Flexural Analysis of a Zee-Section Straight Beam

1.1 Introduction

In this study you will validate your understanding of the use of Cosmos by solving a cantilever beam and comparing the FEA results to that predicted by mechanics of materials theory. The constant cross-section is a zee-shape in the x-y plane as seen in Figure 1. It extends in the z-direction for a length of \( L = 500 \text{ mm} \). The thickness of the section is \( t = 5 \text{ mm} \), each flange has a length of \( a = 20 \text{ mm} \), and the web has a depth of \( h = 2a = 40 \text{ mm} \). At the free end it is loaded by a distributed force parallel to the y-axis (i.e., vertical). A detailed outline of the construction of the solid part model is given in the Appendix.

![Figure 1](image)

1.2 Validation estimate

Before you start a FEA study you should try to get a reasonable approximation of the stresses and deflections to be obtained. This can be an analytic equation for a similar support and loading case, a FEA beam model compared to a continuum solid model, a one or two element model that can be solved analytically, etc.

The cantilever is horizontal and has a vertical load of \( P = 500 \text{ N} \). Therefore, it causes a bending moment, about the x-axis of \( M = P \ (L - z) \), where \( z \) is the distance from the support. Recall (incorrectly, so be alert) that such a loading causes a linear flexural stress (\( \sigma_z \)) that varies linearly through the depth. It is zero at the neutral axis (here parallel to the x-axis at the section centroid) and has a maximum constant tension along the top edge, and a corresponding constant compression along the bottom edge (parallel to x). The load \( P \) causes a varying moment and a transverse shear stress (\( \tau \)) that varies parabolically through the depth and has its maximum at the neutral axis. Those stress behaviors are sketched with the section in Figure 2. The flexural and shear stress equations are \( \sigma_z = M \ y / I_x \) and \( \tau = P \ Q / t \ I_x \) where \( I_x \) is the second moment of the section and \( Q \) is the first moment of the section at \( y \). For this section \( I_x = 8/3 \ t \ a^3 \). The maximum constant tension will occur at \( y = a \), while compression occurs at \( y = -a \). Likewise, the end deflection of the beam in the vertical (y) direction will be \( U_y = PL^3 / 3EI_x \). With that review and predictions you can now proceed with the FEA study.
1.3 Cosmos study
Results comparison

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Re-validation

Of course the FEA and the validation predictions do not agree! The simplified mechanics relations are only valid for straight symmetric sections. Usually those sections have two planes of symmetry. But they must have at least one symmetry plane, so as to make the product of inertia vanish \( (l_{xy} = 0) \). The current section does not even have one symmetry plane. Its geometric properties are \( I_x = 8/3 \ t \ a^3 \), \( I_y = 2/3 \ t \ a^3 \), \( I_{xy} = -t \ a^3 \), and \( A = 4 \ t \ a \).

We should have remembered that any time \( I_{xy} \neq 0 \), one must employ non-symmetric beam theory for bending. That theory states that ...

References

Appendix: Construction of the solid model

![Figure 13](image)

![Figure 14](image)
Figure 16