

Computer-Aided Strategic Reasoning

SUGUMAN BANSAL

BASED ON

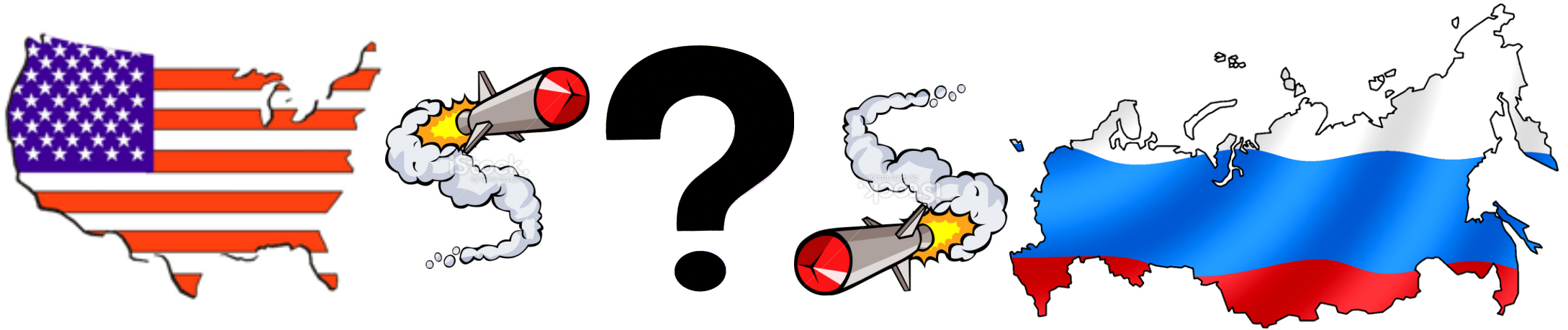
COMPUTING ALL EQUILIBRIA IN WEIGHTED REGULAR GAMES*

BY

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*WORK UNDER SUBMISSION AT AAI-16

Cold War (CW): A Strategic Interaction



Whether to attack or not?
What is the best decision?

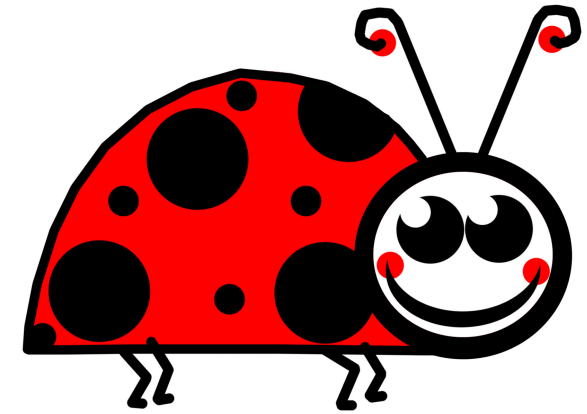
This is a **Strategic Interaction** in
System of Selfish Agents (SSA)

Analysis of SSAs



Manual and Laborious

Time-consuming



Error prone

Can Strategic Interactions be automated?

Formal Modeling of SSA?

How do we quantify payoffs?

What is a good strategy?

What is the mathematical model of such systems?

GAME THEORY

Mathematical study of strategic interactions

War or Peace (WP)



USSR \ USA	Don't Attack (D)	Attack (A)
	Don't Attack (D)	Attack (A)
Don't Attack (D)	2,2	0,3
Attack (A)	3,0	1,1

Strategy - A ,D

Profile - (D,A) USSR plays D, USA plays A

Payoff for USSR from (A,D) = 3

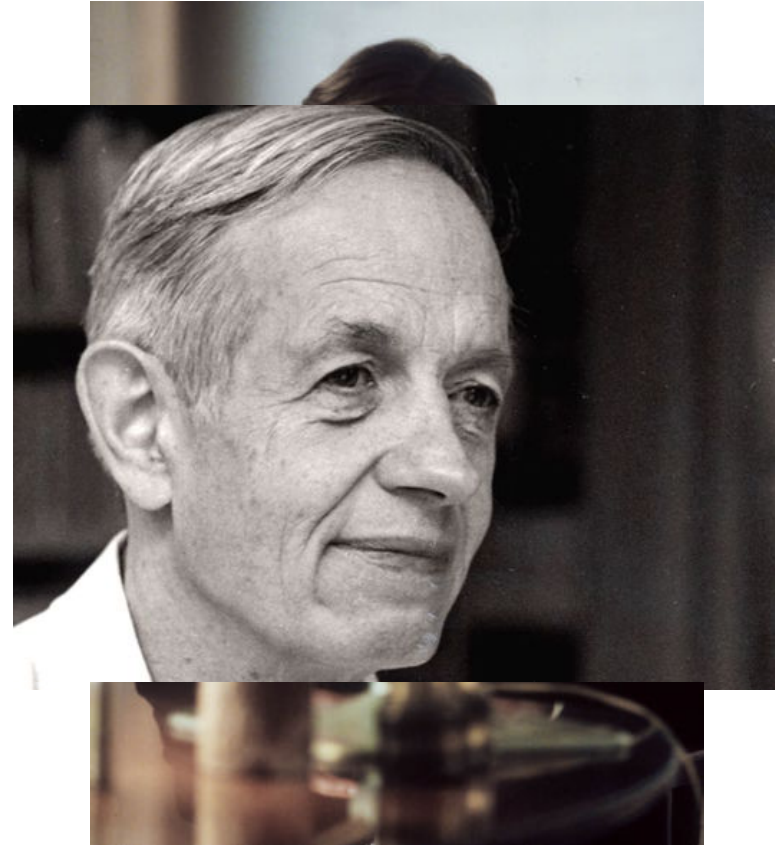
To attack or not to attack is the question!

Nash Equilibria

A profile is in Nash Equilibrium if no agent has an incentive to *unilaterally* deviate from the profile

USA USSR	Don't Attack (D)	Attack (A)
	Don't Attack (D)	Attack (A)
Don't Attack (D)	2,2	0,3
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(A, A) is the only Nash equilibrium



https://www.youtube.com/watch?v=2d_dtTZQyUM

What does Nash Mean?

- Each agent has to decide **independently**
- Each agent tries to **maximize their payoff**
- But agent's reward also **depends on other agents decisions**

USA USSR	Don't Attack (D)	Attack (A)
Don't Attack (D)	2,2	0,3
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Something's fishy...

Nash is when both agents Attack?

But this is not what was observed.

Where are we going wrong?

USA USSR \	Don't Attack (D)	Attack (A)
Don't Attack (D)	2,2	0,3
Attack (A)	3,0	1,1



Repeated Games

- When a one-shot game is played repeatedly
- Captures responses
- One-shot game is inaccurate model for real life scenarios
- Repeated Game are accurate model for real life scenarios
 - **Cold War - Repeated War or Peace**

USA USSR	Don't Attack (D)	Attack (A)
	Don't Attack (D)	Attack (A)
Don't Attack (D)	2,2	0,3
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Cold War (CW)

Profile = (D,A),(A,A),(D,A),(A,D),(A,D)...

Strategy for USSR = D,A,D,A,A ...

Payoff Sequence for USSR = 0, 1, 0, 3, 3 ...

Payoff for USSR = ???

Discount Factor = $1/2$

Payoff for USSR = $0 \cdot (1/2)^0 + 1 \cdot (1/2)^1 + 0 \cdot (1/2)^2 + 3 \cdot (1/2)^3 + 3 \cdot (1/2)^4 + \dots$

USA USSR \	Don't Attack (D)	Attack (A)
Don't Attack (D)	2,2	0,3
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Discounted Sum for sequence M with discount factor $(1/d)$

$$DS(M, d) = \sum_{i=0}^{\infty} m_i / d^i$$

**Model
Game?**

**All Nash Equilibria
In CW?**

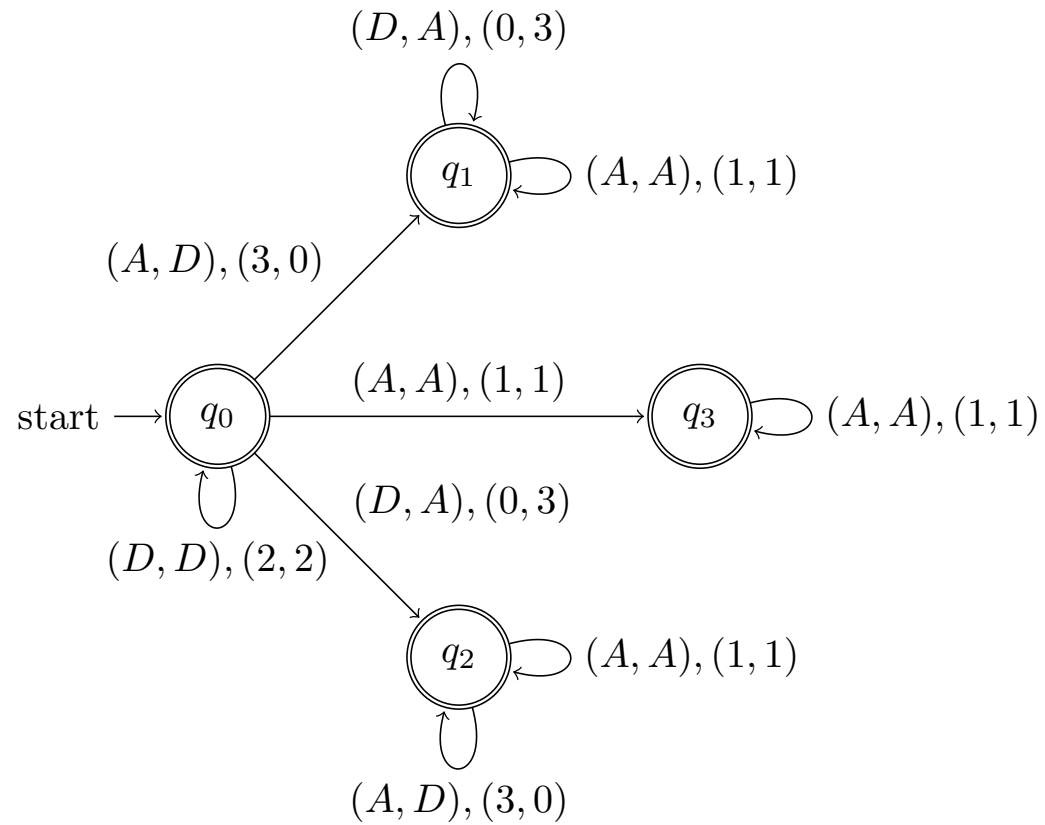
**Find
Nash?**

**Represent
All Nash
Equilibria?**

USA USSR	Don't Attack (D)	Attack (A)
	Don't Attack (D)	Attack (A)
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CW



USA \ USSR	Don't Attack (D)	Attack (A)
Don't Attack (D)	2,2	0,3
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**Model
Game**

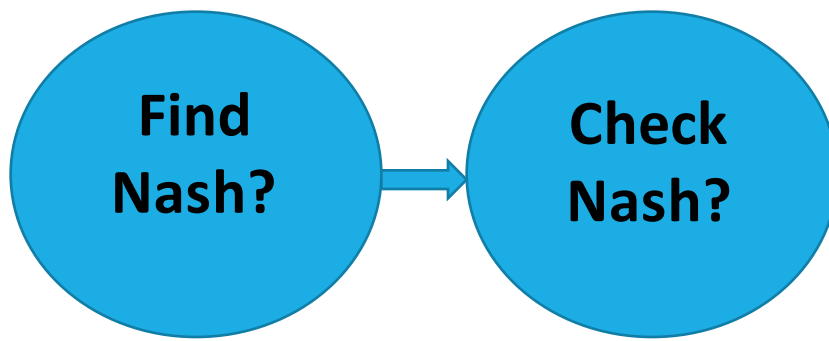
**All Nash Equilibria
In CW?**

**Find
Nash?**

**Represent
All Nash
Equilibria?**

USA USSR	Don't Attack (D)	Attack (A)
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Is (A,A) Nash?

- Check for USSR, try (D, A) - Pass
- Check for USA, try (A,D) - Pass
- If BOTH test pass, YES!

USA \ USSR	Don't Attack (D)	Attack (A)
Don't Attack (D)	2,2	0,3
Attack (A)	3,0	1,1



Is (D, A) Nash?

- Check for USSR, try (A, A) - Fail ← **Witness**
- Check for USA, try (D, D) - Pass
- If ANY ONE test fails, NO!

To do:

- Compare payoffs
- Identifying profiles with unilateral deviation

Find Non-Nash

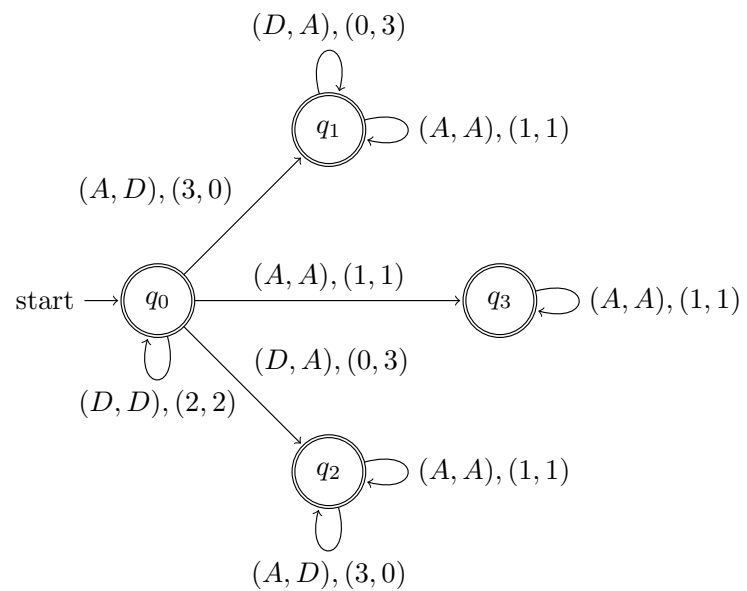
Automata for
**Comparing
Payoffs**

+

Automata for
**Identifying
Profiles with
unilateral
Deviation**

=

Automata for
finding Non-
Nash



×

+

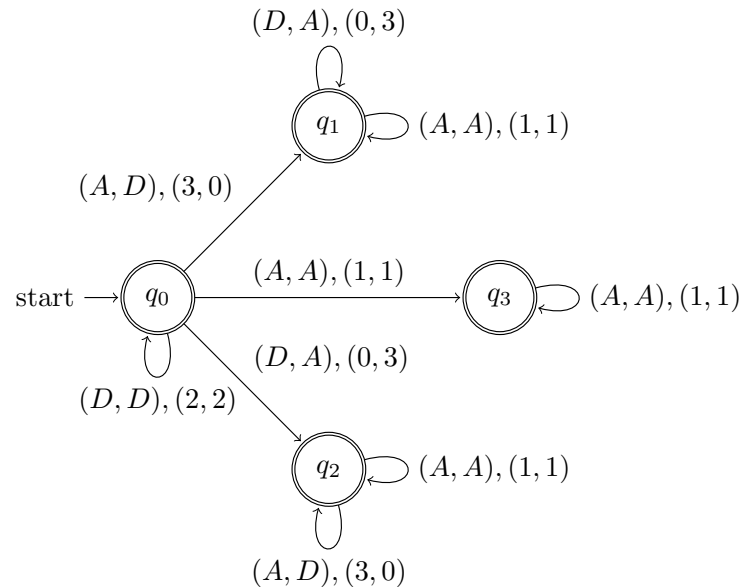
Automata for
finding Non-
Nash

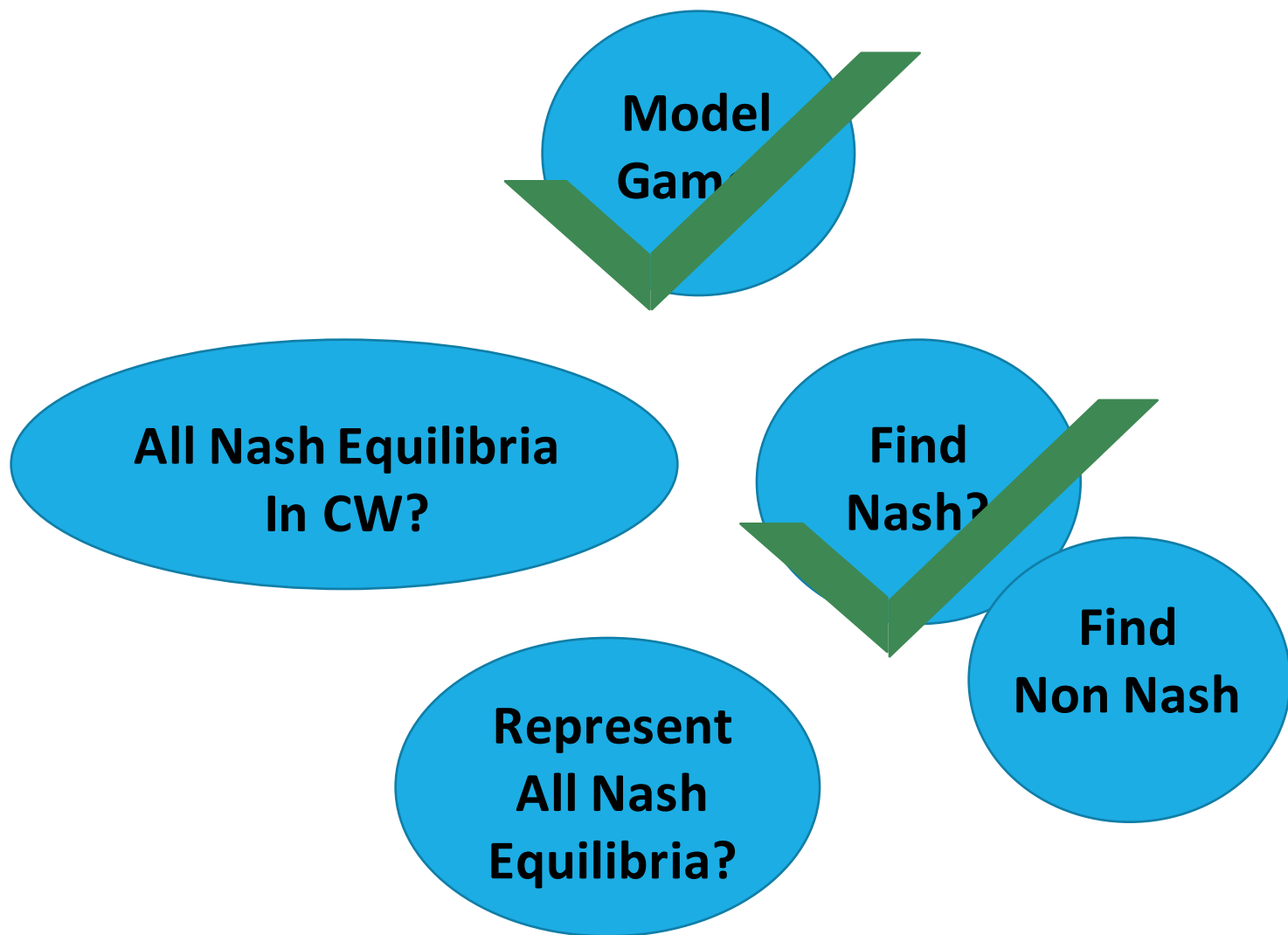
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Non-Nash
Strategy
Profiles

↓ Complement

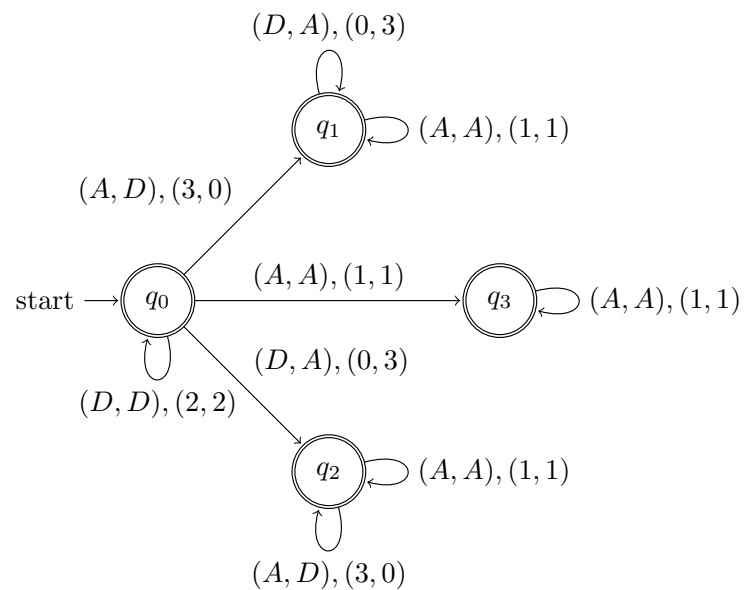
All Nash
Equilibria





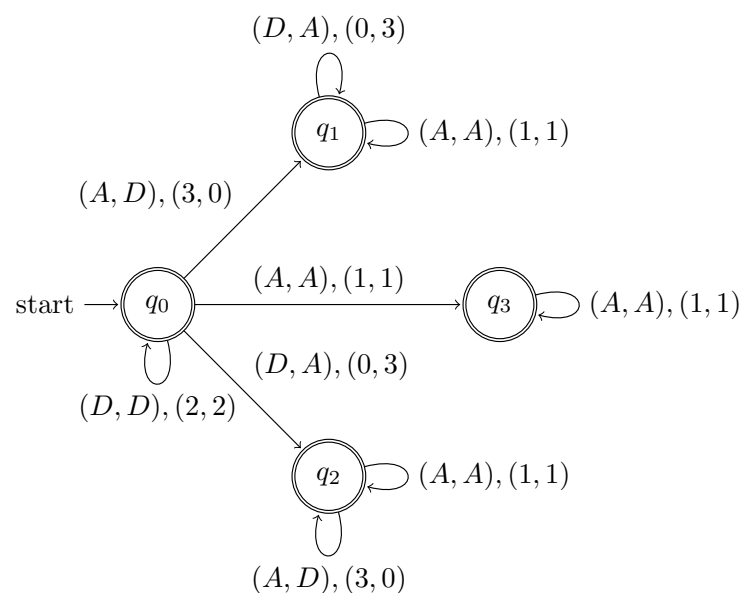
USA USSR	Don't Attack (D)	Attack (A)
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All are Automata operations!

Automata for
finding Non-
Nash

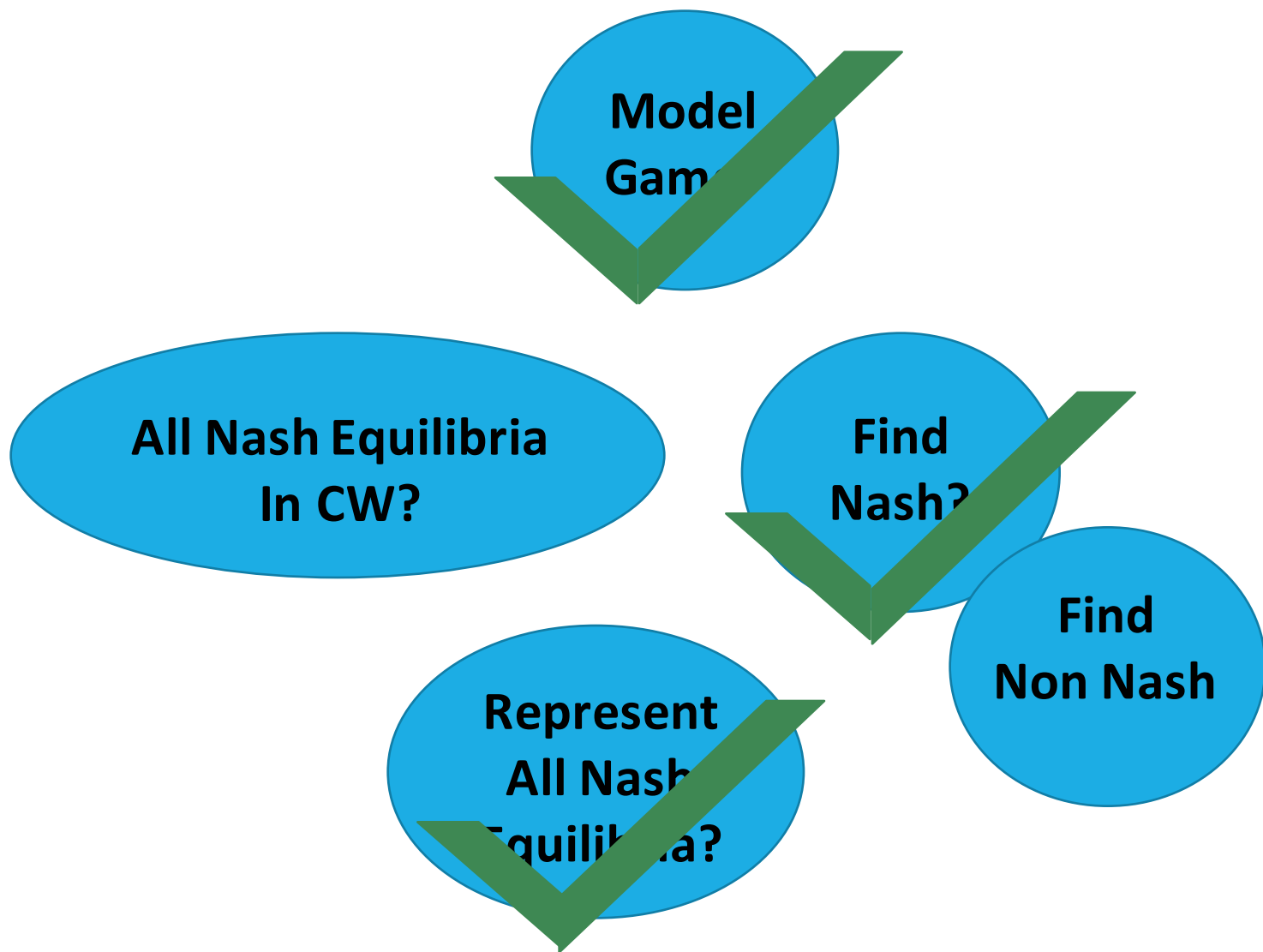
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Non-Nash
Strategy
Profiles

↓ Complement

Automata! →
- Finite representation

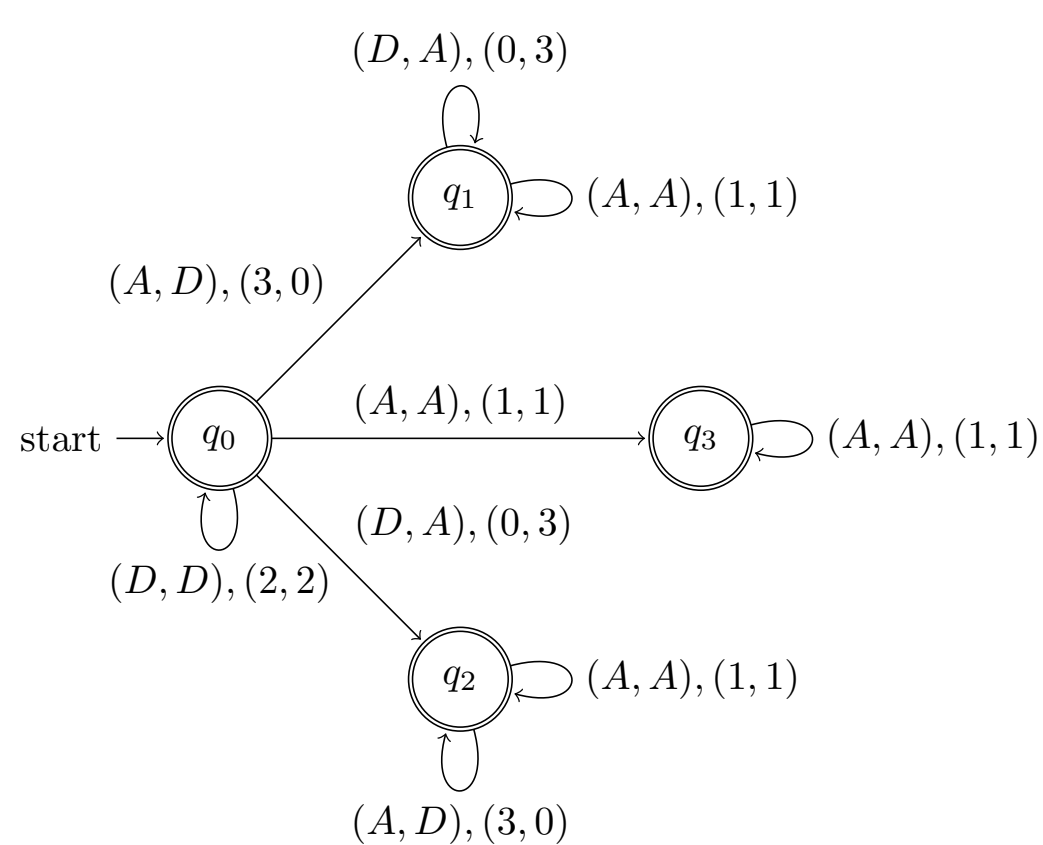
All Nash
Equilibria



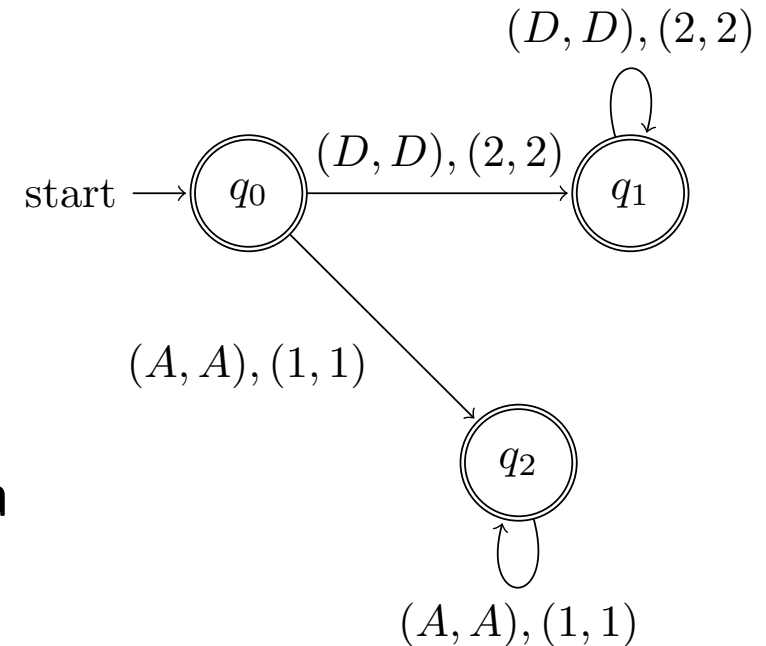
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Nash Equilibria in CW



Nash
Equilibria



Always Not-Attack or Always Attack!
Greater Payoff with Always Not-Attack

In Summary

- Most Real life interactions are Systems of Selfish Agents (SSA)
- Real life games can be studied as Repeated Games
- Repeated Games can be modelled by Weighted Regular Games
- Nash Equilibria of Weighted Regular Games form an automata (regular set)
 - Nash Equilibria are well structured (Rational behavior is not so complex!)
- Fun Fact: We owe our lives to Game Theory!

QUESTIONS?