ENGI 128

INTRODUCTION TO ENGINEERING SYSTEMS

Lecture 4:
Torque,
Gears & Torque ,
Transmissions & Torque,
Bicycles & Torque

"Understand Your Technical World"

Outline

Goal: Add rigor to the concepts of gears and torque that you learned last time.

Gears are in almost every modern vehicle, so we'll use different vehicles as our systems *du jour.*

Torque

Speed and Force, Translation and Rotational

	speed	**force**
linear motion	speed s	force f
rotational motion	angular speed ω	twisting force τ

Torque always makes people unhappy

Torque is simple: It is a “force” that makes anything rotate

It’s just that simple. Any time anything rotates, there is a torque involved.

It’s also called a “rotational force” or a “twisting force”

You can feel torque if you try…

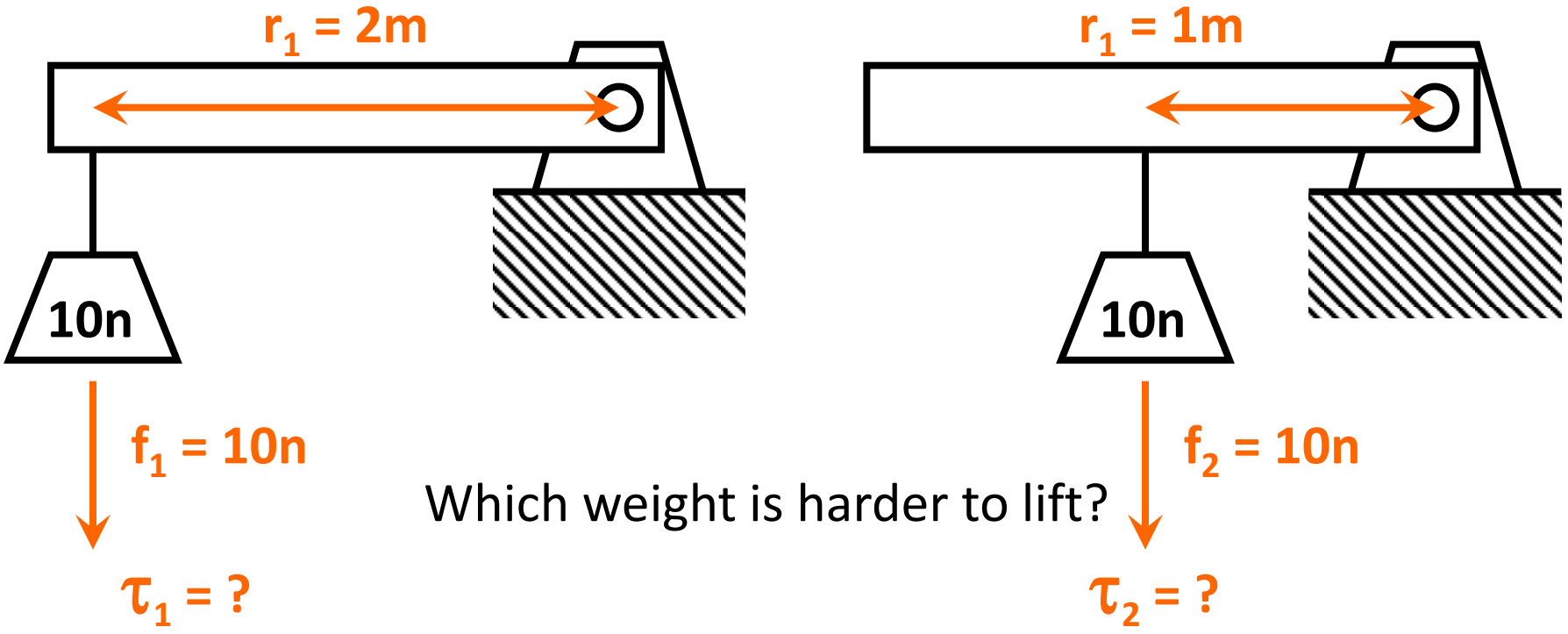
The power of radius:

The equation for torque is simple:

$$\tau = fr$$

But that ‘r’ is what causes all the mental problems

Imagine these two cranes:



Which weight is harder to lift?

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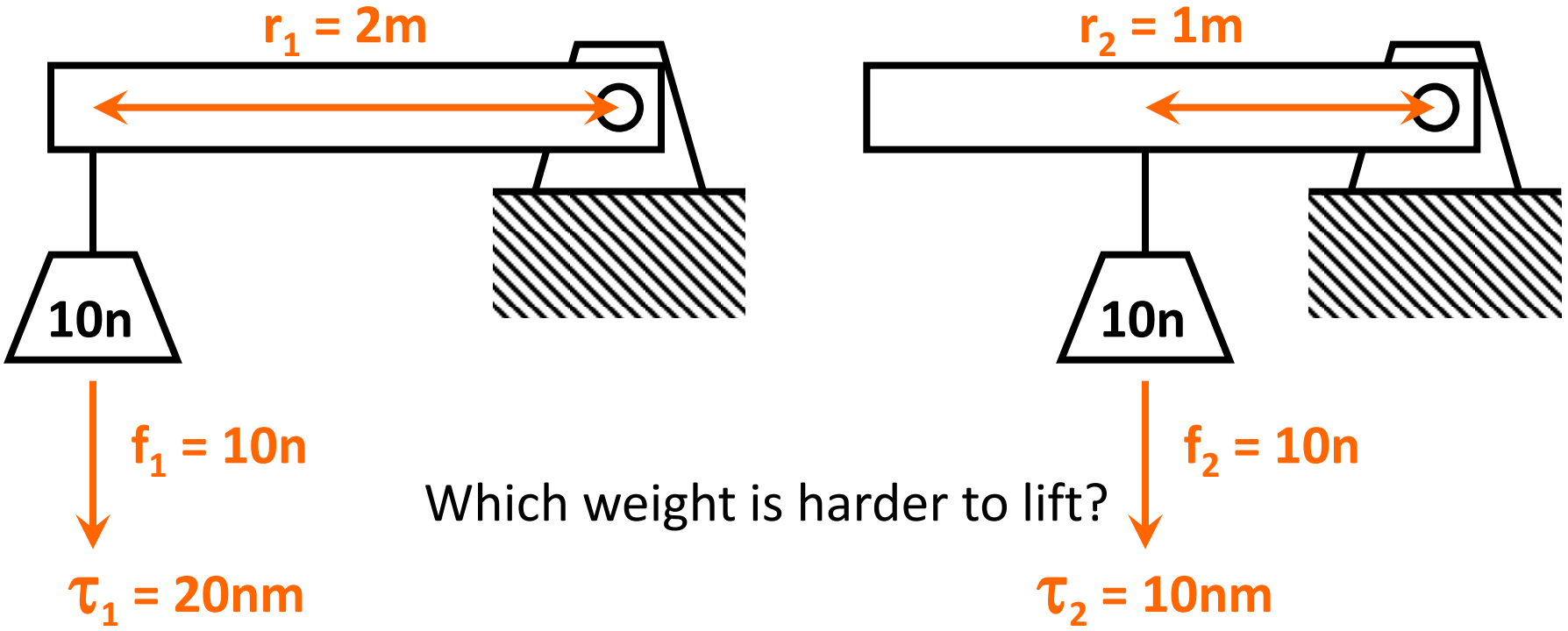


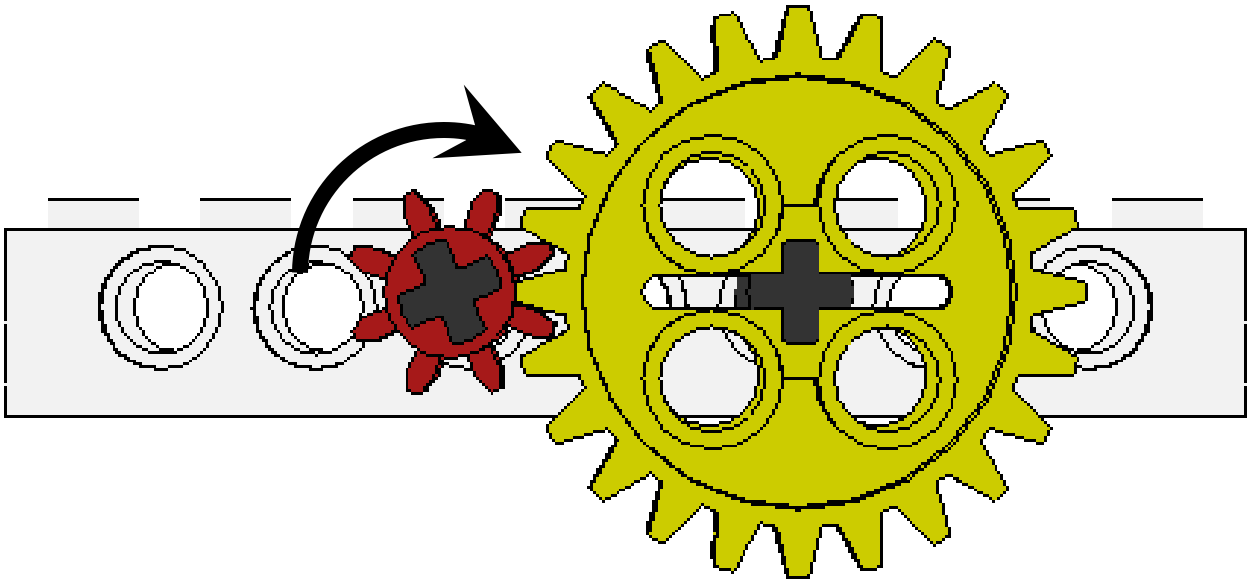
Which weight is harder to lift?

Gears & Torque

LEGO Gears

The red gear has 8 teeth, yellow gear has 24 teeth.

How far will the yellow gear turn per revolution of the red gear?



The Gear Ratio

The gear ratio is the ratio of input turns to output turns

$$g = \frac{\theta_1}{\theta_2}$$

Ratios can be written as fractions, or with a colon

$$g = 3{:}1$$

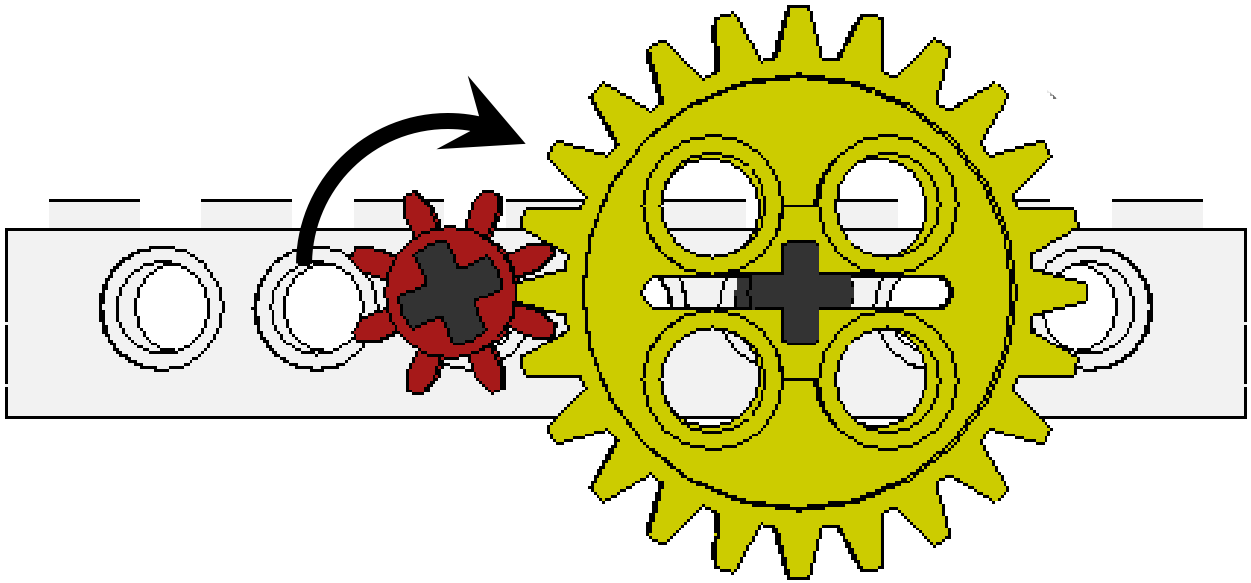
Is usually written with the right hand side normalized (set to) 1

How do we compute the gear ratio?

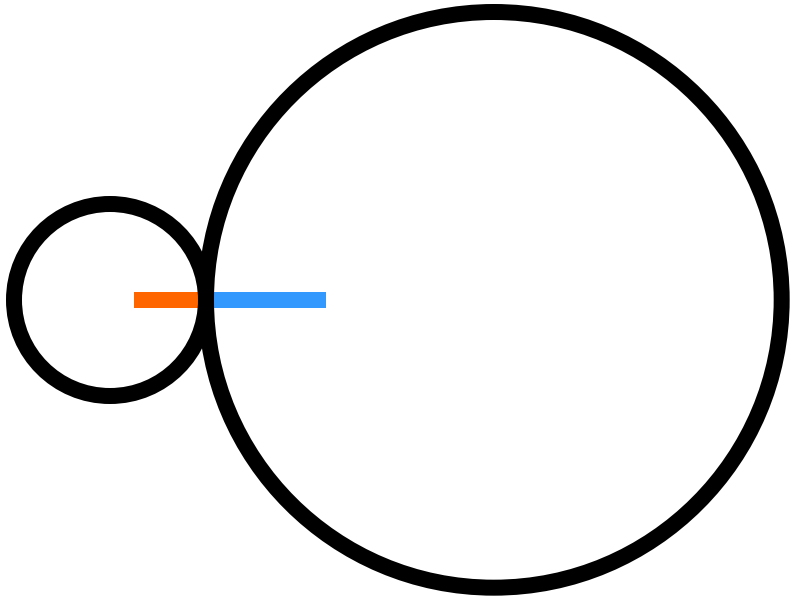
Gear Ratio

LEGO Gear reduction

The red gear has 8 teeth, yellow gear has 24 teeth

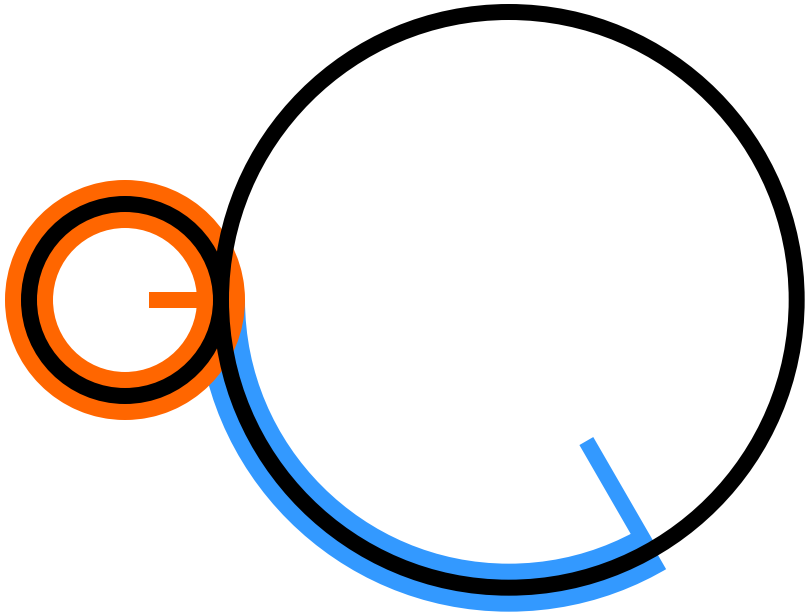


Gear Ratio



The “no slip condition”

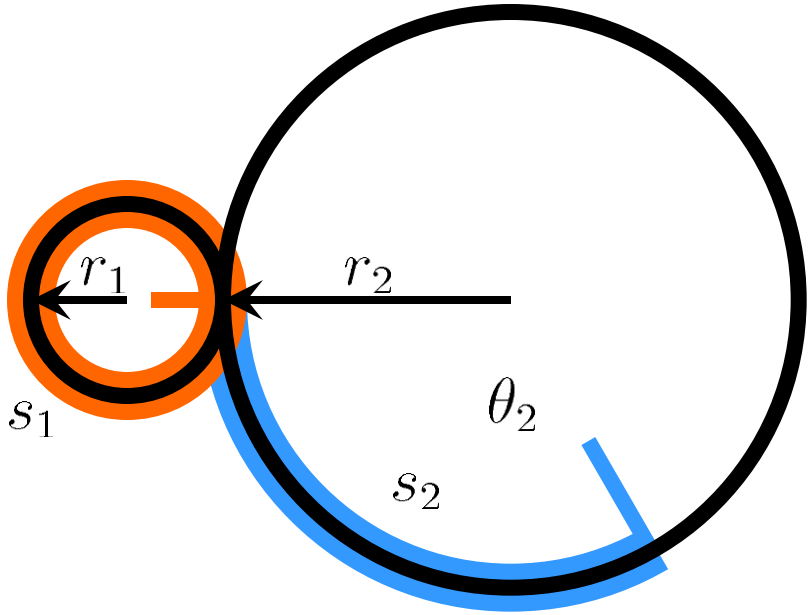
Gears can’t slip. They have teeth.



Gear Ratio

Basic equation for distance around a circle

$$s = \theta r$$



Gear Ratio

We want to compute g in terms of r_1 and r_2

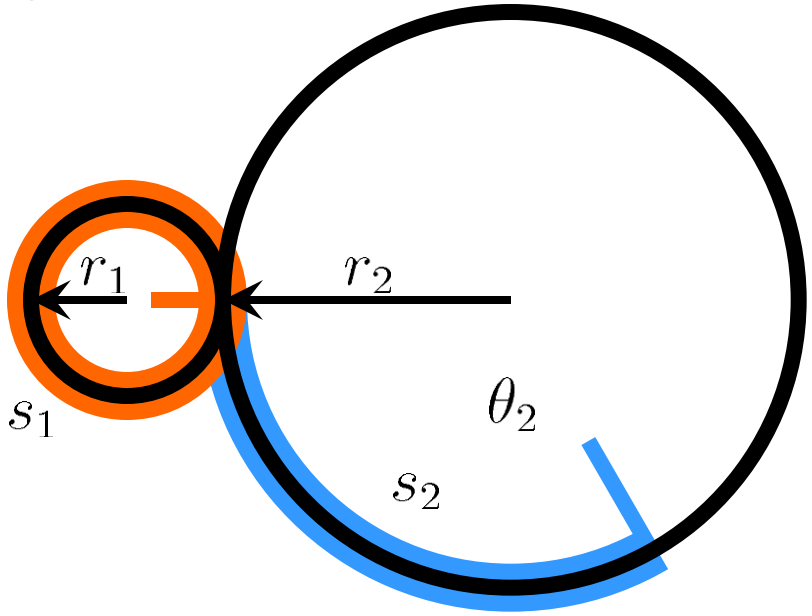
Definition: The gear ratio is the ratio of input turns to output turns:

$$g = \frac{\theta_1}{\theta_2}$$

Constants:

$r_1 = 1$

$r_2 = 3$



$$s = \theta r$$

$$s_1 = \theta_1 r_1$$

$$s_2 = \theta_2 r_2$$

$$s_1 = s_2$$

$$\theta_1 r_1 = \theta_2 r_2$$

$$\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1}$$

$$\boxed{g = \frac{\theta_1}{\theta_2} = \frac{r_2}{r_1}}$$

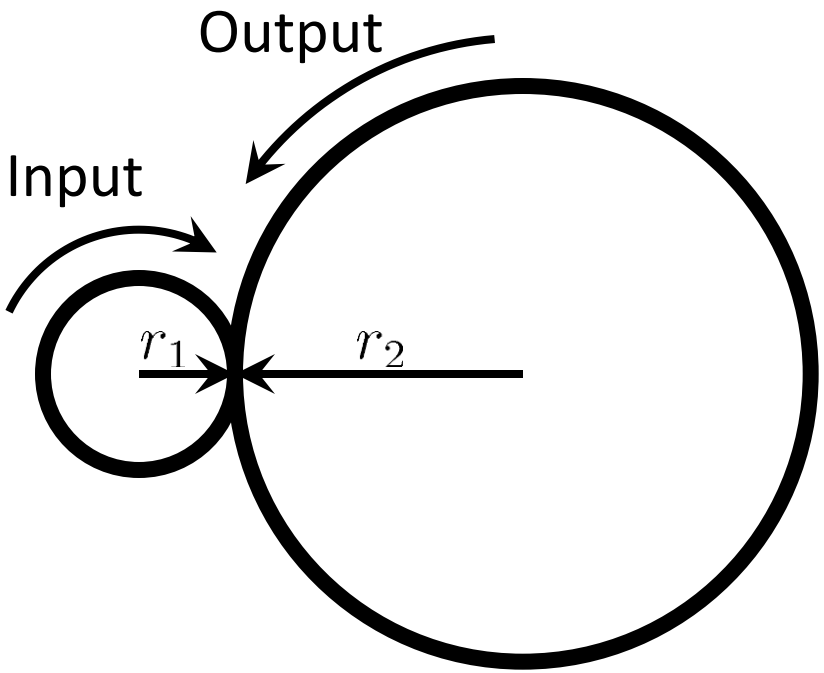
$$g = \frac{3}{1} = 3:1$$

Gears & Speed

Rotational (angular) Speed

First think: Should the output gear be going faster or slower?

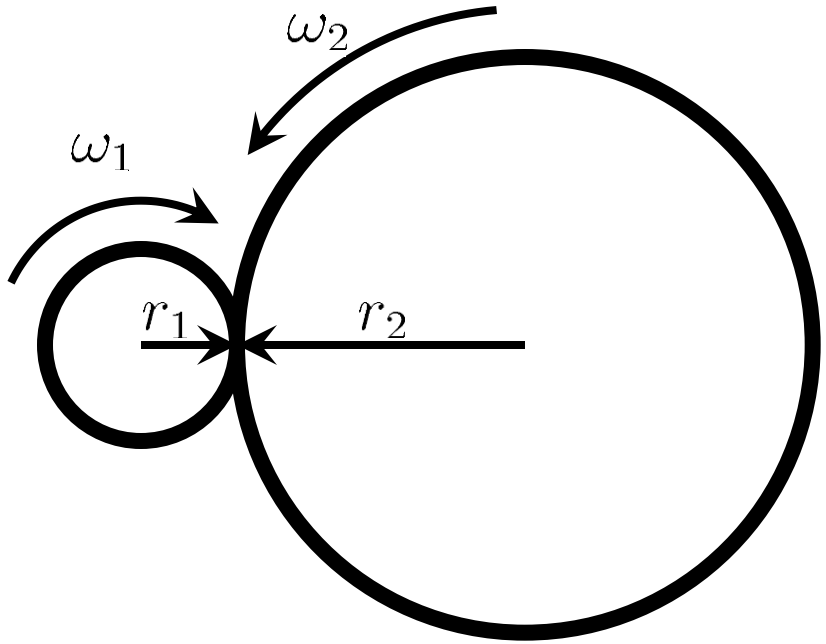
By how much?



Rotational (angular) Speed

Definition of angular speed:

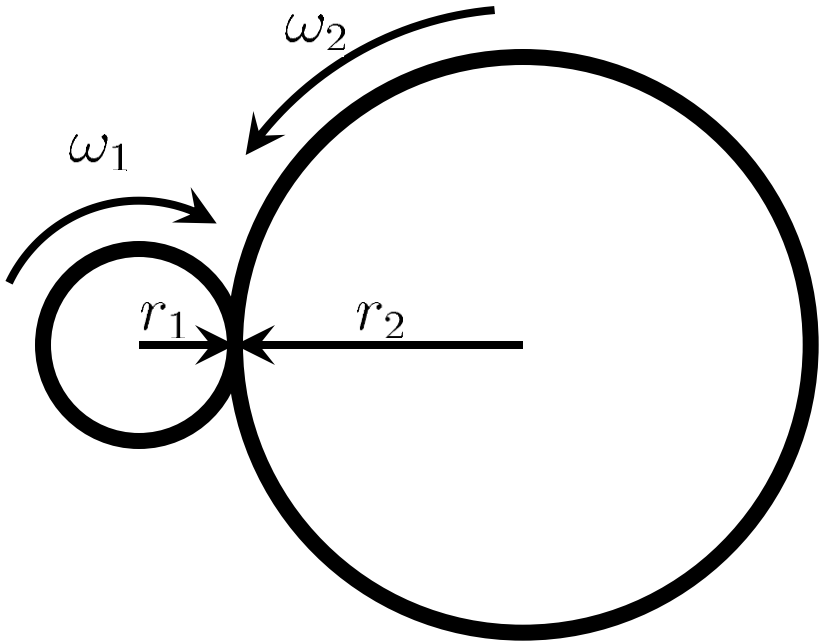
$$\omega = \frac{\theta}{t}$$



Angular Speed

We want to compute ω_2 in terms of ω_1 and g

$$g = \frac{\theta_1}{\theta_2}$$



$$\omega = \frac{\theta}{t}$$

$$\omega_1 = \frac{\theta_1}{t}$$

$$\omega_2 = \frac{\theta_2}{t}$$

$$t = \frac{\theta_2}{\omega_2} = \frac{\theta_1}{\omega_1}$$

$$\omega_2 = \frac{\theta_2}{\theta_1}\omega_1$$

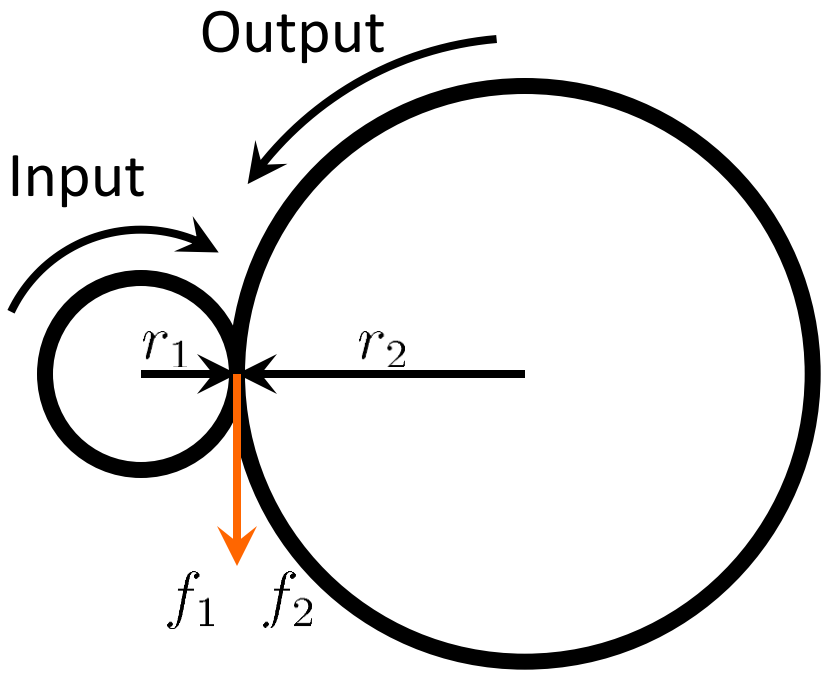
$$\boxed{\omega_2 = \frac{\omega_1}{g}}$$

Gears & Torque

Torque and gears

First think: Should the output gear have more or less torque?

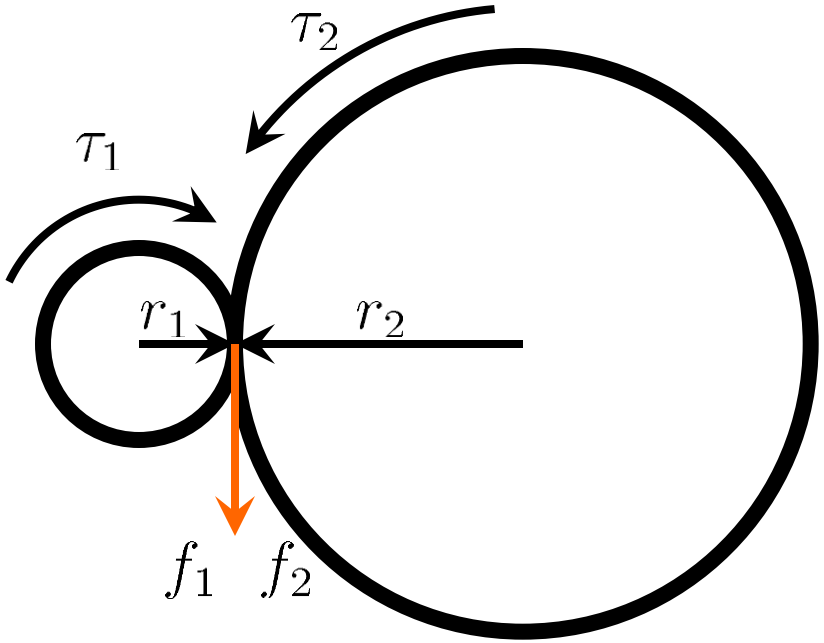
By how much?



Gears and torque

$$\tau = fr$$

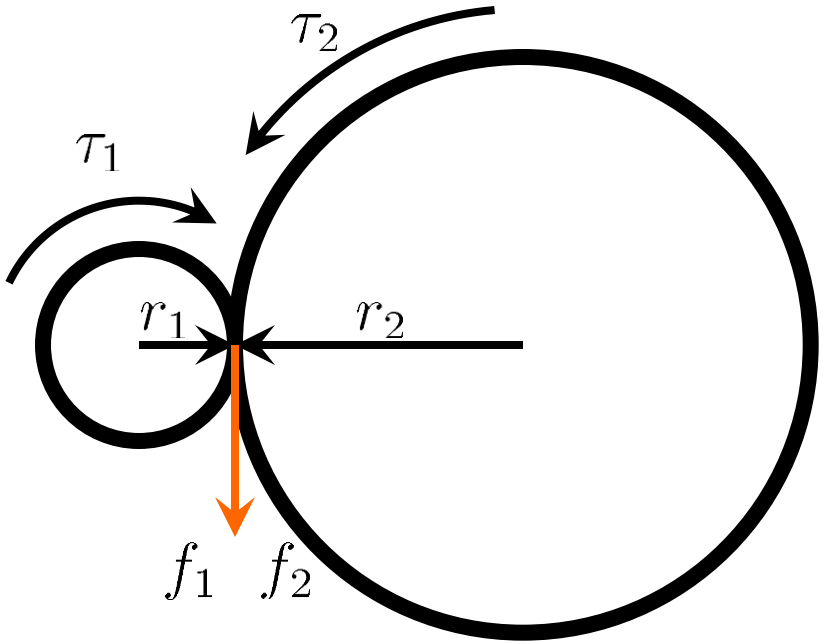
Let's start with our torque equation:



Gears and torque

We want to compute τ_2 in terms of τ_1 and g

$$g = \frac{r_2}{r_1}$$



$$\tau = fr$$

$$\tau_1 = f_1 r_1$$

$$\tau_2 = f_2 r_2$$

$$f_1 = f_2$$

$$\tau_1 = f r_1$$

$$\tau_2 = f r_2$$

$$f = \frac{\tau_1}{r_1}$$

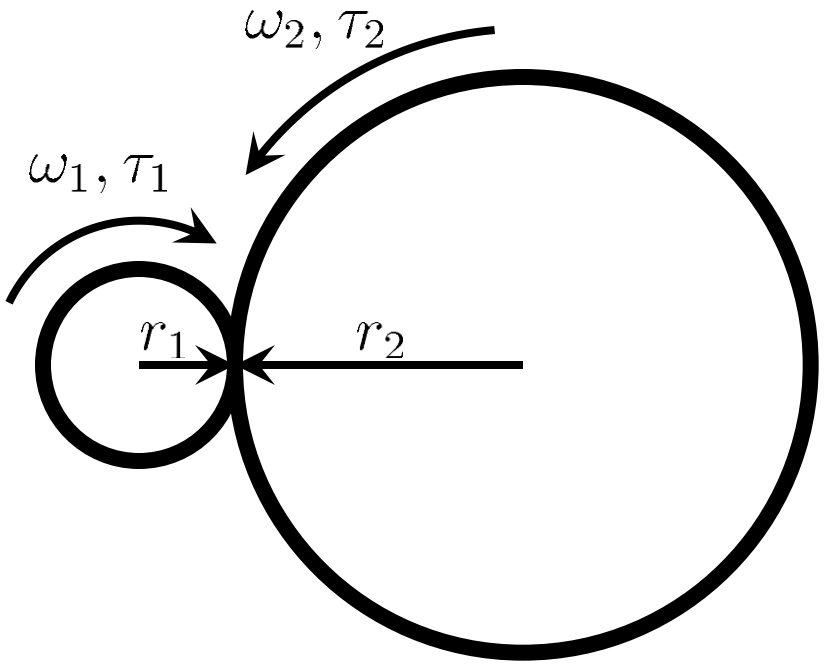
$$\tau_2 = \frac{\tau_1}{r_1} r_2$$

$$\tau_2 = \tau_1 \frac{r_2}{r_1}$$

$$\boxed{\tau_2 = \tau_1 g}$$

Summary

$$g = \frac{r_2}{r_1} \qquad \omega_2 = \frac{\omega_1}{g} \qquad \tau_2 = \tau_1 g$$



No free lunch: If you want more torque, you have to give up speed (and vice versa)

When trading torque for speed, something is conserved…

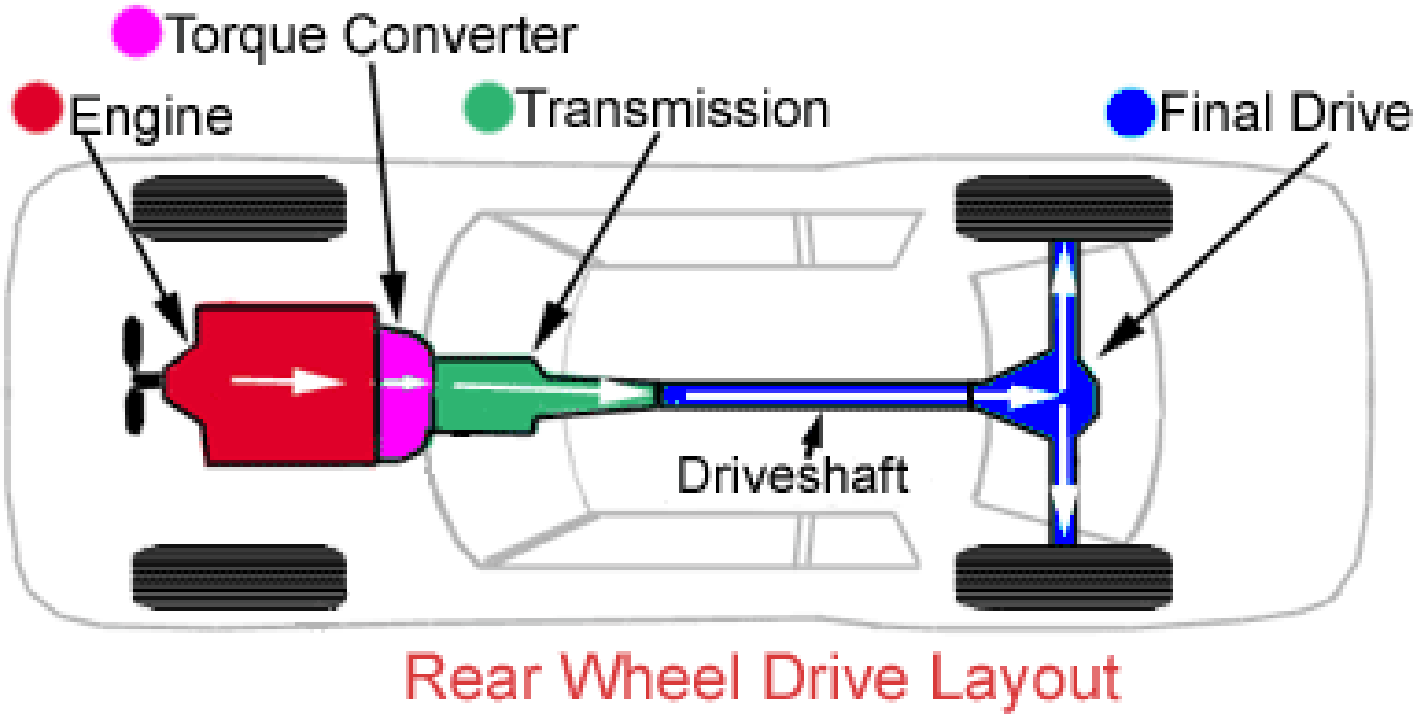
Transmissions

Transmissions

What is a transmission?

Where can you find them?

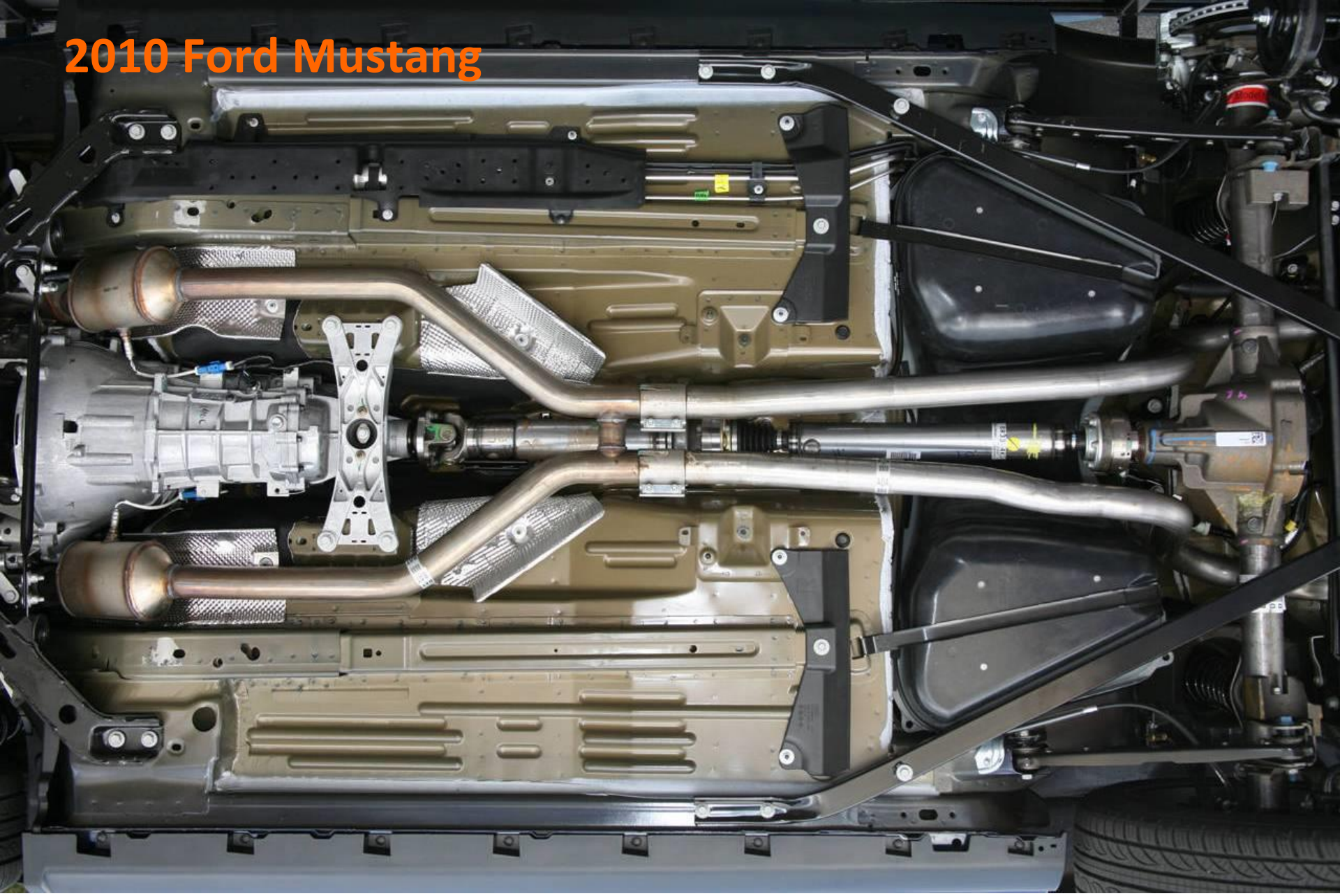
Car Transmission



http://www.familycar.com/transmission.htm

2010 Ford Mustang

2010 Ford Mustang



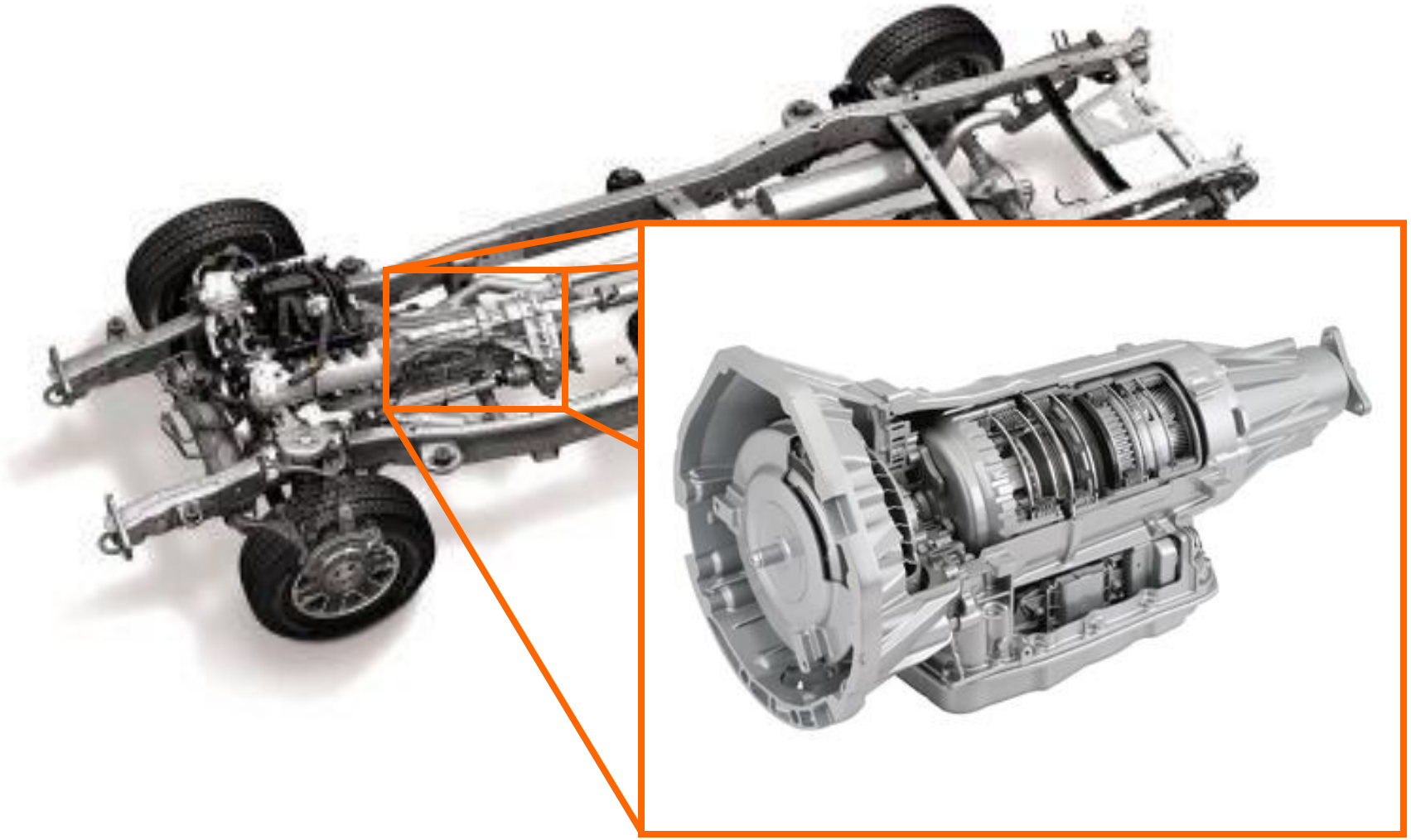
2004 Ford F-150

2009 Ford F-150

Automatic Transmission

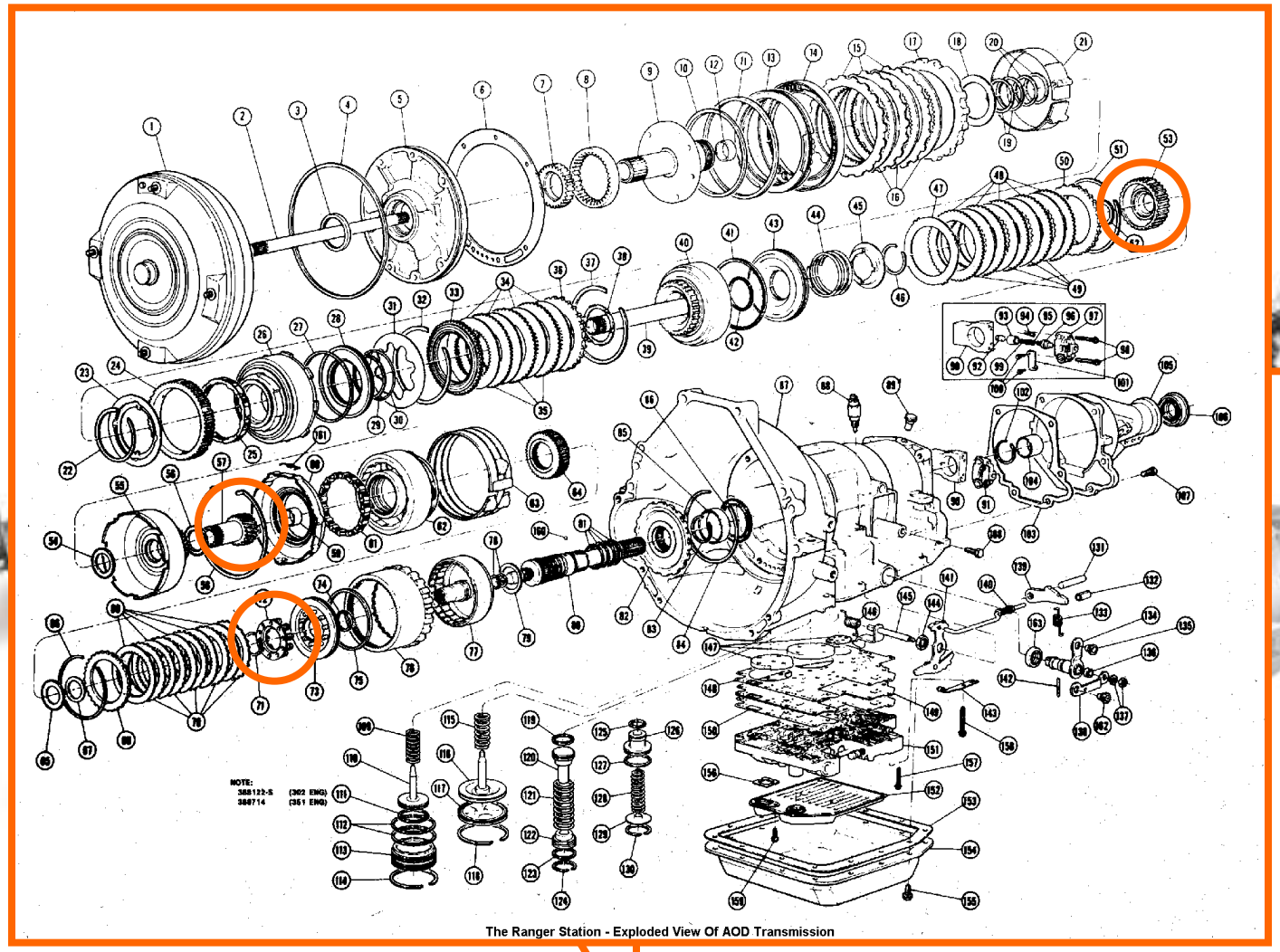
Different sets of gears trade torque for speed

Most cars have 4-6 gear ratios + reverse



Automatic Transmission

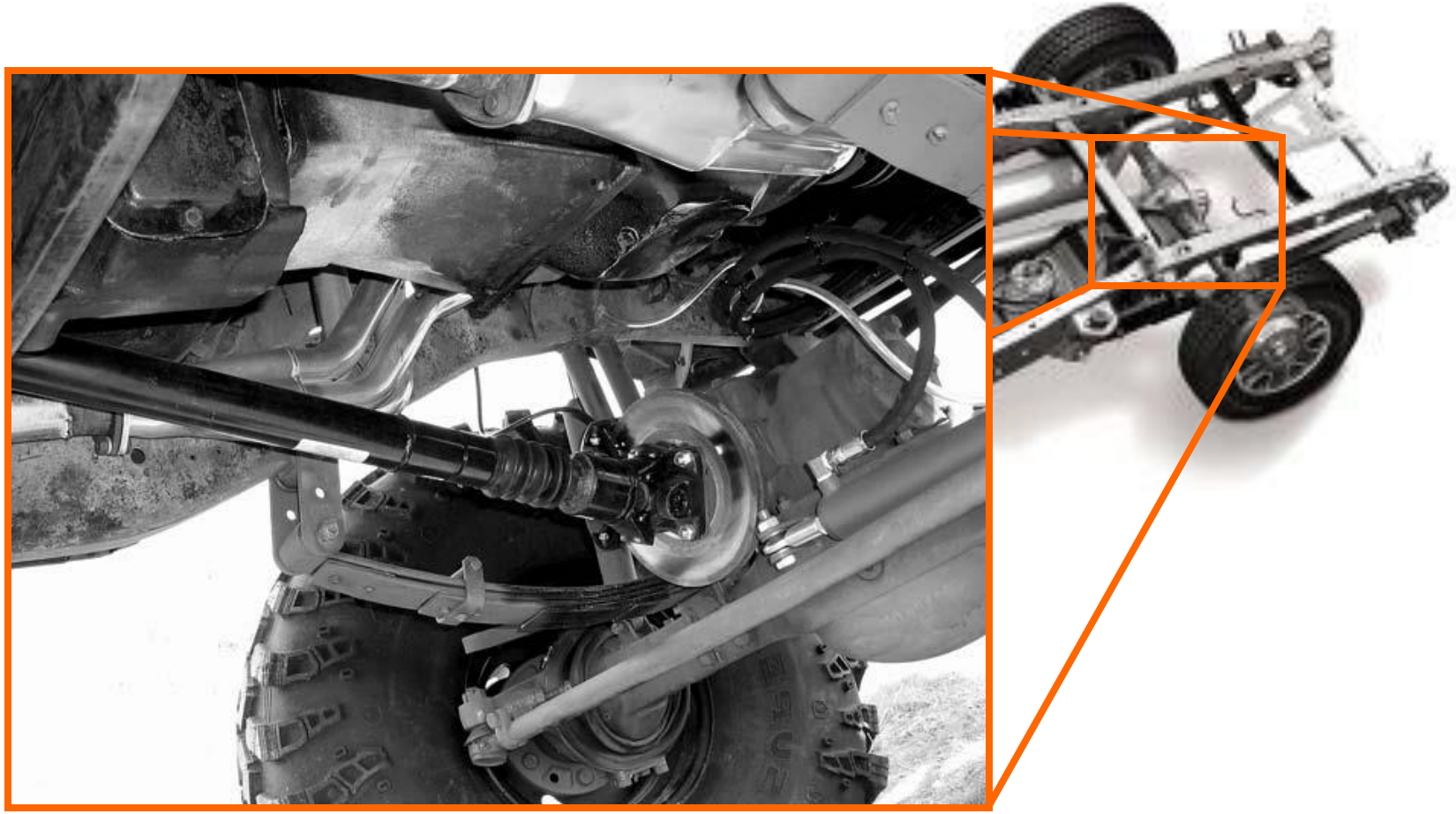
They’re ridiculously complicated, but look, they have gears!



Final Drive

A set of *bevel gears* rotate the power 90 degrees, …

And a *differential* splits it between the two rear wheels



2009 Honda CBR 600RR

Where is the transmission? How many gears? Why?

2009 Honda CBR 600RR

Where is the transmission? How many gears? Why?

2014 Specalized Stumpjumper FSR Comp Evo

Where is the transmission? How many gears? Why?

[Transmission Demo]

Other types of transmissions?

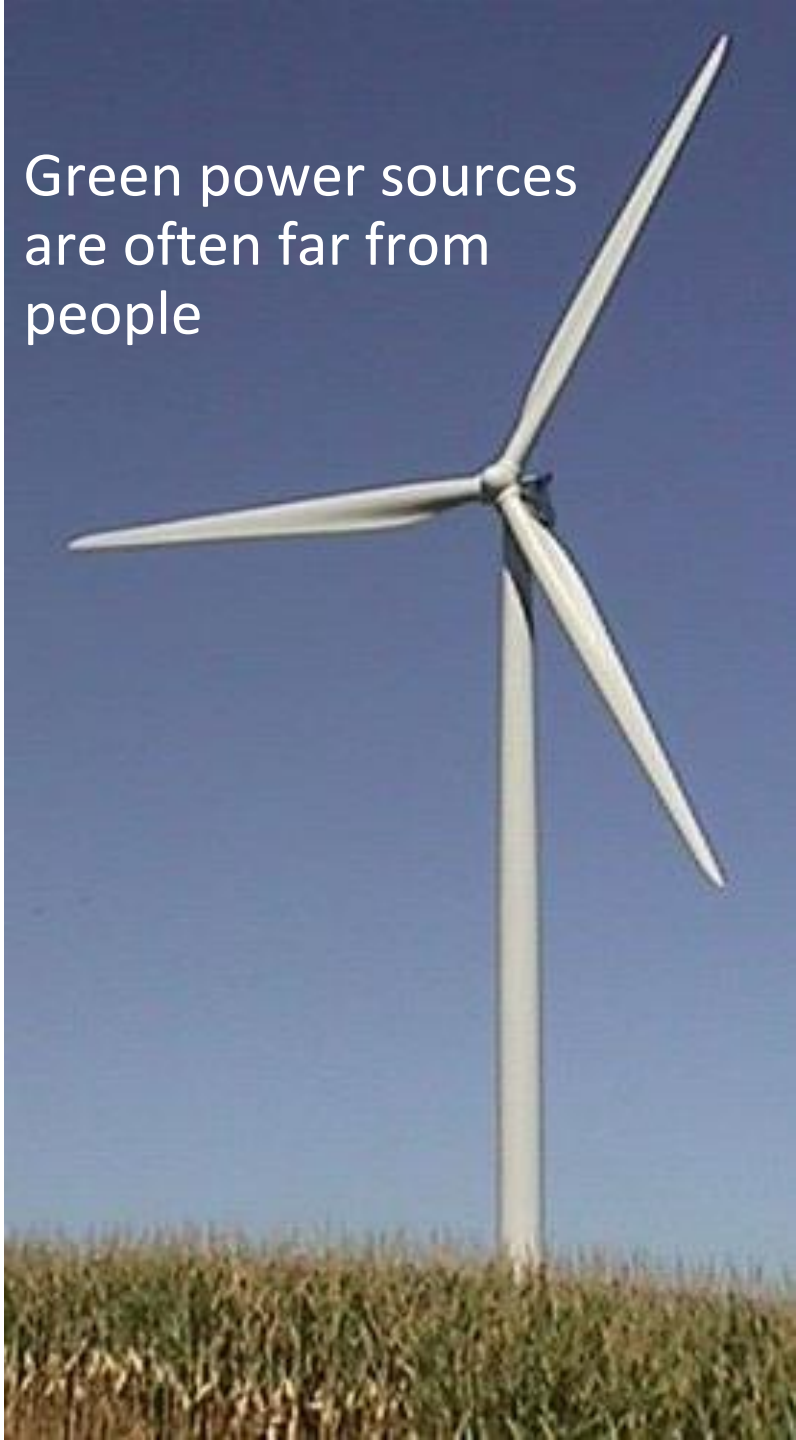
Or maybe just things that use the word “transmission”?

[Hoover Dam]

Transmission Lines

Critical for Green Energy

Green power sources are often far from people



Abstraction

Transmissions in Block Diagrams

What do all these transmission have in common?

What is a transmission block diagram element?

What is its interface?

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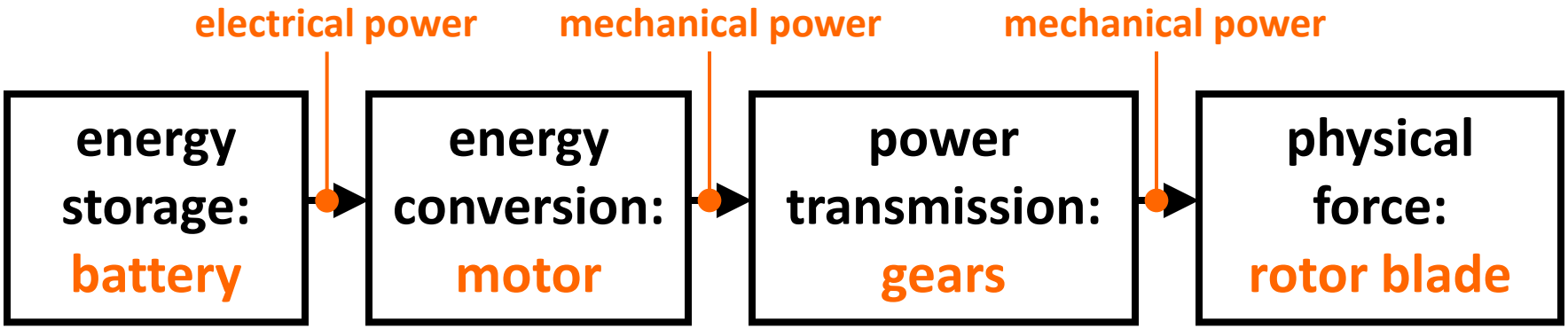

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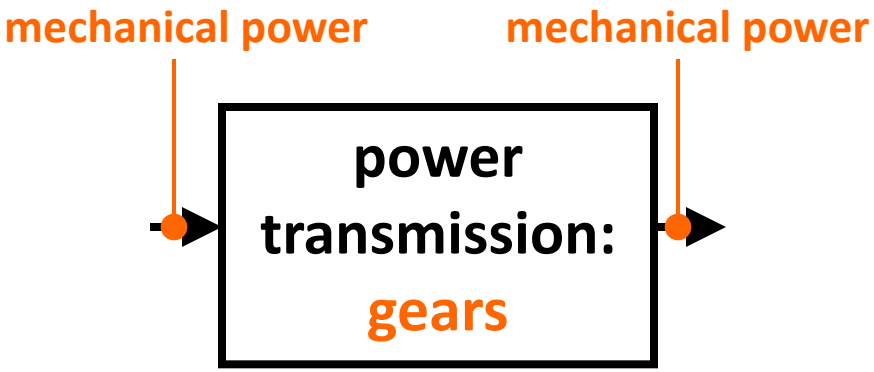

Transmissions in Block Diagrams

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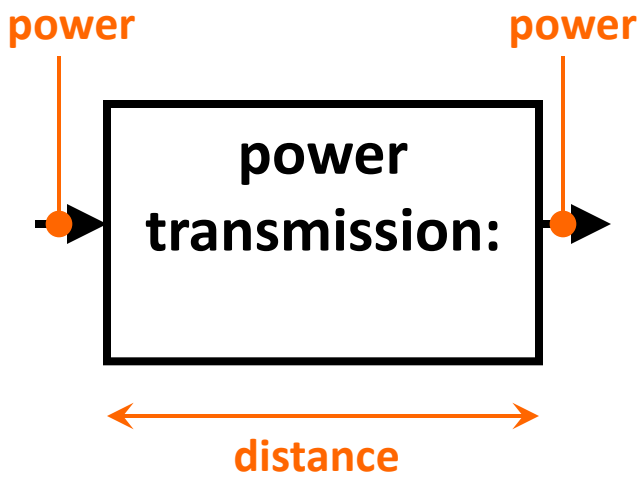
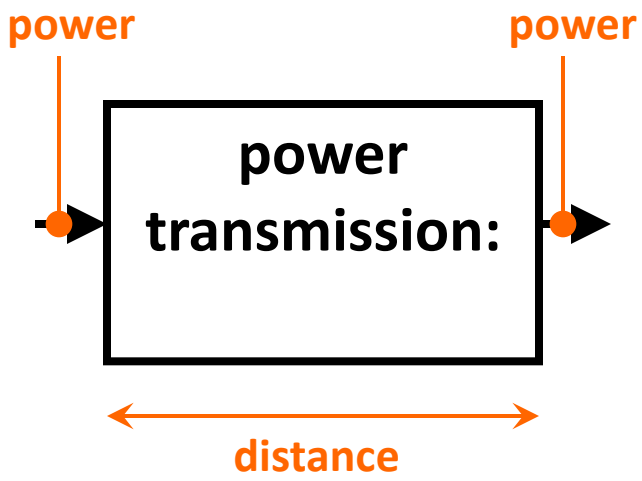


Wait a minute…

Power in and power out…

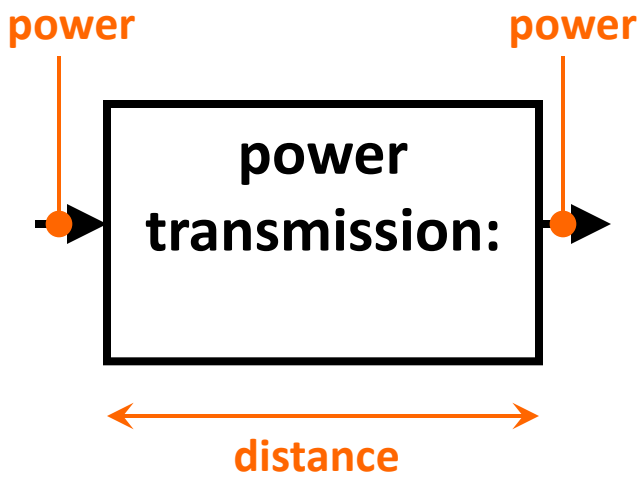
[wait for the realization from previous “no free lunch” comment]



Conservation of Energy

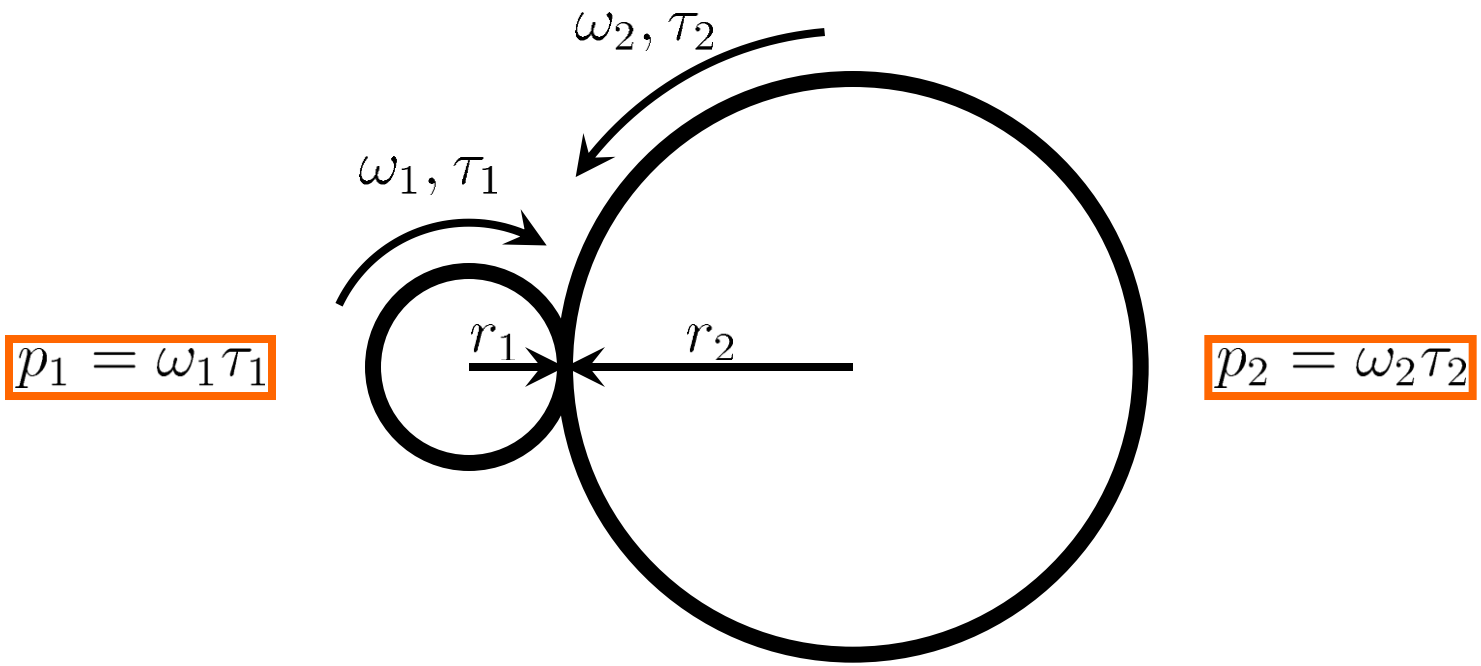
Power in = power out!

How do you compute power in a system of gears?



Summary, part II

$$g = \frac{r_2}{r_1} \qquad \omega_2 = \frac{\omega_1}{g} \qquad \tau_2 = \tau_1 g \qquad \boxed{p = \omega\tau}$$



No free lunch: If you want more torque, you have to give up speed (and vice versa)

When trading torque for speed, something is conserved…

Summary, part II

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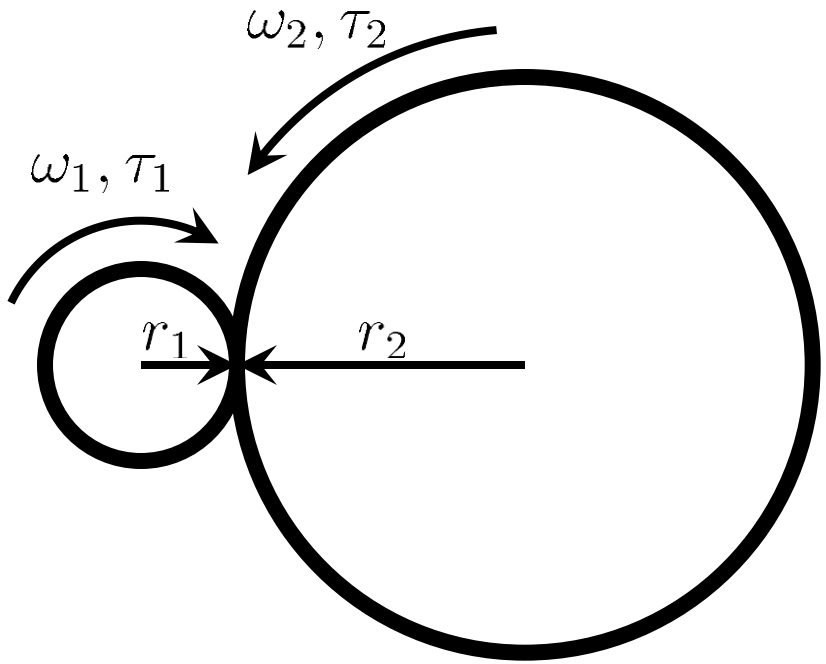


$p_1 = \omega_1\tau_1$

$p_2 = \omega_2\tau_2$

$$p_2 = \frac{\omega_1}{g}(\tau_1 g)$$

$$\boxed{p_2 = \omega_1\tau_1}$$

No free lunch: If you want more torque, you have to give up speed (and vice versa)

When trading torque for speed, **power** is conserved!

Speed and force and power

	speed	**force**
linear motion	speed s	force f
rotational motion	angular speed ω	twisting force τ

Fun with Torque: Physics of Two-Wheeled Vehicles

The “Brakie”

r_1

r_2

+

mg

μN

$$mgr_1 - \mu N r_2 = 0$$

2014 Specalized Stumpjumper FSR Comp Evo

What is this bike designed for?

Slickrock Trail, Moab, UT

[slidewhow]

Question 1: Torque and Brakes

Which has more stopping power?

A. Front brakes

B. Rear brakes

C. Both are equal

Braking Torque

[todo: finish slide: add torque arrows]

Braking Torque

Application of brakes creates a torque

- This torque increases the normal force on the front wheel
- With more normal force, the front wheel can generate more friction
- Almost ¾ of braking force comes from the front wheels!

2009 Honda CBR 600RR

Question 2: Torque and Steering

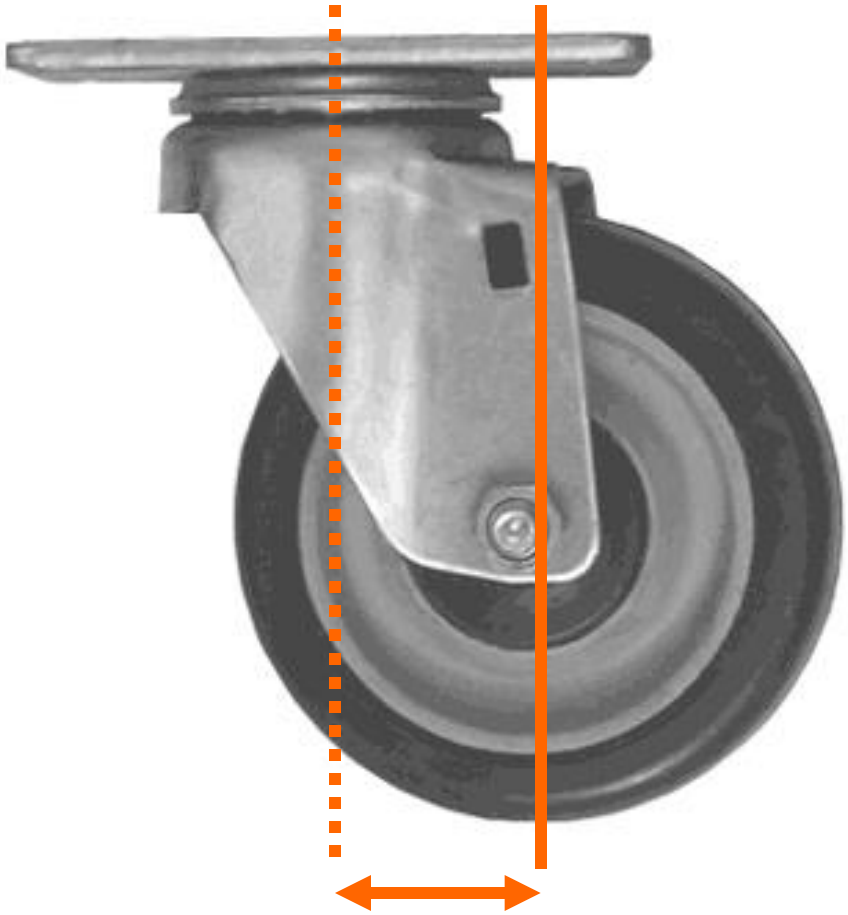
What makes a bicycle go straight?

A. Rider balance

B. Gyroscopic force

C. Torque around front wheel axis

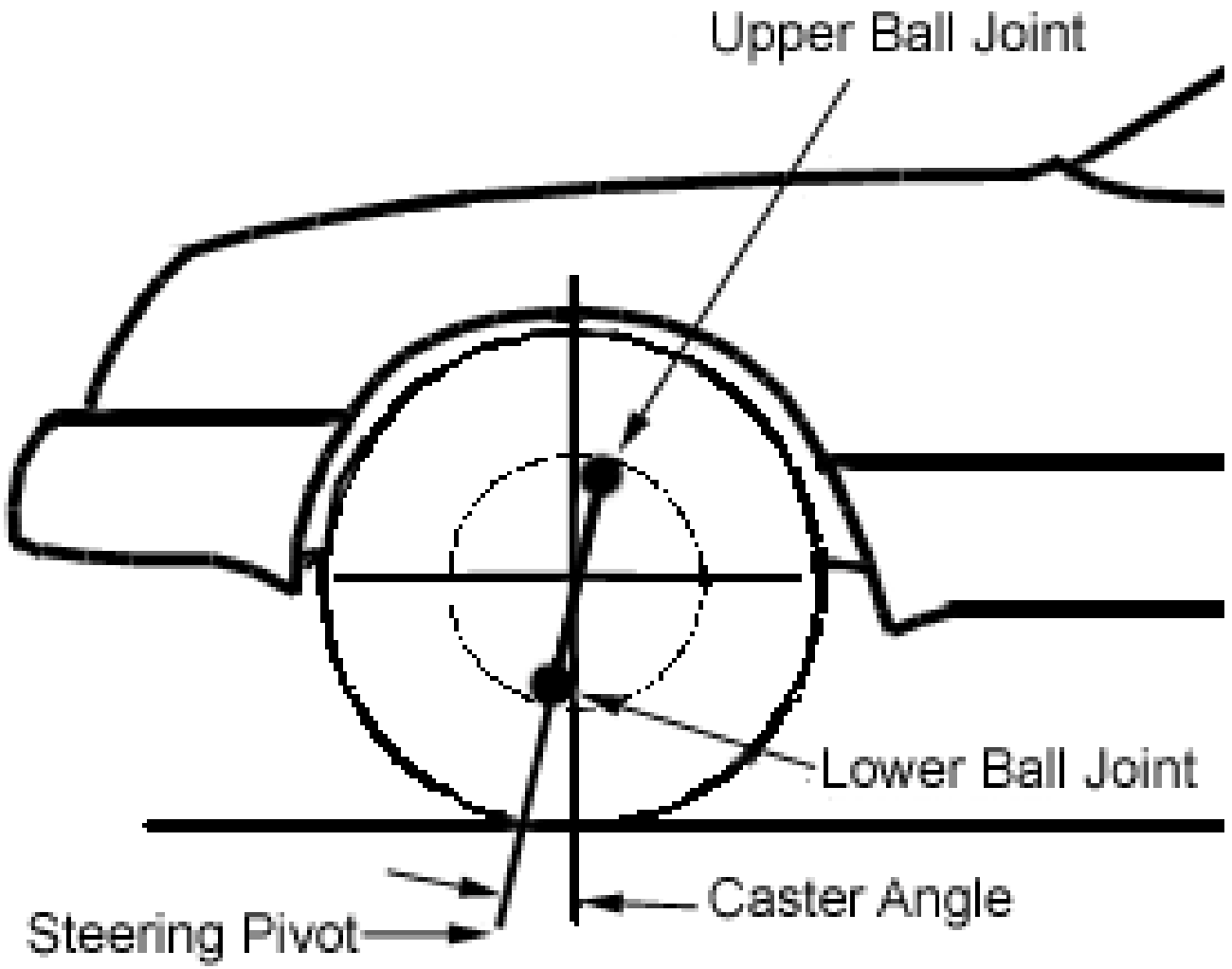
Caster



Caster angle in Bicycles

Caster angle in cars

www.familycar.com



Question 3: Torque and Bunny Hops

How does a “bunny hop” work?

A. Toe clips

B. Torque around the center of mass

C. Bouncing off of the tires

The Bunny Hop

Try This: Shopping Cart Bunnyhops!