8.1 Image Restoration

Nuclear medicine imaging systems can be modeled as a blurring process (lowpass filtering) plus additive noise.

\[ x(m, n) = s(m, n) \otimes g(m, n) + w(m, n) \]

with \( g(m, n) \) the unit-sample response of the blurring process, \( s(m, n) \) the true image, and \( w(m, n) \) white noise. A sample image (spine.mat) and the DFT of \( g(m, n) \) (G.mat) are shown.

The spine image is blurred and noisy and the blurring system is indeed lowpass.

(a) Assume that the power density function of medical images has the form

\[ P_s(e^{j2\pi f_1}, e^{j2\pi f_2}) = \frac{\sigma_s^2}{\sqrt{1 + k \cdot (2\pi f_1)^2 + k \cdot (2\pi f_2)^2}} \quad \frac{1}{2} \leq f_1, f_2 \leq \frac{1}{2} \]

where \( \frac{1}{2} \leq \alpha \leq \frac{3}{2} \), \( \sigma_s^2 \) is the DC power, and \( k \) is a constant related to the spatial sampling interval. By examining the power spectrum of the spine image, determine reasonable values for \( \sigma_s^2, k, \) and \( \alpha \).

(b) What is the optimal mean-squared error transfer function? Express it in terms of the ratio \( R = \sigma_s^2 / \sigma_w^2 \).

(c) Implement the optimal restoration filter using Matlab. Experiment with various choices for \( \alpha \) and \( R \), and pick the best restored image you found.