Reading: OSB: Chapter 3

2.1 $z$-Transforms
Find the causal inverse $z$-transforms of the following.

(a) $X(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}$

(b) $X(z) = \frac{1}{1 - \frac{1}{3}z^{-3}}$

(c) $X(z) = \exp\{z^{-1}\}$

(d) $X(z) = \frac{z^3 - 2z}{z - 2}$

2.2 Causal and Noncausal Filtering
A system is described by the transfer function

$$H(z) = \frac{(z + 1)^2}{(z - 1/2)(z + 3/4)}$$

(a) Find the causal and purely noncausal unit-sample responses corresponding to this transfer function.

(b) What is the difference equation in each case?

(c) Which of these filters is stable? Why?

2.3 Making filters work
An ELEC 431 student found a filter (by trial-and-error) that has the frequency response he wants.

$$H(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - 2\sqrt{2}z^{-1} + 4z^{-2}}$$

(a) Is his proposed filter be causal and stable?

(b) Find a better filter that has the same magnitude as his, but with nicer properties?

2.4 Autocorrelation
The correlation function of the signal $x(n)$ is defined to be

$$R_x(l) = \sum_{n=-\infty}^{\infty} x(n)x(n + l)$$

The index $l$ is known as the lag.

(a) Show that correlation functions are even functions.
(b) Show that the $z$-transform of the correlation function equals

$$X(z)X(z^{-1}).$$

(c) If $x(n) = a^n u(n)$, find its correlation function. Sketch the pole-zero plot of the correlation function and indicate the region of convergence.

(d) Find a signal not equal to the signal in the previous part but which has the same correlation function.

(e) Define the causal part of an even signal to be that signal having an even part equal to the given signal. In other words, if $s(n)$ is an even signal, its causal part, $s_c(n)$, satisfies

$$s(n) = \frac{s_c(n) + s_c(-n)}{2}, \quad s_c(n) = 0, \quad n < 0$$

Show that the $z$-transform of $s_c(n)$ has poles located where the $z$-transform of $s(n)$ has poles, but that the zeros don’t necessarily agree. Which poles are shared?