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Self-healing for Mobile Robot Networks with Motion Synchronization

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***Abstract*—The objective of self-healing in mobile robot networks is to maintain not only logical topology but also physical one of a network when robots fail. An interaction dynamics model is first established to describe both logical and physical topologies of the network. Considering the mobility of mobile robot networks, we propose a recursive, distributed topology control for self-healing when mobile robots fail, and give a metric of the topology structure for evaluating the performence of recovered network topologies. Then, we prove the stability of motion synchronization with the topology control based on Lyapunov exponent. Finally, the results of simulations have demonstrated the validity of the proposed modeling and control methods.**

I. Introduction

MOTION synchronization of mobile robots has been defined as follows: a large number of mobile robots with local interacting could achieve a global objective of synchronizing speeds of all robots [1]. It is mainly inspired by the research on animal behavior [2-5], for example, self-organizing flocks, schools and swarms in nature always tend to keep synchronized motions. Synchronization is also a fundamental concept, and is a universal phenomenon in many research fields of science and technology, including sensor networks [6], complex dynamical networks [7] and communication networks [8]. In multi-robotics, many complex practical tasks and applications can be decomposed into a series of motion synchronization, for example large object transportation and manipulation [9], formation generation and maintenance [10], exploration and map-building in an unknown environment [11], and reconnaissance and surveillance for military missions.

Self-healing is viewed as one of these complex tasks based on motion synchronization. It refers to the process of recovering topologies and system performances of networks from failed robots. Clearly, the disadvantages of only self-healing communication topologies include more energy consumption with the enlarged communicating range in wireless networks, and more blind zones with the limited sensing range in sensor networks. Different with general networks, mobile robot networks possess the feature of the mobility of physical topologies. Therefore, the objective of self-healing in mobile robot networks is to maintain not only logical topology but also physical one of the network to solve the problem of energy consumption and blink zones.

The self-healing of sensor networks has been studied in sensor networks for many years. However, many literatures were focused on wireless sensor networks without mobility. Boonma *et al*. [6] proposed a biologically-inspired, decentralized architecture of sensor networks to self-heal false positive sensor data, with the mobility of mobile robot networks ignored. Furthermore, distributed control is one of the most challenging problems of designing self-healing control. Zhang *et al*. [1] proposed a topology control for self-healing and showed that the control can improve the robustness of motion synchronization, but its control was not completely distributed and the stability analysis of self-healing was not provided.

Stability of motion synchronization is one of the most important characteristics in groups of mobile robots, and is widely studied in the related research. Lyapunov function is the main method to analyze the stability. Gazi *et al*. [12] investigated the stability properties of swarm aggregation and showed that the individual agents will form a cohesive swarm in a finite time. Olfati-Saber *et al*. [13] gave the Lyapunov stability of disagreement functions for consensus problems in networks of agents with switching topology and time-delays. Although these conclusions about stability conditions have been successfully applied in their corresponding models and controls, it is necessary for other controls to search for the suitable Lyapunov functions. Therefore, we expect to provide a more general tool of analyzing the stability of motion synchronization for mobile robot networks.

In recent years, the research on synchronization in complex dynamical networks based on an interaction dynamics model has achieved many important conclusions on the stability and robustness [7, 14-16]. They have shown that synchronization of a complex dynamical network can be decided by the network topology and the maximum Lyapunov exponent of the individual nodes. However, the research on this topic has not involved in self-healing of mobile robot networks.

The main contribution of this paper is to provide a theoretical modeling and control method for self-healing of mobile robot networks with motion synchronization. We propose a fully distributed topology control for self-healing, and give a metric of the topology structure to evaluate the performance of the network topology. Based on Lyapunov exponent, we also prove the global stability of the network with the proposed topology control.

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The paper is organized as follows. Section II establishes an interaction dynamics model of mobile robot network. In Section III, we propose a recursive, distributed topology control for self-healing. Section IV proves the stability of the network with the topology control. The results of simulations in Section V have demonstrated the validity of the topology control for self-healing. Conclusions are given in Section VI.

II. MODEL OF MOBILE ROBOT NETWORK

Now we establish an interaction dynamics model describing logical and physical topologies of a network. In this paper, we assume that each robot only knows the information about its neighboring ones without any time delay of communication.

A. Logical Topology of Mobile Robot Network

Consider a network having n robots, whose logical topology can be viewed as a graph G defined by a set of vertices V and a set of edges E connecting the vertices. The vertex $v_i \in V$, $i = 1, 2, \cdots, n$, corresponds to the robot i, and the edge $\left(v_i, v_j\right) \in E$, $i, j = 1, 2, \cdots, n$, corresponds to an active communication connection between the robots i and j. The coupling matrix $A = \left(a_{ij}\right) \in \Re^{n\times n}$ represents the coupling configuration of the network [1, 7], where

$$a_{ij} = \begin{cases} 0, \left(v_i, v_j\right) \notin E \\ 1, \left(v_i, v_j\right) \in E \end{cases}, \quad i \neq j\,, \quad i, j = 1, 2, \cdots, n\,, \tag{1}$$

indicates which vertices are linked by an edge, and thus which robots communicate. Moreover,

$$a_{ii} = -\sum_{j=1, j\neq i}^{n} a_{ij} = -d_i\,, \; i = 1, 2, \cdots, n\,. \tag{2}$$

where d_i denotes the *degree* of the robot i.

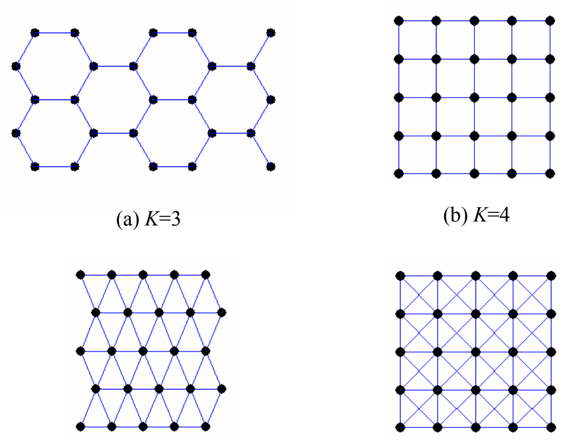
Remark: In this paper, the coupling matrix A is somewhat different with the adjacency matrix in graph theory [17].

Because of bi-directional communication between two robots, the graph G representing the network is undirected, i.e., $A^T = A$. Suppose that the network is connected without isolate clusters. Then G is *connected*. The *neighbors* of the robot i are the robots having active connection with i. Let $N_{\mathrm{e}}(i)$ denote the index set of neighbors of i, i.e.,

$$N_{\mathrm{e}}(i) \triangleq \left\{ j \mid a_{ij} = 1 \right\} \subseteq \left\{1, 2, \cdots, n\right\}. \tag{3}$$

Furthermore, the degree of the robot i is equal to the number of its neighbors.

The topologies of mobile robot networks are often described by K-neighbor models [18], where the degree of each robot is required to be equal to or less than K, i.e., $d_i \leq K$, $i = 1, 2, \cdots, n$. For example, Fig. 1 shows four typical topologies of robot networks with K=3, 4, 6, and 8, respectively.



(a) K=3 (b) K=4 (c) K=6 (d) K=8

Fig. 1 Four typical topologies of mobile robot networks with K-neighbor model.

B. Motion Model of Mobile Robot Network

Consider each robot operating in an N-dimensional space. Let $p_i \in \Re^N$ and $x_i \in \Re^N$ with $i = 1, 2, \cdots, n$ denote the position and speed of the robot i, respectively. According to the motion models in [1, 7, 12, 13], we propose a double integrator system to describe the continuous-time dynamics of each mobile robot:

$$\begin{cases} \dot{p}_i = x_i, \\ \dot{x}_i = u_i^{\mathrm{e}} + u_i^{\mathrm{c}}, \quad i = 1, 2, \cdots, n, \end{cases} \tag{4}$$

where u_i^{e} stands for the individual N-dimensional dynamics of the robot i, expressed as:

$$u_i^{\mathrm{e}} = f(p_i, x_i)\,. \tag{5}$$

For example,

$$f(p_i, x_i) = a_{\mathrm{m}}(v(t) - x_i)\,, \; i = 1, 2, \cdots, n\,, \tag{6}$$

where $a_{\mathrm{m}} > 0$ and $v(t)$ represents the value of a velocity field in the environment. The control u_i^{c} stands for the effect of the neighboring robots upon the robot i. Given a whole number $k > 0$ and a switched matrix $A(k)$, u_i^{c} can be described as:

$$u_i^{\mathrm{c}} = c\sum_{j=1}^{n} a_{ij}(k)\cdot(p_j - q_{ij}) + c\sum_{j=1}^{n} a_{ij}(k) x_j\,, \; i = 1, 2, \cdots, n\,, \tag{7}$$

where $c > 0$ represents the coupling strength of the network, and $q_{ij} = p_j^{\mathrm{d}} - p_i^{\mathrm{d}}$ represents the desired relative position of the robots j and i, with p_j^{d} standing for the synchronized (desired) position of the robot j. In this paper, q_{ij} represents the physical topology of the network.

The benefit of the proposed model is that it allows one can describe not only logical topology but also physical one of a mobile robot network.

III. TOPOLOGY CONTROL FOR SELF-HEALING

Self-healing represents the self-organized collective behavior of mobile robot networks when robots fail. It refers to the process of filling the blank locations of the failed robots and recovering both logical and physical topologies from robot failures. The topology control for self-healing should satisfy:

1) It can prevent the whole network from being separated into two or more small groups.

2) It can recover the network topology from failed robots and keep system properties of the whole network with regard to the stability and robustness of motion synchronization.

A. Network Topology with Failed Robots

Now we study the network topology when robots fail. In [1, 19], the second-largest eigenvalue λ_2 of the coupling matrix A is used to evaluate the stability and robustness of the whole network. A small value of λ_2 indicates the better stability and robustness of motion synchronization of the network [1, 7]. Moreover, $\lambda_2 = 0$, iff the network is broken into two or more groups.

Let $A_{\mathrm{rf}} \in \Re^{n\times n}$ and $\tilde{A}_{\mathrm{rf}} \in \Re^{(n-[\Delta n])\times(n-[\Delta n])}$ denote the coupling matrices of the original network with n robots and the new network after failure of $[\Delta n]$, respectively. Here, $[\Delta n]$ stands for the smaller but nearest integer to the real number Δn. Let also $\lambda_{2\mathrm{rf}}$ and $\tilde{\lambda}_{2\mathrm{rf}}$ denote the second-largest eigenvalues of A_{rf} and $\tilde{A}_{\mathrm{rf}}$, respectively. Moreover, assume that the robots i_1, $i_2, \cdots, i_{[\Delta n]}$ have failed in the network. Then the new coupling matrix $\tilde{A}_{\mathrm{rf}}$ from the original matrix A_{rf} can be formed according to [1, 19].

Ref. [19] guaranteed that if a small fraction $\Delta (0 < \Delta << 1)$ of robots whose degrees are sufficiently small $d_i < d_0, i = 1,2,\cdots,[\Delta n]$ are deleted in the network ($n >> 1$), then

$$\tilde{\lambda}_{2\mathrm{rf}} \approx \lambda_{2\mathrm{rf}} . \tag{8}$$

It means that the second-largest eigenvalue of the coupling matrix is held almost fixed so that the synchronization stability of the network also remains. Assume that the degree of each robot in the K-neighbors model is far less than the size of the whole network, i.e., $K \ll n$, then (8) can be also applied in our model, because the degree of each robot $d_i \le K$, $i = 1,2,\cdots,n$ is sufficiently small.

Ref. [1] has shown that the whole network will be broken into two small groups when the robots in one row or column fail, that is $\tilde{\lambda}_{2\mathrm{rf}} = 0$. Therefore, it is necessary to self-heal the network topology from failed robots.

B. Self-healing Rules and Algorithm

Through controlling the network topology [20], the network can fill the blank locations of the failed robots. Let $A(1) = \tilde{A}_{\mathrm{rf}}$ denote the coupling matrix after robots fail. Assume that the robot i_{f} has failed with its neighbors $N_{\mathrm{e}}(i_{\mathrm{f}})$. Our topology control for self-healing can be described through the following rules.

Rule 1: The neighbors $j \in N_{\mathrm{e}}(i_{\mathrm{f}})$ of i_{f} with $d_j \le d_{i_{\mathrm{f}}}$ become the candidate of filling the failed robot, whose the index set is denoted by:

$$C_{\mathrm{a}}(i_{\mathrm{f}}) \triangleq \left\{ j \mid d_j \le d_{i_{\mathrm{f}}}, j \in N_{\mathrm{e}}(i_{\mathrm{f}}) \right\}. \tag{9}$$

Rule 2: The robot in $C_{\mathrm{a}}(i_{\mathrm{f}})$ with the smallest degree will fill the blank location of the failed robot, denoted by i_{s}.

Rule 3: If there exist two or more robots with the same, smallest degree, then one robot is randomly chosen to be the filling robot i_{s}. Furthermore, a mark $M = q_{i_{\mathrm{s}} i_{\mathrm{f}}}$ is generated and sent to the robot i'_{s}, $q_{i'_{\mathrm{s}} i_{\mathrm{s}}} = M$, $i'_{\mathrm{s}} \in N_{\mathrm{e}}(i_{\mathrm{s}})$. In next step, the robot i'_{s} will fill the new blank of i_{s}, if the neighbors $j \in C_{\mathrm{a}}(i_{\mathrm{s}})$ does not satisfy *Rule 2*. The mark M will also be sent to the next robot until not finding the corresponding robot.

Note that the mark is used to avoid self-healing repeatedly for one robot. Thus, it forms a recursive, distributed topology control algorithm of self-healing as follows.

Self-healing Algorithm:

Step 1: $k = 1$. Calculate the coupling matrix $A(k)$.

Step 2: $k = k+1$. If one robot obtains the information about its neighbor's failure, it will compare its degree with $d_{i_{\mathrm{f}}}$ and judge whether it belongs to $C_{\mathrm{a}}(i_{\mathrm{f}})$.

Step 3: If there exists a robot with the smallest degree in $C_{\mathrm{a}}(i_{\mathrm{f}})$, it is the filling robot i_{s}. Go to Step 6.

Step 4: If there exist two or more robots with the same, smallest degree, the robot with the mark M in $C_{\mathrm{a}}(i_{\mathrm{f}})$ will be the filling robot i_{s}. Moreover, the robot i_{s} will send M to the corresponding robot in its neighbors. Go to Step 6.

Step 5: If there exist two or more robots with the same, smallest degree and without a mark, then one robot is chosen randomly in $C_{\mathrm{a}}(i_{\mathrm{f}})$ with the smallest degree to be the filling robot i_{s}. A mark M is generated according to *Rule 3*.

Step 6: The action of filling the blank location of the failed robot is described here. The filling robot i_{s} cuts off the connections with its origin neighbors $N_{\mathrm{e}}(i_{\mathrm{s}})$, and creates new connections with $N_{\mathrm{e}}(i_{\mathrm{f}})$. Thus, the coupling matrix $A(k)$ is obtained by:

$$\begin{cases} a_{ij}(k) = a_{ij}(k-1),\ i, j = 1,2,\cdots,n-[\Delta n], \\ a_{i_{\mathrm{s}}j}(k) = a_{ji_{\mathrm{s}}}(k) = 0,\ j \in N_{\mathrm{e}}(i_{\mathrm{s}}), \\ a_{i_{\mathrm{s}}j}(k) = a_{ji_{\mathrm{s}}}(k) = 1,\ j \in N_{\mathrm{e}}(i_{\mathrm{f}}), \\ a_{ii}(k) = -\sum_{j=1, j\ne i}^{n} a_{ij}(k), \quad i = 1,2,\cdots,n. \end{cases} \tag{10}$$

Other failed robots can also be substituted by (10).

Step 7: If a new blank location appears, then go to Step 2. Otherwise, self-healing finishes.

C. Properties of the Switched Topology with Self-healing

Now we study the properties of the switched topology with *Self-healing Algorithm*. Assume that the network is connected after robots fail, i.e., $\lambda_2(1) \neq 0$.

The aim of *Self-healing Algorithm* is to substitute the filling robots i_{s_1}, i_{s_2},…, $i_{s_{[\Delta n]}}$ for the failed ones i_1, i_2,…, $i_{[\Delta n]}$ with $d_{s_j} \leq d_j$, $j = 1, 2, \cdots, [\Delta n]$. In *K*-neighbors models, the process of self-healing is to move the blank locations of the failed robot out of the network. Let k_{s} denote the final step number of self-healing.

Lemma: Consider the mobile robot network (4). It follows with *Self-healing Algorithm* when robots fail. Then, the following statements hold:

1) The whole network is prevented from being separated into two or more small groups, i.e., $\lambda_2(k_{\mathrm{s}}) \neq 0$.

2) The network topology recovers from failed robots and the second-largest eigenvalue of the coupling matrix of the network is kept, i.e., $\lambda_2(k_{\mathrm{s}}) \approx \lambda_2(k_{\mathrm{s}}-1) \approx \cdots \approx \lambda_2(1)$.

Proof: First, to prove statement 1), suppose that $\lambda_2(k_{\mathrm{p}}) = 0$ with $k_{\mathrm{p}} > 0$. Because $\lambda_2(1) \neq 0$, it is clear that the new blank positions causes the broken network. Then the neighbors of the filling robot will fill the new blank positions, and therefore self-healing will continue, i.e., $k_{\mathrm{p}} < k_{\mathrm{s}}$. According to *Rule 1-3*, the blank locations of the failed robots are moved out of the network finally. Consequently, $\lambda_2(k_{\mathrm{s}}) \neq 0$.

Second, prove statement 2). Because the robot with the small degree substitutes the failed robot for each step, i.e., $d_{s_j} \leq d_j$, $j = 1, 2, \cdots, [\Delta n]$, based on (8) we have

$$\lambda_2(k_{\mathrm{s}}) \approx \lambda_2(k_{\mathrm{s}}-1) \approx \cdots \approx \lambda_2(1). \tag{11}$$

Consequently, *Self-healing Algorithm* is effective. □

IV. Stability Analysis

Hereafter, the mobile robot network (4) is said to achieve synchronous speeds if

$$x_1(t) = x_2(t) = \cdots = x_n(t) \to v(t), \quad t \to \infty, \tag{12}$$

where $v(t) \in \Re^N$ can be an equilibrium speed, a leader's speed or a value of a velocity field, given by (6). Now we investigate the stability condition of motion synchronization with *Self-healing Algorithm*.

First, the model (4) can be expressed as:

$$\begin{bmatrix} \dot{p}_i \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} x_i \\ f(p_i, x_i) \end{bmatrix} + c\sum_{j=1}^{n} a_{ij}(k) \begin{bmatrix} 0 & 0 \\ I_N & I_N \end{bmatrix} \begin{bmatrix} (p_j - q_{ij}) \\ x_j \end{bmatrix}, \tag{13}$$
$$i = 1, 2, \cdots, n, \quad k = 1, 2 \cdots, k_{\mathrm{s}},$$

where I_N denotes the *N*-dimensional identity matrix. Let $q_i = p_i^{\mathrm{d}} - p_1^{\mathrm{d}}$ denote the desired relative distance between the robot i and 1. Note that q_i just simplifying the following analysis does not affect the distributed control for self-healing.

Denote $\Gamma = \begin{bmatrix} 0 & 0 \\ I_N & I_N \end{bmatrix}$ and $y_i = \begin{bmatrix} p_i - q_i & x_i \end{bmatrix}^T \in \Re^{2N}$, $i = 1, 2, \cdots, n$, and rewrite (13) as:

$$\dot{y}_i = F(y_i) + c\sum_{j=1}^{n} a_{ij}(k)\Gamma y_j, \; i = 1, 2, \cdots, n, \tag{14}$$
$$k = s(t), \; A(k) \in \Omega,$$

which is a *hybrid system* with a continuous-state y_i and a discrete-state $A(k)$ that belongs to a finite set of switched coupling matrices $\Omega = \{A(k), k = s(t)\}$. Let $s(t): \Re_{\geq 0} \to \Lambda_\Omega$ denote a switching signal where $\Lambda_\Omega \subset \mathbb{N}$ is the index set associated with the elements of Ω.

Note that the relative positions of the neighboring robots are also stable when achieving the synchronized state (12). Let $p_i' = p_i - q_i$, and the state p_i' is synchronized at the stable states:

$$p_1'(t) = p_2'(t) = \cdots = p_n'(t) = p_1(t), \quad t \to \infty. \tag{15}$$

Denote $S(t) = \begin{bmatrix} p_1(t) & v(t) \end{bmatrix} \in \Re^{2N}$. Thus, the synchronized states of the dynamical network (14) becomes

$$y_1(t) = y_2(t) = \cdots = y_n(t) = S(t), \quad t \to \infty. \tag{16}$$

Assume that the network is connected without isolate robots. Then its coupling matrix $A(k)$ is therefore a symmetric irreducible matrix. In this case, zero is an eigenvalue of $A(k)$ with multiplicity 1 and all the other real eigenvalues of $A(k)$ are strictly negative [7], denoted by

$$0 = \lambda_1(k) \geq \lambda_2(k) \geq \lambda_3(k) \geq \cdots \geq \lambda_n(k), k = s(t). \tag{17}$$

Denote by $h_{\max} = h_1, h_2, \cdots, h_N$ the corresponding Lyapunov exponents of each individual *N*-dimensional mobile robot. If it is stable, the maximum Lyapunov exponents $h_{\max}$ will be negative. In the following, assume all the individual robots are stable with $h_{\max} < 0$. Assume that any interswitching time is enough large during switching topologies.

Theorem: Consider the network (14) with identical stable robots. It follows with *Self-healing Algorithm* when robots fail. Then, the synchronized states (16) are exponentially stable.

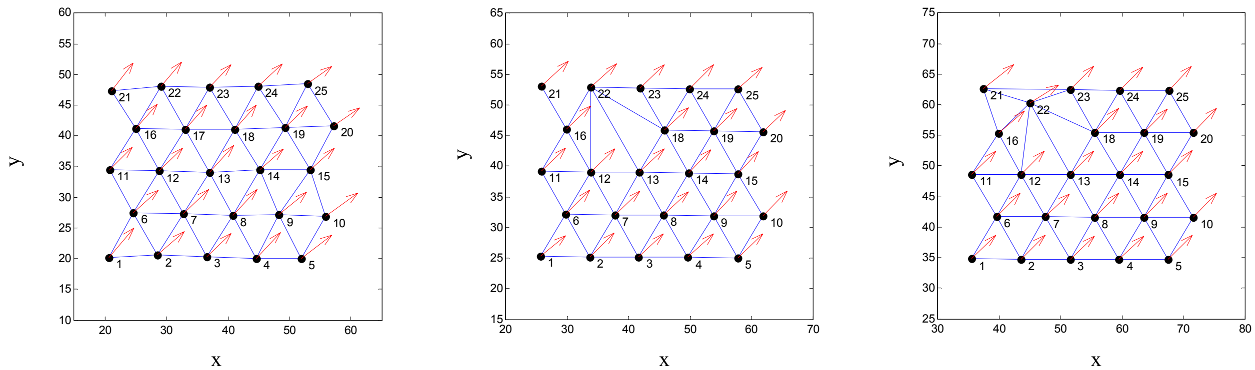
Proof: The exponential stability of (16) is transformed to the exponential stability of the following systems [7]:

$$\dot{\omega} = [DF(s) + c\lambda_i(k)\Gamma]\omega, i = 1, 2, \cdots, n, k = s(t), \tag{18}$$

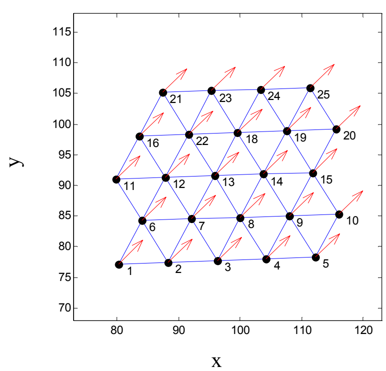
where $DF(s)$ represents the Jacobian of $F(s)$ about *s*. To prove its stability, recall the concept of *transversal Lyapunov exponents* [21]. For every eigenvalue $\lambda_i(k), i = 2, \cdots, n$, $k = s(t)$, the corresponding transversal Lyapunov exponents for (18) is a function of $\lambda_i(k)$, denoted as $L_j(\lambda_i(k))$, $j = 1, 2, \cdots, N$, and given by

$$L_j(\lambda_i(k)) = h_j + c\lambda_i(k), \; j = 1, 2, \cdots, N, \; k = s(t). \tag{19}$$

To stabilize the synchronized states, if suffices to require that



(a) Topology at t=1.5 (s) (b) Topology at t=2.0 (s) (c) Topology at t=3.5 (s)



(d) Topology at t=7.5 (s)

Fig. 2 Simulation results of self-healing for one failed robot.

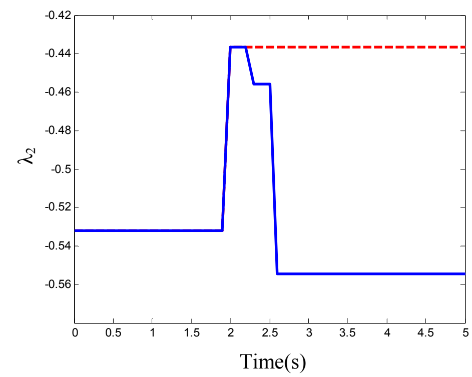


Fig. 3 Second-largest eigenvalue of coupling matrix in the case of self-healing for one failed robot.

these $L_j(\lambda_i(k))$ for each $\lambda_i(k), i = 2, \cdots, n, k = s(t)$ be negative. According to *Lemma*, when satisfying (17) and $h_{\max} < 0$, it indicates that the following equivalent inequality should hold:

$$L_{\max}(\lambda_i(k)) = h_{\max} + c\lambda_2(k) < 0\,, k = s(t)\,. \qquad (20)$$

The minimum in (20) always exists and is achieved because Ω is a finite set. That is, the synchronized states (16) are exponentially stable with *Self-healing Algorithm*. □

V. SIMULATIONS

The simulations utilized the programming of MATLAB, and performed the process of self-healing in mobile robot networks. The operating space of robots was a 2-dimensional plane $\Re^2$ in simulations, i.e., $N = 2$. The step length in simulations was 0.05 (s). The initial positions were selected randomly within the interval [-1, 1] around the corresponding positions according to K-neighbor model with K=6, and the initial speeds were chosen randomly with [-5, 5].

A. Self-healing for One Failed Robot

We performed two simulations of the proposed topology control for self-healing in mobile robot networks when robots fail. In the first simulation, one robot failed in a network of 25 robots ($n = 25$). The total time of experiments was 10 seconds and other parameters were configured as above. The results of the simulation were shown in Fig. 2 and 3.

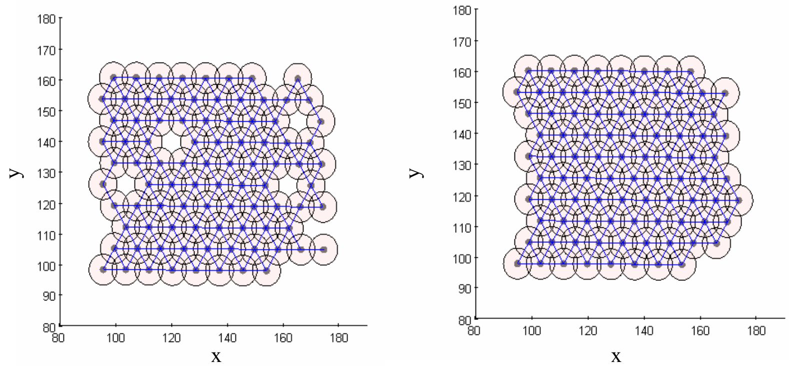
In Fig. 2, the plots of (a)-(d) represent the topologies of the mobile robot network at the time t=1.5, 2.0, 3.0 and 7.5 (s), respectively, where the black points marked by numbers and the red arrows are the positions and speeds of the robots, correspondingly. Fig. 2(a) shows the original topology of the network. Robot 17 was failed at the time t=2.0. Because robot 22 and 23 had the same, smallest degree (d=3) in the neighbors of 17. The robot 22 was randomly chosen to fill in the blank position of 17 in Fig. 2(b) according to *Self-healing Algorithm*. Meanwhile, robot 21 filled in the new blank position of 22 in Fig. 2(c). Because there was no robot with the smaller degree than 21, self-healing finished. Fig. 2(d) shows the final topology after self-healing.

In Fig. 3, the second-largest eigenvalue of the coupling matrix decreased from -0.4367 to -0.5889 with the topology control. It means that the topology control is effective to keep system performance of the network fixed.

B. Self-healing for Failed Robots

Now we illustrate the second simulation to demonstrate the self-healing behavior in a robot network of 100 robots with K=6 for failed robots. In the simulation, 10 robots failed randomly in the network. The total time of experiments was 10 seconds and other parameters were configured as above.

In Fig. 4, the plots of (a) and (b) represent the topologies



(a) Topology before self-healing (b) Topology after self-healing

Fig. 4 Simulation results of self-healing for failed robots.

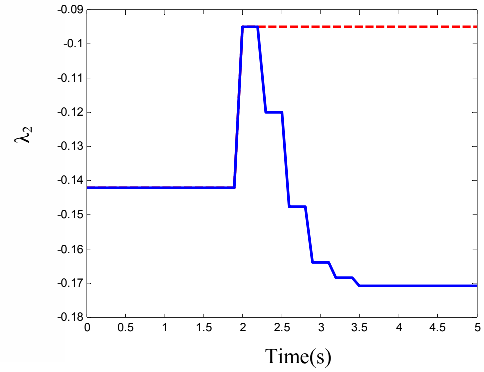


Fig. 5 Second-largest eigenvalue of coupling matrix in the case of self-healing for failed robots.

before and after self-healing, respectively, where the circle represents the sensing range of the robot. Obviously, self-healing has decreased the blind zones of sensing in the network. The dashed and the solid lines in Fig. 5 represent the variation of the second-largest eigenvalue of the coupling matrix for Fig. 4(a) and (b) with the time, respectively. Fig. 5 shows that the whole network spends six steps for self-healing, and the second-largest eigenvalue of the coupling matrix decreases, which indicated that the proposed topology control for self-healing is effective.

VI. Conclusions

Self-healing in mobile robot networks refers to the process of maintaining not only logical topology but also physical one of the network. We first establish an interaction dynamics model describing both logical and physical topologies of the network. Considering the mobility of mobile robot network, we provide a recursive, distributed topology control for self-healing when mobile robots fail, and give a metric of the topology structure for evaluating the performance of recovered network topologies. Based on Lyapunov exponent, we prove the global stability of the network with the fully distributed topology control. Finally, the results of simulations have demonstrated the validity of the proposed modeling and control methods.

The conclusions about self-healing in mobile robot networks can be also applied in mobile sensor networks.

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