

# Local Buckling of a Cantilever (draft 2 Oct 25 06)

## Background

You previously went through the analysis of a horizontal tapered cantilever subject to a transverse load distributed over its free end face. The fixed support at the wall included a semi-circular section of the supporting vertical section. The member was  $L = 50$  inch long,  $t = 2$  inch thick, and the depth,  $d$ , tapered from 3 inch at the load, to 9 inch at the support. A complete plane stress analysis was conducted. The computed stresses were relatively low. It was decided to save material costs by reducing the thickness of the beam.

## Factor of Safety

For the ductile material used here the factor of safety (FOS) is defined as the material yield stress divided by the von Mises' effective stress. To view its distribution the default results plot is opened with **Design check**→**Plot 1**, which is shown in Figure 1.

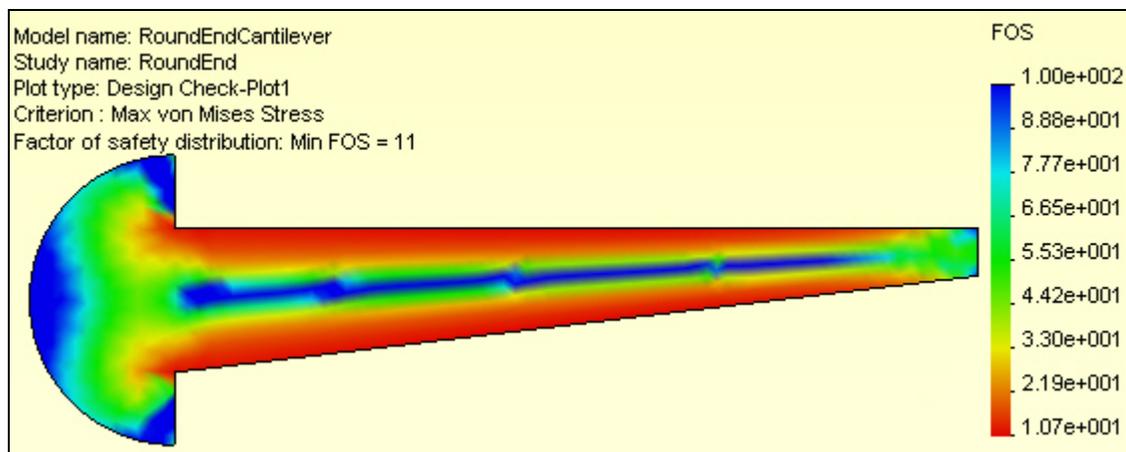


Figure 1 Original factor of safety in bending

The FOS is also quite high, ranging from a low value of about 10 to a high value of about 100. This probably suggests (incorrectly) that a simple redesign will save material, and thus money. The load carrying capacity of a beam is directly proportional to its geometric moment of inertia,  $I_z = t d^3 / 12$ . Thus, it also is proportional to its thickness,  $t$ . Therefore, it appears that you could simply reduce the thickness from  $t = 2$  to 0.2 inches and your FOS would still be above unity. If you did that then the “thickness to depth ratio” would vary from  $0.2 / 3 = 0.067$  at the load to  $0.2 / 9 = 0.022$  at the wall, a range of about 1/15 to 1/45. The edited extrusion feature in Figure 2 employs that thickness.

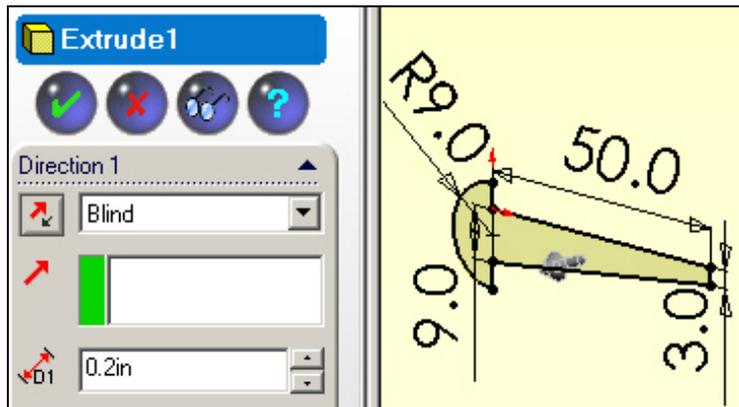


Figure 2 The reduced thickness extrusion

## Local buckling

If component has a region where the relative thickness to depth ratio of less than 1/10 you should consider the possibility of “*local buckling*”. It usually is a rare occurrence, but when it does occur the results can be sudden and catastrophic. To double check the safety of reducing the thickness you should add a second study that utilizes the CosmosWorks buckling feature to determine the lowest buckling load. To do that:

1. Right click on the **Part name**→**Study** to open the **Study panel**.
2. Assign a new **Study name**, select **Buckling** as the **Type of analysis**, and use the thin shell as the **Model type**, **OK**.
3. To use the same loads and restraints **drag** the **Load/Restraints** from the first study and **drop** them into the second one.
4. Likewise, **drag and drop** the first shell **Materials** into the second study.
5. Create a new finer mesh, or drag and drop the first mesh.
6. Right click on the **Part name**→**Run**.

## Buckling mode

A buckling, or stability, analysis is an eigen-problem. The magnitude of the scalar eigen-value is called the “*buckling load factor*”, BLF. The computed displacement eigen-vector is referred to as the “*buckling mode*” or mode shape. They are only relative displacements. Usually they are presented in a non-dimensional fashion where the displacements range from zero to  $\pm 1$ . In other words, the actual value or units of a buckling mode shape are not important. Still, it is wise to carry out a visual check of the first buckling mode:

1. When the solution completes, pick **Displacements**→**Plot1** and examine the resultant displacement URES. Note that the displacement contour curves in Figure 3 are inclined to the long axis of the beam instead of being vertical as before.
2. Use **Edit Definition**→**Vector**→**Line** to get a plot of the displacement vectors, and rotate to an out-of-plane view, as shown in Figure 4.

From Figure 4 you see that under the vertical load the (very thin) beam deflected mainly sideways (perpendicular to the load) rather than downward. This is an example of lateral buckling. That is typical of what can happen to very thin regions. Next, the question is: how large must the end load be to cause such motion, and failure?

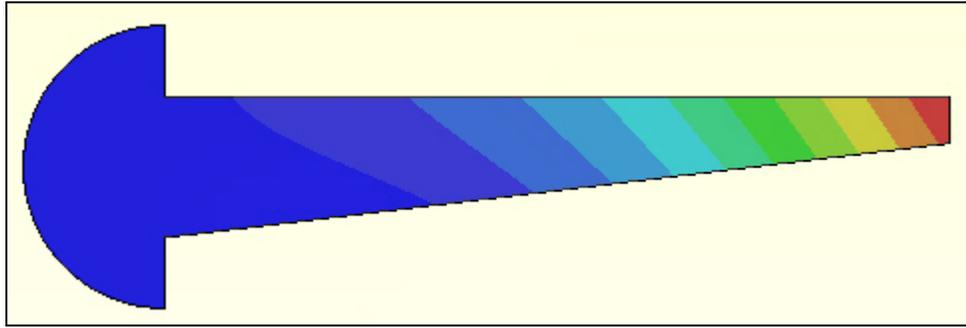


Figure 3 Relative buckling mode displacement values

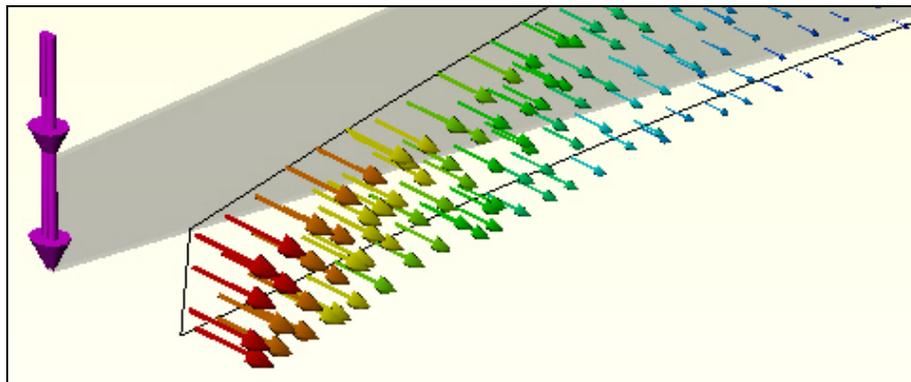
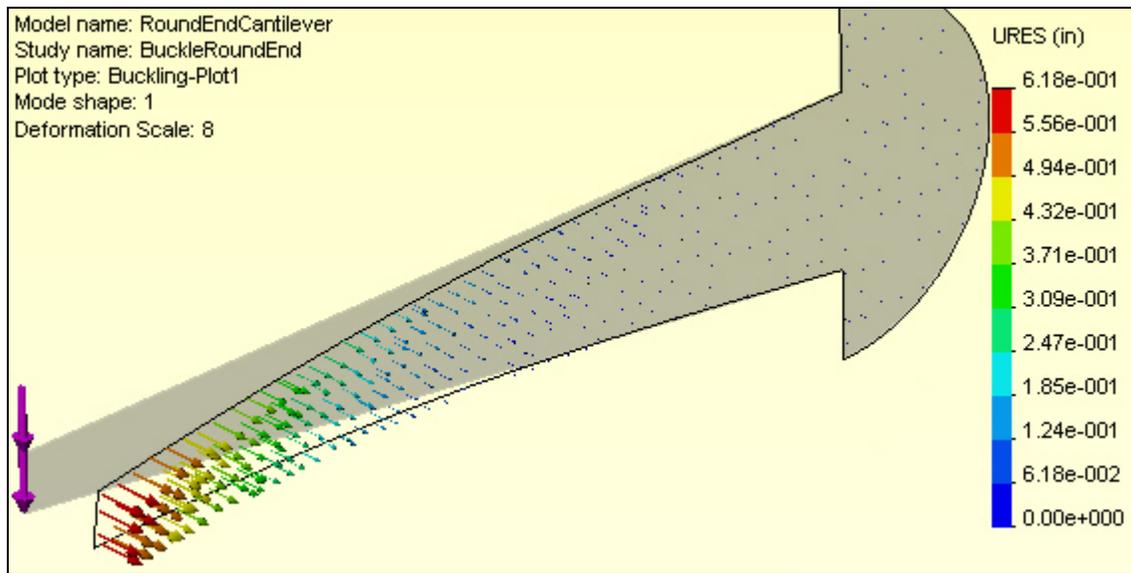


Figure 4 Relative lateral buckling mode displacement vectors

## Buckling Load Factor

To see the magnitude of the BLF (eigen-value):

1. Right click on **Deformation** → **List Mode Shape**.
2. In the **Mode Shape panel**, Figure 5, read the BLF value of about 0.03.

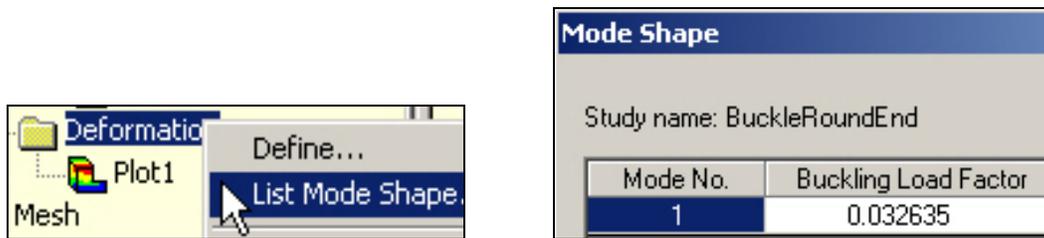


Figure 5 First buckling mode load factor

You want the BLF to be quite a bit higher than unity. Instead, the study shows that only about 3% of the planned load will cause this member to fail by lateral buckling due to loss of stiffness in the out of plane direction. Thus, you must re-consider the thickness reduction. Remember that the geometric moment of inertia about the vertical ( $y$ ) axis is  $I_y = d t^3 / 12$ . It is a measure of the lateral bending resistance. By reducing the thickness,  $t$ , by a factor of 10 the original  $I_z$  (and the in-plane bending resistance) went down by the same factor of 10, but  $I_y$  (and the out-of-plane bending resistance) went down by a factor of 1,000.

The buckling load factor is an indicator of the factor of safety against buckling or the ratio of the buckling loads to the currently applied loads. Since buckling often leads to bad or even catastrophic results, you should utilize a high factor of safety ( $>3$ ) for buckling loads.

## References

1. A. Dimarogonas, *Computer Aided Machine Design*, Prentice Hall, 1989.
2. W.C. Young, R.G. Budynas, *Roark's Formulas for Stress and Strain*, 7-th Ed., McGraw-Hill, 2002.
3. W.C. Young, R.G. Budynas, *Roark and Young on TK*, Universal Technical Systems Rockford, IL, 2002. (All 1,400 design cases and 5,000 equations are available with TK Solver.)
4. W.C. Young, R.G. Budynas, *Roark's Formulas on Excel*, Universal Technical Systems Rockford, IL, 2005.