrow \ldots ;$
  $t_2 \leftarrow c+d$
  $t_3 \leftarrow b+t_2$
  $t_4 \leftarrow t_1 + t_2$
  do j …
    … t_1 …
    … t_3 …
    … t_3 …

- In a + b + c, the operands may vary in different loops
- Need two or more operations in a subexpression to make distribution over two levels of loops profitable

Briggs & Cooper proposed a ranking to address this problem

"Best" ranking might assign different ranks to "x" in different loop nests. (⇒ SSA names?)

Opportunities

Operator Strength Reduction

```
subroutine dmxpy (n1, y, n2, ldm, x, m)
double precision y(*), x(*), m(ldm,*)
...
jmin = j+16
do 60 j = jmin, n2, 16
    do 50 i = 1, n1
        y(i) = (((((((((((((((( (y(i))
            + x(j-15)*m(i,j-15)) + x(j-14)*m(i,j-14)) + x(j-13)*m(i,j-13))
            + x(j-12)*m(i,j-12)) + x(j-11)*m(i,j-11)) + x(j-10)*m(i,j-10))
            + x(j- 9)*m(i,j- 9)) + x(j- 8)*m(i,j- 8)) + x(j- 7)*m(i,j- 7))
            + x(j- 6)*m(i,j- 6)) + x(j- 5)*m(i,j- 5)) + x(j- 4)*m(i,j- 4))
            + x(j- 3)*m(i,j- 3)) + x(j- 2)*m(i,j- 2)) + x(j- 1)*m(i,j- 1))
            + x(j) *m(i,j)
50    continue
60 continue
    ...
    end
```

The largest version of the hand-optimized loop in dmxpy.

33 distinct addresses (+ i & j)

Done poorly, this loop can easily generate 33 or more distinct induction variables.

With some care (and reassocation of the address expressions), the compiler might get that down to two or three.

Opportunities

Operator Strength Reduction

- A reference, such as V[i], translates into an address expression

  $@V_0 + (i\text{-}low) * w$

- A loop with references to V[i], V[i+1], & V[i-1] generates

  $@V_0 + (i\text{-}low) * w$

  $@V_0 + (i\text{-}(low\text{-}1)) * w$

  $@V_0 + (i\text{-}(low\text{+}1) * w$

V is declared V[low:high].
Elements are w bytes wide.
Constants have been folded.

- **OSR** may create distinct induction variables for these expressions, or it may create one common induction variable
  - ♦ ***Matter of code shape in the expression***
  - ♦ ***Difference between 33 induction variables in the dmxpy loop and one or two***
- Situation gets more complex with multi-dimensional arrays

Opportunities

Operator Strength Reduction

- Consider references to A[i,j], B[i+1,j], and C[3*I,j-1]
  - $@A_0 + (i * len_2^A + j) * w$
  - $@B_0 + ((i+1) * len^B + j) * w$
  - $@C_0 + ((3*i) * len_2^A + j) * w$

Assume A, B, & C may have different bounds but all have element width w.

Row major order.

- The diversity of address expressions may increase likelihood of generating too many induction variables in OSR
- Want to canonicalize their shape in a way that minimizes the number of induction variables.
- Problem has been known for a long time. See, for example, Markstein, Markstein & Zadeck.

P. Markstein, V. Markstein, & F.K. Zadeck, “Strength Reduction,” Chapter 9 in *Optimization in Compilers* (F.E. Allen, B.K. Rosen, & F.K. Zadeck, editors). The book was not published, but the draft chapter bears a 1992 ACM Copyright.

Challenges

Expressions are small (in real code)

- In IR from human-written code, many expressions are small
  - ♦ Frequent assignment to variables breaks up computation
    - → *May be cognitive reasons for this style of code*
  - ♦ More operations and operands means more opportunity for reassociation
- May want to transform code to build larger expressions

Complexity grows with number of operands

- Pairwise commutativity is easy to handle (*think* ***LVN***)
- With 5, 6, … operands, the number of orders is large
- Suggests a "rank & sort" methodology (*Briggs*)
  - ♦ Need to derive a rank scheme that achieves desired result

Any algorithmic approach to reassociation must cope with these challenges

The Running Example (*from* [BC 94])

```
FUNCTION foo(y, z)
  s = 0
  x = y + z
  DO i = x, 100
    s = 1 + s + x
  ENDDO
  RETURN s
END foo
```

$\text{enter}(r_y, r_z)$
$r_s \leftarrow 0$
$r_x \leftarrow r_y + r_z$
$r_i \leftarrow r_x$
if $r_i > 100$ branch

$r_1 \leftarrow r_s + 1$
$r_s \leftarrow r_1 + r_x$
$r_i \leftarrow r_i + 1$
if $r_i \leq 100$ branch

return r_s

Fortran 90 Source Code **Intermediate Code**

Briggs-Cooper Approach

To improve results out of LCM

1. Reassociation

- Discover facts about global code shape
- Reorder subexpressions based on that knowledge

2. Renaming

- Use redundancy elimination to find equivalences
- Rename virtual registers to reflect equivalences, and to conform to the code shape constraints for **LCM**
- Encode value equality into the name space

3. LCM

- Run **LCM** unchanged on the result
- Performs code placement, partial redundancy elimination
- Run it anywhere, anytime, on any code

This lecture focuses on reassociation & renaming

Reassociation

Simple Idea

- Use algebraic properties to rearrange expressions
- Hard part is to choose <u>one</u> shape quickly

The Approach

1. Compute a rank for each expression
2. Propagate expressions forward to their uses
3. Reorder by sorting operands into rank order

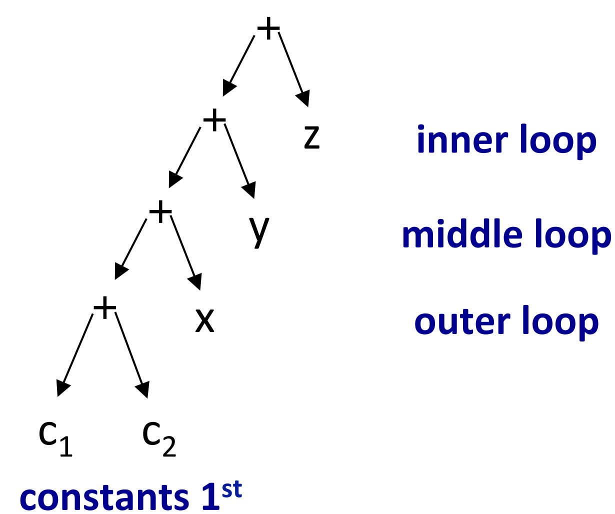
The algorithm needs a guiding principle

- Order subscripts to improve code motion & constant propagation

1. Compute Ranks

The Intuitions

- Each expression & subexpression assigned a rank
- Loop-invariant’s rank < loop-variant’s rank
- Deeper nesting ⇒ higher rank
- Invariant in 2 loops < invariant in 1 loop
- All constants assigned the same rank
- Constants should sort together



1. Compute Ranks

The Algorithm

1. Build <u>pruned</u> **SSA** form & fold copies into ϕ-functions
2. Traverse **CFG** in reverse postorder (**RPO**)
   a. Assign each block a rank number as visited
   b. Each expression in block is ranked
      i. x is constant ⇒ rank(x) is 0
      ii. result of ϕ-function has block's RPO number
      iii. x <op> y has rank max(rank(x),rank(y))

This numbering produces the "right" intuitive properties

Recall that pruned **SSA** form only inserts phi-functions that are **LIVE** — that is, whose results are actually used.

Example

$\text{enter}(r_0, r_1)$
$r_2^{0} \leftarrow 0$
$r_3^{1} \leftarrow r_0^{1} + r_1^{1}$
$\text{if } r_3 > 100 \text{ branch}$

$r_4^{2} \leftarrow \phi(r_3, r_8)$
$r_5^{2} \leftarrow \phi(r_2, r_7)$
$r_6^{2} \leftarrow r_5 + 1$
$r_7^{2} \leftarrow r_6 + r_3$
$r_8^{2} \leftarrow r_4 + 1$
$\text{if } r_8 \leq 100 \text{ branch}$

$r_9^{3} \leftarrow \phi(r_7, r_2)$
$\text{return } r_9$

The example is shown in pruned **SSA** form

- Use Ø functions to compute ranks
- Name space of **SSA** form is important

Rank computation:

- Ø's rank & parameter rank is **RPO** number of its block
- Constant's rank is 0
- Rank(*x op y*) is *max*(rank(*x*),rank(*y*))

Pruned SSA Form, With Computed Ranks

2. Propagate Expressions Forward to Their Uses

The Intuition

- Copy expressions forward to their uses
- Build up large expression trees from small ones

The Implementation

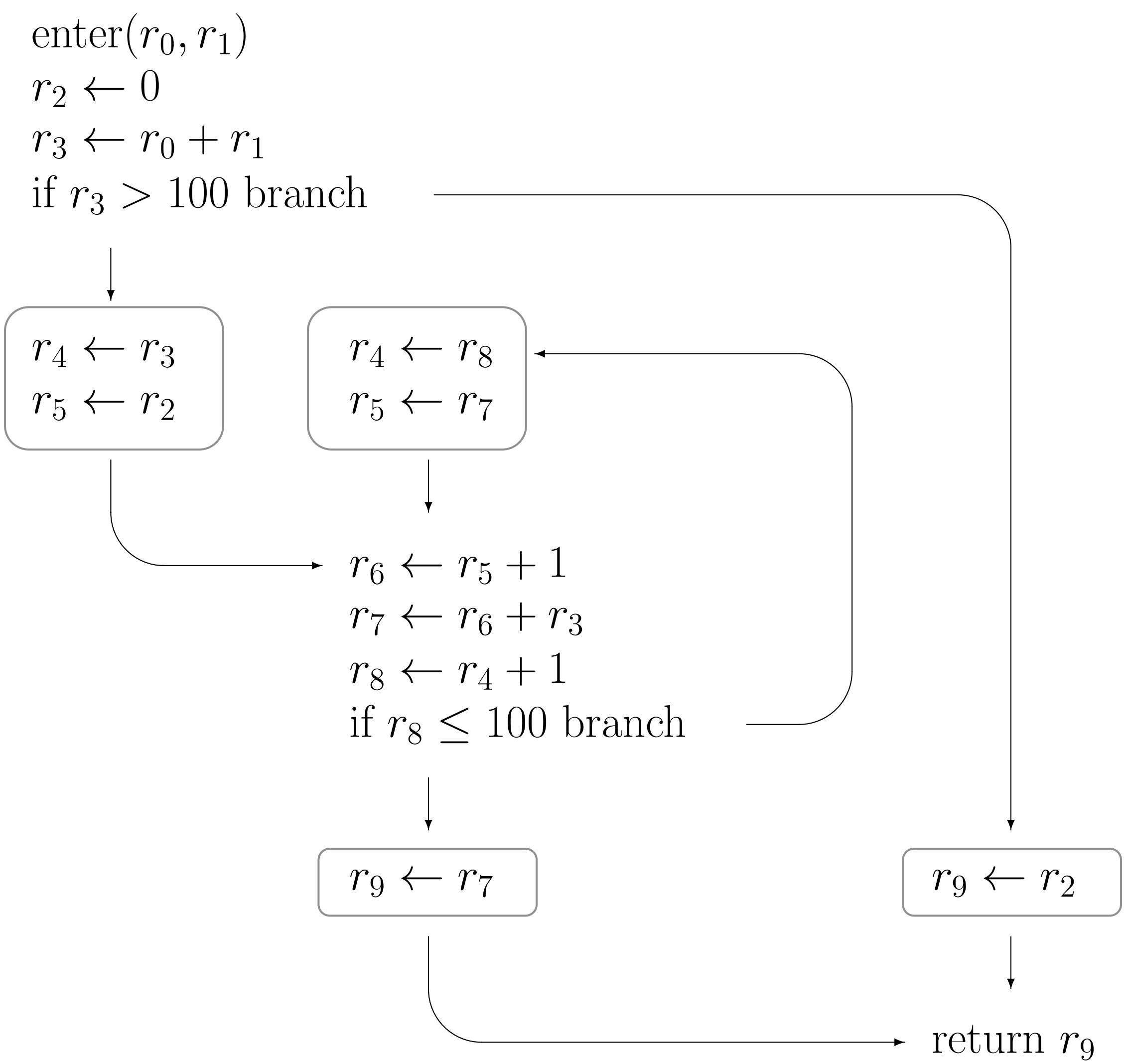
Split them here, not during ranking!

- Split critical edges to create appropriate predecessors
- Replace ϕ-functions with copies in predecessor blocks†
- Trace back from copy to build expression tree

Notes

- Forward propagation <u>does not</u> improve the code
- Addresses a subtle limitation in **PRE** and **LCM** (*expr live across > 1 block*)
- Eliminates some <u>partially-dead</u> expressions

† Based on Briggs et al.'s algorithm for out-of-SSA translation [50].



Example

PRE/LCM operate on the code in conventional (non-**SSA**) form

- Split critical edges
- Use any out-of-**SSA** translation technique
- Chain of copies preserves name space for forward propagation

Example

$\text{enter}(r_0, r_1)$
$r_3 \leftarrow r_0 + r_1$
$\text{if } r_3 > 100 \text{ branch}$

$r_2 \leftarrow 0$
$r_3 \leftarrow r_0 + r_1$
$r_4 \leftarrow r_3$
$r_5 \leftarrow r_2$

$r_7 \leftarrow 1 + r_0 + r_1 + r_5$
$r_8 \leftarrow r_4 + 1$
$r_4 \leftarrow r_8$
$r_5 \leftarrow r_7$

$r_8 \leftarrow r_4 + 1$
$\text{if } r_8 \leq 100 \text{ branch}$

$r_7 \leftarrow 1 + r_0 + r_1 + r_5$
$r_9 \leftarrow r_7$

$r_2 \leftarrow 0$
$r_9 \leftarrow r_2$

$\text{return } r_9$

Replace uses with the defining expressions

- Move immediate values
- Builds up larger expressions
- Removes partially dead expressions
- Technical issue with **PRE**
  — *expr live across >1 block*

After Forward Propagation

3. Reorder Operands

The Intuition

- Rank shows how far **LCM** can move an expression
- Sort subexpressions into ascending rank order
- Allows **LCM** to move subexpression each as far as possible

The Implementation

- Rewrite x - y + z as x + (-y) + z [Frailey 1970]
- Sort operands of associative ops by rank
- Distribute operations where both legal & profitable

Distribution

Room for more work on this issue

- Sometimes pays off, sometimes does not
- We explored one strategy: low rank x over high-rank +

Example

$\text{enter}(r_0, r_1)$
$r_3 \leftarrow r_0 + r_1$
$\text{if } r_3 > 100 \text{ branch}$

$r_2 \leftarrow 0$
$r_3 \leftarrow r_0 + r_1$
$r_4 \leftarrow r_3$
$r_5 \leftarrow r_2$

$r_a \leftarrow r_0 + 1$
$r_b \leftarrow r_a + r_1$
$r_7 \leftarrow r_b + r_5$
$r_8 \leftarrow r_4 + 1$
$r_4 \leftarrow r_8$
$r_5 \leftarrow r_7$

$r_8 \leftarrow r_4 + 1$
$\text{if } r_8 \leq 100 \text{ branch}$

$r_c \leftarrow r_0 + 1$
$r_d \leftarrow r_c + r_1$
$r_7 \leftarrow r_d + r_5$
$r_9 \leftarrow r_7$

$r_2 \leftarrow 0$
$r_9 \leftarrow r_2$

$\text{return } r_9$

After Reordering

Rewrite the code

- Rank expressions
- Sort operands of associative operations by their rank
- Convert back to binary operators

In example, r_7 was already in sorted order.

Name	*Rank*	*Name*	*Rank*
r_0	1	r_5	2
r_1	1	r_6	2
r_2	0	r_7	2
r_3	1	r_8	2
r_4	2	r_9	3

Making It Work with Lazy Code Motion

What have we done to the code, so far?

- Rewritten every expression based on global ranks
  - ♦ and local concerns of constant propagation ...
- Tailored order of evaluation for **LCM**
- Broken the name space that **LCM** needs
  - ♦ so, we cannot possibly run **LCM**

Undoing the damage

- Must systematically rename values to create **LCM** name space
- Can improve on the original name space, if we try
  - ♦ Choose names in a way that encodes values
- Need a global renaming phase

Renaming

The intuition

- Use Alpern *et al.*'s partitioning method
- Rename <u>every</u> value to expose congruences found by **AWZ**

The implementation

- x, y ∈ same congruence class ⇒ use same name
- Use hash table to regenerate consistent names
- Reserve variable names & insert copies

Reconstruct the 4 magic naming rules

Notes

- Clever implementation might eliminate some stores
- Variables become obvious from conflicting definitions

Any renaming scheme that builds the right name space will work.
We will see **AWZ** in a couple of lectures.

Example

$\text{enter}(r_0, r_1)$
$r_3 \leftarrow r_0 + r_1$
if $r_3 > 100$ branch

$r_2 \leftarrow 0$
$r_3 \leftarrow r_0 + r_1$
$r_4 \leftarrow r_3$
$r_5 \leftarrow r_2$

$r_6 \leftarrow r_0 + 1$
$r_7 \leftarrow r_6 + r_1$
$r_8 \leftarrow r_7 + r_5$
$r_9 \leftarrow r_4 + 1$
$r_4 \leftarrow r_9$
$r_5 \leftarrow r_8$

$r_9 \leftarrow r_4 + 1$
if $r_9 \leq 100$ branch

$r_6 \leftarrow r_0 + 1$
$r_7 \leftarrow r_6 + r_1$
$r_8 \leftarrow r_7 + r_5$
$r_{10} \leftarrow r_8$

$r_2 \leftarrow 0$
$r_{10} \leftarrow r_2$

return r_{10}

Now, reconstruct the **PRE** name space

- Use some global value numbering technique (**AWZ**, Simpson)
- Encode value identity in lexical identity

After renaming, compiler can run **PRE/LCM**

After Renaming

Results

What do we gain from all this manipulation?

- Can run **LCM** (or **PRE**) at any point in the optimizer
  - ♦ Can reconstruct the name space
  - ♦ Makes results independent of choices made in front end

- More effective redundancy elimination
  - ♦ Measured with **PRE** (not **LCM**)
  - ♦ Reductions of up to 40% in total operations (over **PRE**)

- Sometimes, code runs more slowly
  - ♦ Forward propagation moves code into loop
  - ♦ **PRE** cannot move it back out of the loop

Stronger methods can remove them, but this is a minor effect and ...

Example

$\mathrm{enter}(r_0, r_1)$
$r_3 \leftarrow r_0 + r_1$
if $r_3 > 100$ branch

$r_2 \leftarrow 0$
$r_4 \leftarrow r_3$
$r_5 \leftarrow r_2$
$r_6 \leftarrow r_0 + 1$
$r_7 \leftarrow r_6 + r_1$

$r_8 \leftarrow r_7 + r_5$
$r_4 \leftarrow r_9$
$r_5 \leftarrow r_8$

$r_9 \leftarrow r_4 + 1$
if $r_9 \leq 100$ branch

$r_8 \leftarrow r_7 + r_5$
$r_{10} \leftarrow r_8$

$r_2 \leftarrow 0$
$r_{10} \leftarrow r_2$

return r_{10}

PRE/**LCM** move code out of the loop

- Landing pad grows
- Loop body shrinks
  → *In this case, the split block in the back edge*

Role of **PRE** is placement

Name space trick makes redundancy aspect more effective, too.

- Example does not highlight that effect

After PRE

Example

$\text{enter}(r_0, r_1)$
$r_4 \leftarrow r_0 + r_1$
$\text{if } r_4 > 100 \text{ branch}$

$r_5 \leftarrow 0$
$r_6 \leftarrow r_0 + 1$
$r_7 \leftarrow r_6 + r_1$

$r_5 \leftarrow r_7 + r_5$

$r_4 \leftarrow r_4 + 1$
$\text{if } r_4 \leq 100 \text{ branch}$

$r_{10} \leftarrow r_7 + r_5$

$r_{10} \leftarrow 0$

$\text{return } r_{10}$

Chaitin-Briggs coalescing cleans up the copies

- Note the clean, small loop body
- Of course, Briggs & Cooper recommend aggressive coalescing
  - → *Despite what other authors say*
- Result is code that you might write yourself

After coalescing

Other Issues

Code Size

- Forward propagation has the potential for exponential growth in size
- Measured results
  - ♦ Average was 1.27x; maximum was 2.488; 1 of 50 was ≥ 2
- Stronger LCM methods avoid this problem by cloning, so …

Distribution

- Can destroy common subexpressions
- Has choice of shapes & can pick less profitable one

Interaction with other transformations

- Shouldn't turn multiplies into shifts until later
- Reassociation should let **OSR** find fewer induction variables

Issues Related to Lazy Code Motion

Lazy code motion makes significant improvements

- Sometimes, it misses opportunities
- Can only find textual subexpressions
- Array subscripts are a particular concern

LCM has its limitations

- Requires strict naming scheme
    - ♦ Can only run it once, early in optimization
    - ♦ Other optimizations will destroy name space
- Relies on lexical identity (*not value identity*)

Would like version of **LCM** that fixes these problems

Should be fast, easy to implement, & simple to teach …

⇒ *And, as long as I am wishing, it should operate directly on **SSA***

What is Left in Reassociation?

This approach works well for code motion, but …

- The Briggs scheme may not extend well to other problems
  - ♦ For example, it maximizes code motion but may eliminate some redundancies
  - ♦ Simple rank order is not enough; need consistent orders
- Not clear how to extend it for strength reduction
  - ♦ Want to reorganize in a way that minimizes the number of induction variables (demand for registers) and updates (arithmetic operations)
  - ♦ May need to solve an offline problem to choose best shape
- Eckhardt took a more general approach
  - ♦ Reassociation to help scalar replacement & cross-iteration redundancies
  - ♦ Much more involved approach
  - ♦ We will see this algorithm in the next lecture