Electrical Engineering 431
Problem Set VI
Due: March 19, 2004

Reading: OSB: Appendix A

6.1 A Simple Random Process

$X(n)$ is defined by the following equally likely sample functions.

\[ X(\omega_1, n) = +1 \quad X(\omega_3, n) = +\sin 2\pi f_0 n \]
\[ X(\omega_2, n) = -1 \quad X(\omega_4, n) = -\sin 2\pi f_0 n \]

(a) Determine the mean and correlation function of the process $X(n)$.
(b) What is the marginal probability density of $X(n)$?
(c) Is this process second-order stationary? Is it stationary?

6.2 Friend or Foe?

(a) Which of the following are valid correlation functions? Indicate your reasoning.

\[
R_X(l) = a^{||l||} - a^{2||l||} \\
R_X(l) = \begin{cases} 1 - \frac{||l||}{L} & ||l|| \leq L \\ 0 & \text{otherwise} \end{cases} \\
R_X(l) = \delta(l) + 25 \\
R_X(l) = \delta(l + 1) + \delta(l) + \delta(l - 1)
\]

(b) Which of the following are valid power density spectra? Indicate your reasoning.

\[
P_X(e^{j2\pi f}) = \frac{\sin N\pi f}{\sin \pi f} \quad P_X(e^{j2\pi f}) = \left(\frac{\sin N\pi f}{\sin \pi f}\right)^2 \\
P_X(e^{j2\pi f}) = \exp\left\{-\frac{\cos 2\pi(f - f_0)}{4}\right\} \quad P_X(e^{j2\pi f}) = e^{e^{\cos 2\pi f} - e^{2\cos 2\pi f}} \\
P_X(e^{j2\pi f}) = 1 + 0.25e^{-j2\pi f} \quad P_X(e^{j2\pi f}) = \begin{cases} 1 & |f - 2n\pi| \leq f_0, n = 0, \pm 1, \ldots \\ 0 & \text{otherwise} \end{cases}
\]

6.3 Generating Random Processes

It is desired to generate a second-order stationary process with correlation function

\[ R_X(l) = a^{||l||} \]

Two methods are proposed.

1. Produce $X(n)$ by filtering white noise with a stable and causal filter.
2. Let $X(n) = A\cos(2\pi F n + \theta)$ where $A$, $F$, and $\theta$ are statistically independent random variables.

(a) Find at least two unit-sample responses $h(n)$ that will work in method 1.
(b) Specify densities for $A$, $F$, $\theta$ in method 2 that yield the desired results.
(c) Generate sample functions by each method using Matlab. Interpret your result.