CAD and Finite Element Analysis

- **Most ME CAD applications require a FEA in one or more areas:**
  - **Stress Analysis**
  - **Thermal Analysis**
  - **Structural Dynamics**
  - **Computational Fluid Dynamics (CFD)**
  - **Electromagnetics Analysis**
  - **...**

General Approach for FE, 1

- **Select verification tools**
  - **analytic, experimental, other fe method, etc.**
- **Select element type(s) and degree**
  - **3-D solid, axisymmetric solid, thick surface, thin surface, thick curve, thin curve, etc.**
- **Understand primary variables (PV)**
  - **Statics: displacements & (maybe) rotations**
  - **Thermal: temperature**
  - **CDF: velocity & pressure**

General Approach for FE, 2

- **Understand source (load) items**
  - Statics: point, line, surface, and volume forces
  - Thermal: point, line, surface, and volume heat generation
- **Understand secondary variables (SV)**
  - Statics: strains, stresses, failure criterion, error
  - Thermal: heat flux, error

General Approach for FE, 3

- Understand boundary conditions (BC)
  - Essential BC (on PV)
    - Statics: displacement and/or (maybe) rotation
    - Thermal: temperature
  - Natural BC (on SV)
    - Statics: null surface traction vector
    - Thermal: null normal heat flux
- One or the other at a boundary point.

General Approach for FE, 4

- **Understand reactions at Essential BC**
  - **Statics:**
    - **Force at given displacement**
    - **Moment at given rotation (if active)**
  - **Thermal:**
    - **Heat flux at given temperature**

General Approach for FE, 5

- 1. Estimate the solution
- 2. Select an acceptable error (1 %)
- 3. Mesh the model
- 4. Solve the model (PV), post-process (SV)
- 5. Estimate the error
  - A. Unacceptable error: Adapt mesh, go to 3
  - B. Acceptable error: Validate the analysis

FE Mesh (FEM)

- Crude meshes that “look like” a part are ok for mass properties but not for FEA.
- Local error is proportional to product of the local mesh size (h) and the gradient of the secondary variables.
- PV piecewise continuous polynomials of degree p, and SV are discontinuous polynomials of degree (p-1).

FEA Stress Models

- **3-D Solid, PV: 3 displacements (no rotations), SV: 6 stresses**
- **2-D Approximations**
  - **Plane Stress ($\sigma_{zz} = 0$) PV: 2 displacements, SV: 3 stresses**
  - **Plane Strain ($\varepsilon_{zz} = 0$) PV: 2 displacements, SV: 3 stresses (and σ_{zz})**
  - **Axisymmetric ($\partial/\partial\theta = 0$) PV: 2 displacements, SV: 4 stresses**

FEA Stress Models, 2

- **2-D Approximations**
  - **Thick Shells, PV: 3 displacements (no rotations), SV: 5 (or 6) stresses**
  - **Thin Shells, PV: 3 displacements and 3 rotations, SV: 5 stresses (each at top, middle, and bottom surfaces)**
  - **Plate bending PV: normal displacement, in-plane rotation vector, SV: 3 stresses (each at top, middle, and bottom surfaces)**

FEA Stress Models, 3

- 1-D Approximations
  - Bars (Trusses), PV: 3 displacements (1 local displacement), SV: 1 axial stress
  - Torsion member, PV: 3 rotations (1 local rotation), SV: 1 torsional stress
  - Beams (Frames), PV: 3 displacements, 3 rotations, SV: local bending and shear stress
    - Thick beam, thin beam, curved beam
    - Pipe element, pipe elbow, pipe tee

FEA Accuracy

- **PV are most accurate at the mesh nodes.**
- **SV are least accurate at the mesh nodes.**
  - **(SV are most accurate at the Gauss points)**
  - **SV can be post-processed for accurate nodal values (and error estimates)**

Local Error

- **The error at a (non-singular) point is the product of the element size, h, the gradient of the secondary variables, and a constant dependent on the domain shape and boundary conditions.**
  - **Large gradient points need small h**
  - **Small gradient points can have large h**
- **Plan local mesh size with engineering judgement.**

Error Estimators

- **Global and element error estimates are often available from mathematical norms of the secondary variables. The energy norm is the most common.**
- **It is proven to be asymptotically exact for elliptical problems.**
- **Typically want less than 1 % error.**

Error Estimates

- **Quite good for elliptic problems (thermal, elasticity, ideal flow), Navier-Stokes, etc.**
- **Can predict the new mesh size needed to reach the required accuracy.**
- **Can predict needed polynomial degree.**
- **Require second post-processing pass for localized (element level) smoothing.**

Primary FEA Assumptions

- **Model geometry**
- **Material Properties**
  - **Failure Criterion, Factor of safety**
- **Mesh(s)**
  - **Element type, size, degree**
- **Source (Load) Cases**
  - **Combined cases, Factors of safety, Coord. Sys.**
- **Boundary conditions**
  - **Coordinate system(s)**

Primary FEA Matrix Costs

- **Assume sparse banded linear algebra system of E equations, with a half-bandwidth of B. Full system if B = E.**
  - **Storage required, $S = B * E$ (Mb)**
  - **Solution Cost, $C \propto B * E^2$ (time)**
  - **Half symmetry: $B \leftarrow B/2$, $E \leftarrow E/2$, $S \leftarrow S/4$, $C \leftarrow C/8$**
  - **Quarter symmetry: $B \leftarrow B/4$, $E \leftarrow E/4$, $S \leftarrow S/16$, $C \leftarrow C/64$**
  - **Eighth symmetry, Cyclic symmetry, ...**

Symmetry and Anti-symmetry

- **Use symmetry states for the maximum accuracy at the least cost in stress and thermal problems.**
- **Cut the object with symmetry planes (or surfaces) and apply new boundary conditions (EBC or NBC) to account for the removed material.**

Symmetry (Anti-symmetry)

- **Requires symmetry of the geometry and material properties.**
- **Requires symmetry (anti-symmetry) of the source terms.**
- **Requires symmetry (anti-symmetry) of the essential boundary conditions.**

Structural Model

- Symmetry
  - Zero displacement normal to surface
  - Zero rotation vector tangent to surface
- Anti-symmetry
  - Zero displacement vector tangent to surface
  - Zero rotation normal to surface

Thermal Model

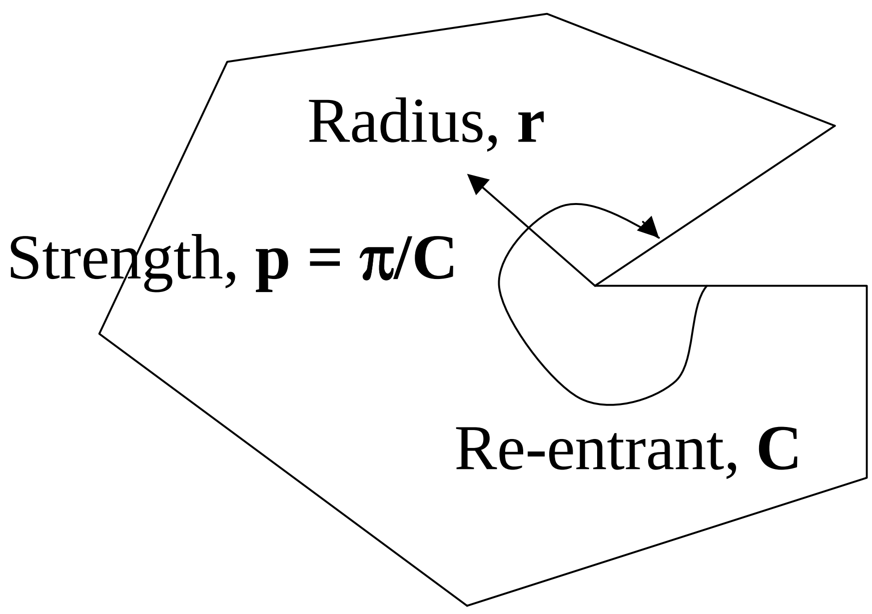
- Symmetry
  - Zero gradient normal to surface (insulated surface, zero heat flux)
- Anti-symmetry
  - Average temperature on surface known

Local Singularities

- **All elliptical problems have local radial gradient singularities near re-entrant corners in the domain.**

$$\mathbf{u = r^{p} f(\theta)}$$

$$\mathbf{\partial u/\partial r = r^{(p-1)} f(\theta)}$$



Corner: p = 2/3, weak

Crack: p = 1/2, strong

$$\mathbf{\partial u/\partial r \Rightarrow \infty \text{ as } r \Rightarrow 0}$$

Stress Analysis Verification, 1

- **Prepare initial estimates of deflections, reactions and stresses.**
- **Eyeball check the deflected shape and the principal stress vectors.**

Stress Analysis Verification, 2

- The stresses often depend only on the shape of the part and are independent of the material properties.
- You must also verify the displacements which almost always depend on the material properties.

Stress Analysis Verification, 3

- **The reaction resultant forces and/or moments are equal and opposite to the actual applied loading.**
- **For pressures or tractions remember to compare their integral (resultant) to the solution reactions.**
- **Reactions can be obtained at elements too.**

Stress Analysis Verification, 4

- Compare displacements, reactions and stresses to initial estimates. Investigate any differences.
- Check maximum error estimates, if available in the code.

Thermal Analysis Verification, 1

- **Prepare initial estimates of the temperatures, reaction flux, and heat flux vectors.**
- **Eyeball check the temperature contours and the heat flux vectors.**
- **Temperature contours should be perpendicular to an insulated boundary.**

Thermal Analysis Verification, 2

- **The temperatures often depend only on the shape of the part.**
- **Verify the heat flux magnitudes which almost always depend on the material properties.**

Thermal Analysis Verification, 3

- **The reaction resultant nodal heat fluxes are equal and opposite to the applied heat fluxes.**
- **For distributed heat fluxes remember to compare their integral (resultant) to the solution reactions.**
- **Reactions can be obtained at elements too.**

Thermal Analysis Verification, 4

- **Compare temperatures, reactions and heat flux vectors to initial estimates. Investigate any differences.**
- **Check maximum error estimates, if available in the code.**