7.1 Other Optimal Filters

The Eckhart filter is an optimal linear filter that maximizes the signal-to-noise ratio of its output. The filter’s output is an estimate of the signal. To find the unit-sample response of the FIR Eckhart filter, consider observations of the form \( x(n) = s(n) + N(n) \) where the correlation matrix of the noise is known. The signal-to-noise ratio is computed according to \( \mathcal{E} \left[ ||h's||^2 \right] / \mathcal{E} \left[ ||h'N||^2 \right] \), where \( h \) is the desired unit-sample response.

(a) Assuming the signal is nonrandom, find the Eckhart filter’s unit-sample response.
(b) What is the signal-to-noise ratio produced by the Eckhart filter? How does it compare with that produced by the corresponding Wiener filter?
(c) Now assume the signal is random, having correlation matrix \( K_s \). Characterize the Eckhart filter.

7.2 Statistical structure and data hiding

Data-hiding means sending data in such a way that it is difficult to determine that a transmission is even present. To test your statistical analysis skills, a dataset transmission.mat located in the course’s Matlab directory contains two vectors. One, \( x_{\text{nd}} \), contains no transmitted data; the second, \( x_{\text{d}} \), contains a bit stream. These data were produced by very similar statistical models, differing only in that second included a bit stream.

(a) Characterize the stochastic signal \( x_{\text{nd}} \) as much as you can. Estimate the marginal amplitude distribution, the correlation function, and any other statistics you think might be important.
(b) Extra Credit!!
   Decode the bit stream by any technique you like. [A clue: Only 128 bits are contained in this length-256 signal.]
(c) What is the transmitted message?