Electrical Engineering 431  
Problem Set IV  
Due: February 23, 2004

**Reading:** OSB: Chapter 7

### 4.1 Filter Design
Your boss wants you to design a digital filter according to the following specifications.

\[
0.99 \leq |H(e^{j2\pi f})| \leq 1.01, \quad 0 \leq f \leq 0.1  \\
|H(e^{j2\pi f})| \leq 10^{-2}, \quad f \geq 0.15
\]

(a) Design a Chebyshev filter using the bilinear transform that meets these specifications. Use Matlab to show the frequency response and the pole-zero plot.

(b) Use the Parks-McClellan algorithm to design a zero-phase FIR filter that meets these specifications.

(c) Compare the magnitude and phases of your two filters.

(d) What are the implications of the orders of the filters you found? In particular, which is computationally more efficient (in other words, which requires fewer computations per output value)?

### 4.2 Bilinear Transforms
An integrator has the transfer function \( H_A(s) = \frac{1}{s} \). We form a discrete-time filter “equivalent” to the integrator by using the bilinear transform.

(a) What is the discrete-time transfer function \( H_D(z) \) and the corresponding unit-sample response?

(b) Compare the frequency responses of the analog and discrete-time filters. When will the discrete-time system well approximate an integrator?

(c) Suppose instead we have a differentiator and it too is mapped into a discrete-time system using the bilinear transform. Are they inverses of each other? In other words, will “integration” followed by “differentiation” yield the original discrete-time signal? Do you predict numerical problems with this cascade?

### 4.3 Relating Analog Systems to Discrete-Time Ones
One way of relating analog systems to discrete-time ones is through the usual approximation of a derivative by a finite difference.

\[
\frac{dx(t)}{dt} \approx \frac{x(t) - x(t - T)}{T}
\]

Assume that discrete-time signals are related to analog ones by \( x(n) = x(nT) \).

(a) Show that relating the derivative to a finite difference corresponds to mapping the \( s \)-plane onto the \( z \)-plane. What is the mapping?
(b) What contour in the $z$-plane corresponds to $s = j2\pi f$ in the $s$-plane? Do causal and stable analog systems map to causal and stable discrete-time ones?

(c) Can any “off-the-shelf” stable and causal discrete-time system be related to a causal and stable analog system this way?

(d) Suppose you simulated the solution of a differential equation by replacing derivatives by finite-differences? How would you convert the computed solution into the analog solution?