

12 Buckling Analysis

12.1 Introduction

There are two major categories leading to the sudden failure of a mechanical component: material failure and structural instability, which is often called buckling. For material failures you need to consider the yield stress for ductile materials and the ultimate stress for brittle materials.

Those material properties are determined by axial tension tests and axial compression tests of short columns of the material (see Figure 12-1). The geometry of such test specimens has been standardized. Thus, geometry is not specifically addressed in defining material properties, such as yield stress. Geometry enters the problem of determining material failure only indirectly as the stresses are calculated by analytic or numerical methods.

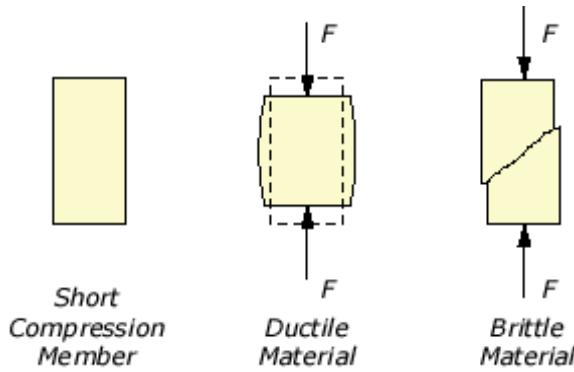


Figure 12-1 Short columns fail due to material failure

Predicting material failure may be accomplished using linear finite element analysis. That is, by solving a linear algebraic system for the unknown displacements, $\mathbf{K} \boldsymbol{\delta} = \mathbf{F}$. The strains and corresponding stresses obtained from this analysis are compared to design stress (or strain) allowables everywhere within the component. If the finite element solution indicates regions where these allowables are exceeded, it is assumed that material failure has occurred.

The load at which buckling occurs depends on the stiffness of a component, not upon the strength of its materials. Buckling refers to the loss of stability of a component and is usually independent of material strength. This loss of stability usually occurs within the elastic range of the material. The two phenomenon are governed by different differential equations [18]. Buckling failure is primarily characterized by a loss of structural stiffness and is not modeled by the usual linear finite element analysis, but by a finite element eigenvalue-eigenvector solution, $\mathbf{K} + \lambda_m \mathbf{K}_F \boldsymbol{\delta}_m = 0$, where λ_m is the buckling load factor (BLF) for the m -th mode, \mathbf{K}_F is the additional “geometric stiffness” due to the stresses caused by the loading, \mathbf{F} , and $\boldsymbol{\delta}_m$ is the associated buckling displacement shape for the m -th mode. The spatial distribution of the load is important, but its relative magnitude is not. The buckling calculation gives a multiplier that scales the magnitude of the load (up or down) to that required to cause buckling.

Slender or thin-walled components under compressive stress are susceptible to buckling. Most people have observed what is called “Euler buckling” where a long slender member subject to a compressive force moves lateral to the direction of that force, as illustrated in Figure 12-2. The force, F , necessary to cause such a buckling motion will vary by a factor of four depending only on how the two ends are restrained. Therefore, buckling studies are much more sensitive to the component restraints than in a normal stress analysis. The

theoretical Euler solution will lead to infinite forces in very short columns, and that clearly exceeds the allowed material stress. Thus in practice, Euler column buckling can only be applied in certain regions and empirical transition equations are required for intermediate length columns. For very long columns the loss of stiffness occurs at stresses far below the material failure.

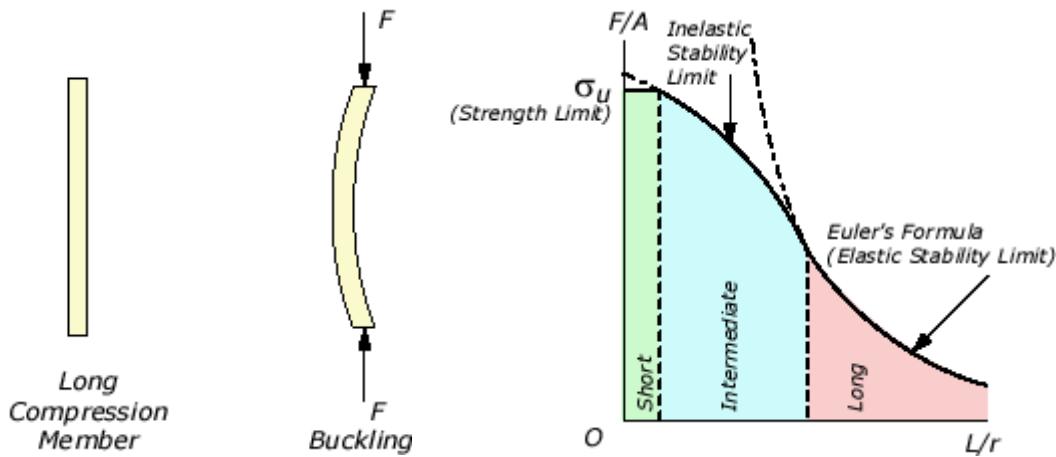


Figure 12-2 Long columns fail due to instability

There are many analytic solutions for idealized components having elastic instability. About 75 of the most common cases are tabulated in the classic reference “Roark’s Formulas for Stress and Strain” [15-17], and in the handbook by Pilkey [11].

12.2 Buckling terminology

The topic of buckling is still unclear because the keywords of “stiffness”, “long” and “slender” have not been quantified. Most of those concepts were developed historically from 1D studies. You need to understand those terms even though finite element analysis lets you conduct buckling studies in 1D, 2D, and 3D. For a material, stiffness refers to either its elastic modulus, E , or to its shear modulus, $G = E / (2 + 2v)$ where v is Poisson’s ratio.

Slender is a geometric concept of a two-dimensional area that is quantified by the radius of gyration. The radius of gyration, r , has the units of length and describes the way in which the area of a cross-section is distributed around its centroidal axis. If the area is concentrated far from the centroidal axis it will have a greater value of r and a greater resistance to buckling. A non-circular cross-section will have two values for its radius of gyration. The section tends to buckle around the axis with the smallest value. The radius of gyration, r , is defined as:

$$r = (I/A)^{1/2},$$

where I and A are the area moment of inertia, and area of the cross-section. For a circle of radius R , you obtain $r = R/2$. For a rectangle of large length R and small length b you obtain $r_{\max} = R/2\sqrt{3} = 0.29R$ and $r_{\min} = 0.29b$. Solids can have regions that are slender, and if they carry compressive stresses a buckling study is justified.

Long is also a geometric concept that is quantified by the non-dimensional “slenderness ratio” L/r , where L denotes the length of the component. The slenderness ratio is defined to be long when it obeys the inequality

$$L/r > (\pi/k)(2E/\sigma_y)^{1/2}$$

where k is a constant that depends on the restraints of the two ends of the column. A long slenderness ratio is typically in the range of >120 . The above equation is the dividing point between long (Euler) columns and

intermediate (empirical) columns. The critical compressive stress that will cause buckling always decreases as the slenderness ratio increases.

Euler long column buckling is quite sensitive to the end restraints. Figure 12-3 shows five of several cases of end restraints and the associated k value used in both the limiting slenderness ratio and the buckling load or stress. The critical buckling force is

$$F_{Euler} = k \pi^2 EI / L^2 = k \pi^2 EA / (L/r)^2$$

So the critical Euler buckling stress is

$$\sigma_{Euler} = F_{Euler} / A = k \pi^2 E / (L/r)^2.$$

Case	1	2	3	4	5
Constraints					
k	4	1	.25	2.046	1

Figure 12-3 Restraints have a large influence on the critical buckling load

12.3 Buckling Load Factor

The buckling load factor (BLF) is an indicator of the factor of safety against buckling or the ratio of the buckling loads to the currently applied loads. Table 12-1 Interpretation of the Buckling Load Factor (BLF) illustrates the interpretation of possible BLF values returned by SW Simulation. Since buckling often leads to bad or even catastrophic results, you should utilize a high factor of safety (FOS) for buckling loads. That is, the value of unity in Table 12-1 Interpretation of the Buckling Load Factor (BLF) should be replaced with the FOS value.

Table 12-1 Interpretation of the Buckling Load Factor (BLF)

BLF Value	Buckling Status	Remarks
>1	Buckling not predicted	The applied loads are less than the estimated critical loads.
= 1	Buckling predicted	The applied loads are exactly equal to the critical loads. Buckling is expected.
< 1	Buckling predicted	The applied loads exceed the estimated critical loads. Buckling will occur.
-1 < BLF < 0	Buckling possible	Buckling is predicted if you reverse the load directions.
-1	Buckling possible	Buckling is expected if you reverse the load directions.
< -1	Buckling not predicted	The applied loads are less than the estimated critical loads, even if you reverse their directions.

12.4 General buckling concepts

Other 1D concepts that relate to stiffness are: axial stiffness, $E A / L$, flexural (bending) stiffness, $E I / L$, and torsional stiffness, $G J / L$, where J is the polar moment of inertia of the cross-sectional area ($J = Iz = Ix + Iy$). Today, stiffness usually refers to the finite element stiffness matrix, which can include all of the above stiffness terms plus general solid or shell stiffness contributions. Analytic buckling studies identify additional classes of instability besides Euler buckling (see Figure 12-4). They include lateral buckling, torsional buckling, and other buckling modes. A finite element buckling study determines the lowest buckling factors and their corresponding displacement modes. The amplitude of a buckling displacement mode, $|\delta_m|$, is arbitrary and not useful, but the shape of the mode can suggest whether lateral, torsional, or other behavior is governing the buckling response of a design

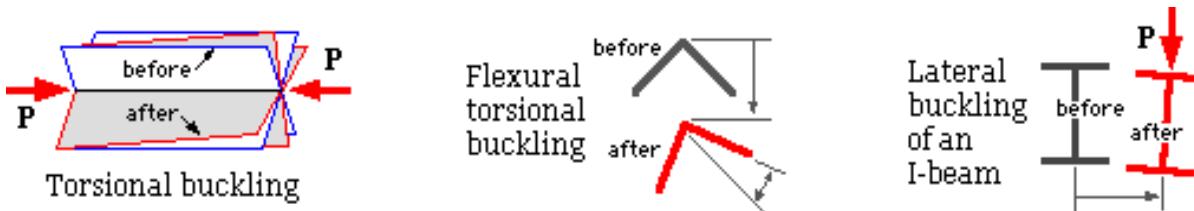


Figure 12-4 Some sample buckling shapes

12.5 Local Buckling of a Cantilever

12.5.1 Background

You previously went through the analysis of a horizontal tapered cantilever subject to a transverse load distributed over its free end face. The fixed support at the wall included a semi-circular section of the supporting vertical section. The member was $L = 50$ inch long, $t = 2$ inch thick, and the depth, d , tapered from 3 inch at the load, to 9 inch at the support. A complete plane stress analysis was conducted. The computed stresses were relatively low. It was decided to save material costs by reducing the thickness of the beam.

12.5.2 Factor of Safety

For the ductile material used here the material factor of safety (FOS) is defined as the material yield stress divided by the von Mises' effective stress. To view its distribution the default results plot is opened with **Results**→**Define Factor of Safety Plot**→**Max von Mises stress**, which is shown in Figure 12-5.

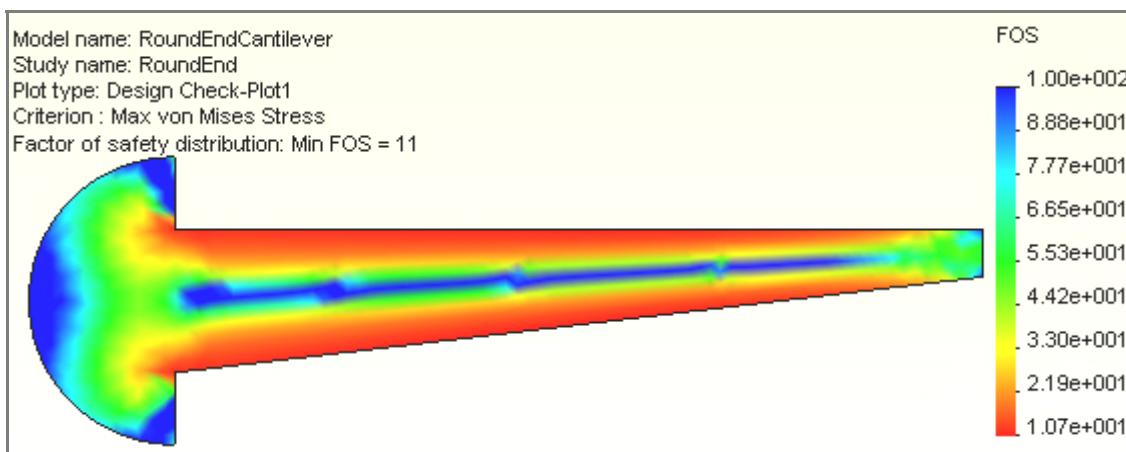


Figure 12-5 Original material factor of safety in bending

The material FOS is also quite high, ranging from a low value of about 10 to a high value of about 100. This probably suggests (incorrectly) that a simple redesign will save material, and thus money. The load carrying capacity of a beam is directly proportional to its geometric moment of inertia, $I_z = t d^3 / 12$. Thus, it also is proportional to its thickness, t . Therefore, it appears that you could simply reduce the thickness from $t = 2$ to 0.2 inches and your material FOS would still be above unity. If you did that then the “thickness to depth ratio” would vary from $0.2 / 3 = 0.067$ at the load to $0.2 / 9 = 0.022$ at the wall, a range of about 1/15 to 1/45.

12.5.3 Local buckling

If component has a region where the relative thickness to depth ratio of less than 1/10 you should consider the possibility of “*local buckling*”. It usually is a rare occurrence, but when it does occur the results can be sudden and catastrophic. To double check the safety of reducing the thickness you should add a second study that utilizes the SW Simulation buckling feature to determine the lowest buckling load. To do that:

1. Right click on the **Part name**→**Study** to open the **Study panel**.
2. Assign a new **Study name**, select Buckling as the **Type of analysis**, and use the thin shell as the **Model type**, click **OK**.
3. To use the same loads and restraints **drag** the **External Loads** from the first study and **drop** them into the second one.
4. Likewise, **drag and drop** the first shell **Materials** into the second study.
5. Create a new finer mesh, or drag and drop the first mesh.
6. Right click on the **Part name**→**Run**.

12.5.4 Buckling mode

A buckling, or stability, analysis is an eigen-problem. The magnitude of the scalar eigen- value is called the “*buckling load factor*”, BLF. The computed displacement eigen-vector is referred to as the “*buckling mode*” or mode shape. They are only relative displacements. Usually they are presented in a non-dimensional fashion where the displacements range from zero to ± 1 . In other words, the actual value or units of a buckling mode shape are not important. Still, it is wise to carry out a visual check of the first buckling mode:

1. When the solution completes, pick **Displacements**→**Plot1** and examine the resultant displacement URES. Note that the displacement contour curves in Figure 12-6 are inclined to the long axis of the beam instead of being vertical as before.
2. Use **Edit Definition**→**Vector**→**Line** to get a plot of the displacement vectors, and rotate to an out-of-plane view, as shown in Figure 12-7.

From Figure 12-7 you see that under the vertical load the (very thin) beam deflected mainly sideways (perpendicular to the load) rather than downward. This is an example of lateral buckling. That is typical of what can happen to very thin regions. Next, the question is: how large must the end load be to cause such motion, and failure?

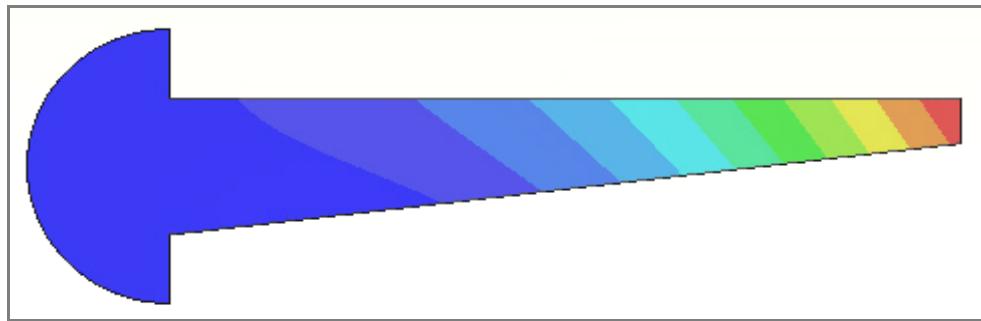


Figure 12-6 Relative buckling mode displacement values (normal to surface)

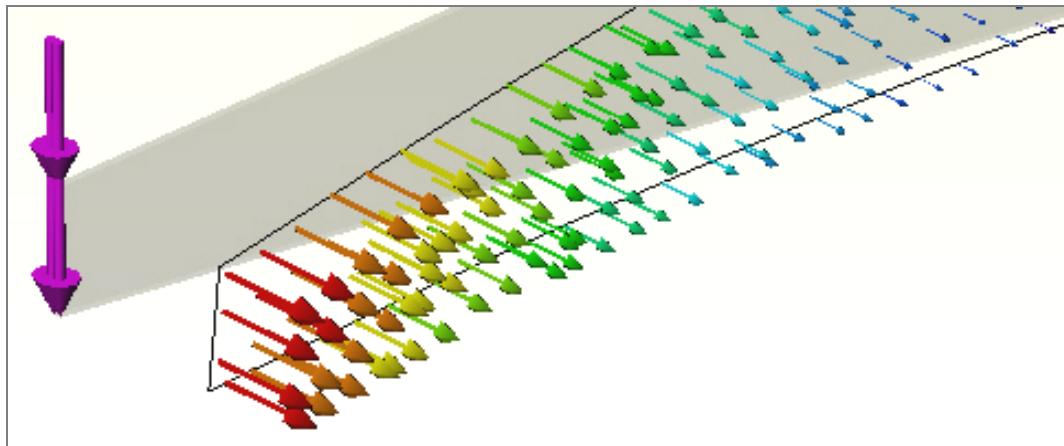
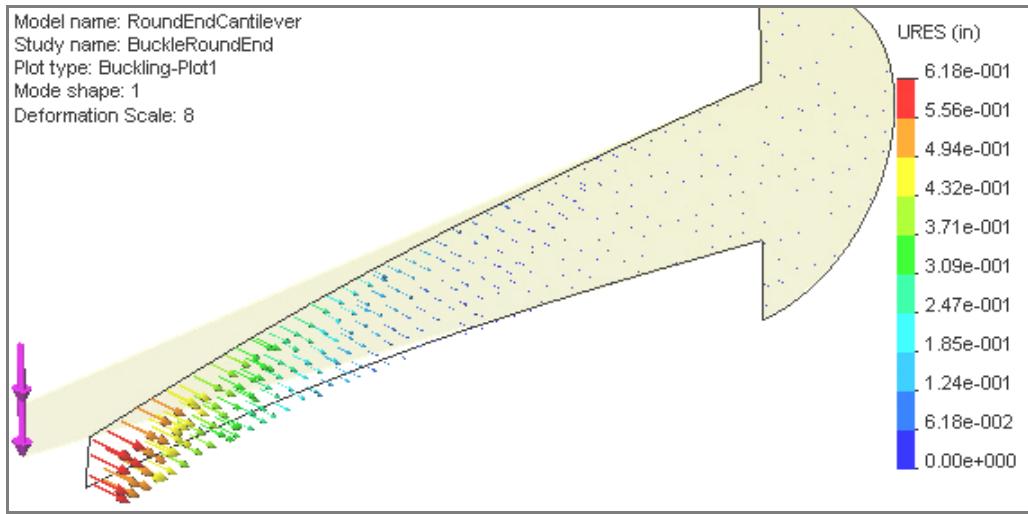


Figure 12-7 Relative lateral buckling mode displacement vectors

12.5.5 Buckling Load Factor

To see the magnitude of the BLF (eigenvalue):

1. Right click on **Deformation** → **List Mode Shape**.
2. In the **Mode Shape** panel, Figure 12-8, read the BLF value of about 0.03.

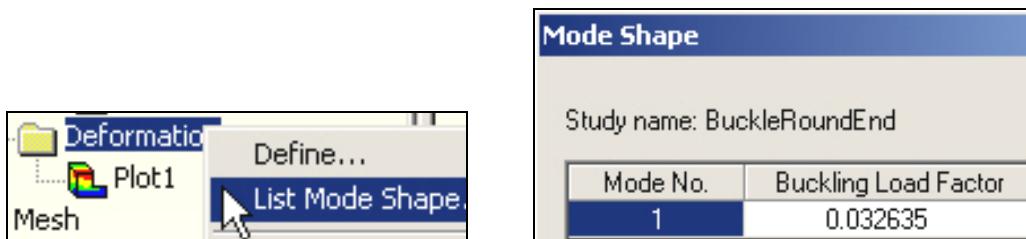


Figure 12-8 First buckling mode load factor

You want the BLF to be quite a bit higher than unity. Instead, the study shows that only about 3% of the planned load will cause this member to fail by lateral buckling due to loss of stiffness in the out of plane direction. Thus, you must re-consider the thickness reduction. Remember that the geometric moment of inertia about the vertical (y) axis is $I_y = d t^3 / 12$. It is a measure of the lateral bending resistance. By reducing the thickness, t , by a factor of 10 the original I_z (and the in-plane bending resistance) went down by the same factor of 10, but I_y (and the out-of-plane bending resistance) went down by a factor of 1,000.

The buckling load factor is an indicator of the factor of safety against buckling or the ratio of the buckling loads to the currently applied loads. Since buckling often leads to bad or even catastrophic results, you should utilize a high factor of safety (at least >3) for buckling loads.