8 Flat Plate Analysis

8.1 Introduction

A flat plate is generally considered to be a thin flat component that is subjected to load conditions that cause deflections transverse of the plate. Therefore, the loads are transverse pressures, transverse forces and moment vectors lying in the plane. Those loads are resisted mainly by bending. It is assumed that in-plane membrane stresses are not present and that the transverse displacements are "small". Generally, "small" is taken to mean a deflection that is less than half the thickness of the plate. If the deflection is larger than that and/or membrane forces are present you have to use a non-linear large deflection solution.

8.2 Rectangular plate

Figure 8-1 shows some of the boundary conditions that can be applied to the edges of a plate. A segment of a plate can be fixed or encastred (left), simply supported (center), or mixed supported (right), or have a free edge. A simply supported condition usually means that the transverse displacement is zero on that segment but the rotation tangent to the segment is unknown. A fixed supported condition usually means that the rotation vector tangent to the segment is also zero. A free edge is stress free. That is, it has no moment or transverse shear resultants acting along its length.

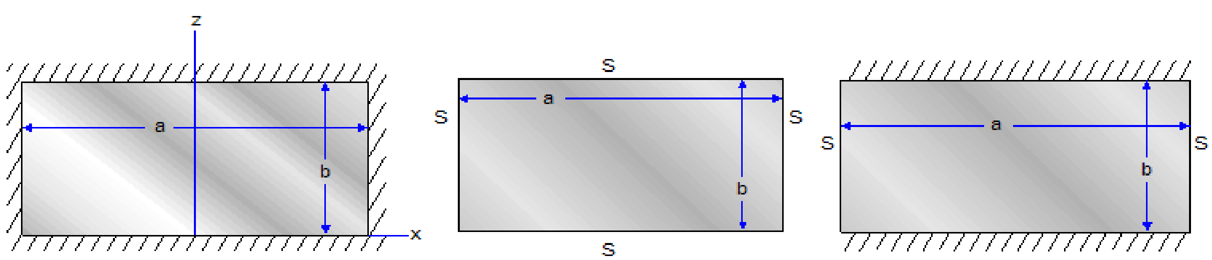


Figure 8-1 Some typical boundary condition options on rectangular plates

In this section the classic example of a simply supported plate subjected to a uniform transverse pressure will be illustrated. Quarter symmetry will be utilized to illustrate symmetry boundary conditions for an element with displacement and rotational degrees of freedom. A short story about this case will be noted at the end. The example plate is AISI 1020 steel with a yield stress of about *51 ksi*. The dimensions of the full plate are *4.68* by *12.68* by *0.08 inches* and it is subjected to a uniform pressure of *25 psi*. The total force is about *371 lb*, so you expect the resultant edge reactions to be equal and opposite to that value. Since external edge effects are usually important, a finer mesh is employed along those edges. The plate is set to be of the "thin" type and the study is executed.

The sketch is built and converted to a planar surface with **Insert→Surface→Planar surface.** A static study is opened and the thickness set in **Part Name→Edit Definition→Shell Definition** and it is marked as a thin shell. The two symmetry edges have no in-plane displace normal to the edge, nor any rotation about the edge. They are invoked with **Fixtures→Advanced→Use Reference Geometry**. The two physical support edges are prevented from translation, but can have a rotation vector tangent to the edge. **They are set with Fixtures→ Standard→Immovable**. Note that the immovable restraint along a planar curve has the effect of indirectly eliminating the rotation vector normal to the plane as well as the in-plane rotation vector normal to the curve. The two classes of displacement restraints are shown in Figure 8-2.

The constant external pressure is set with **External Loads→Pressure→Normal to**, and the value is set at *25 psi.* The loaded and restrained model is shown in Figure 8-3 and the mesh is in Figure 8-4.

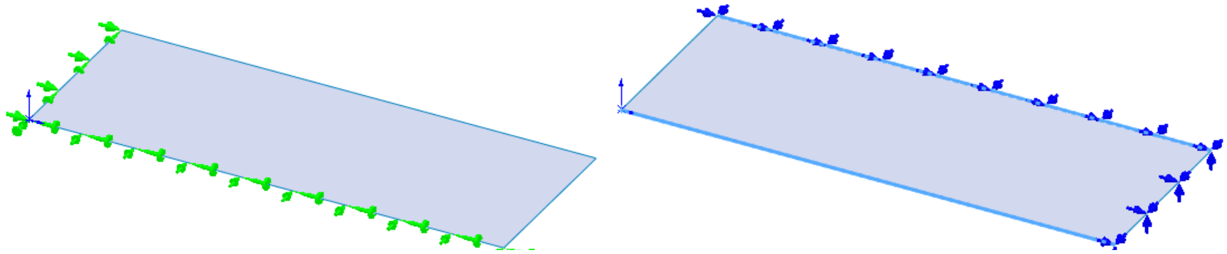


Figure 8-2 Symmetry (left) and simple supported plate restraints

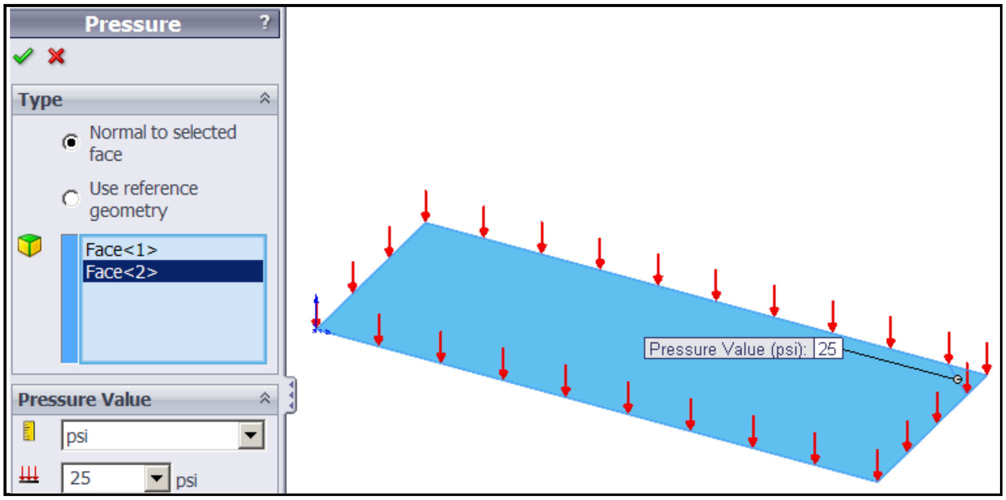


Figure 8-3 Constant normal pressure load on the plate

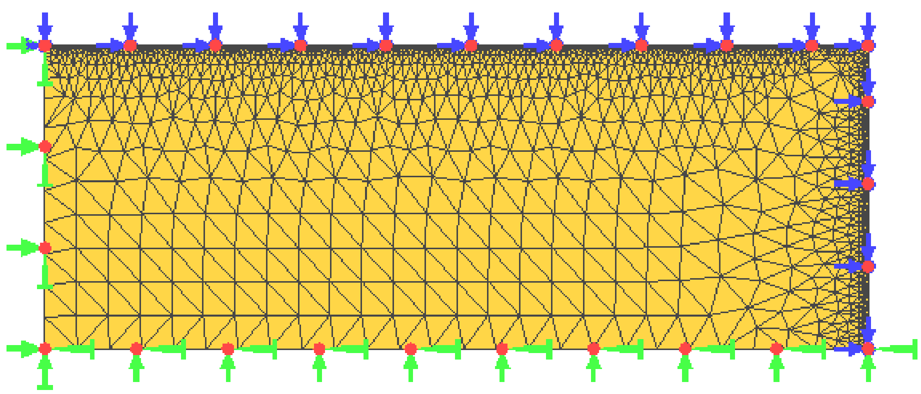


Figure 8-4 Refined edge mesh for a quarter symmetry rectangular plate

8.3 Post-processing

8.3.1 Displacements and rotations

The plate deflections are given in Figure 8-5. The surface deflections are given as contours. The short symmetry edge deflection is graphed for more detail in the lower image. The graph starts at the outer (zero deflection) edge and goes to the maximum deflection at the plate center (zero rotation) point. It serves to verify that the restraints were properly applied. The center point deflection can also be compared to analytic estimates [14, 17]. Here the maximum computed deflection is more than half the thickness of the plate, therefore the small deflection assumption appears questionable. SW Simulation did not issue a warning about the change in stiffness due to perceived large deflections, but a re-run will be considered later.

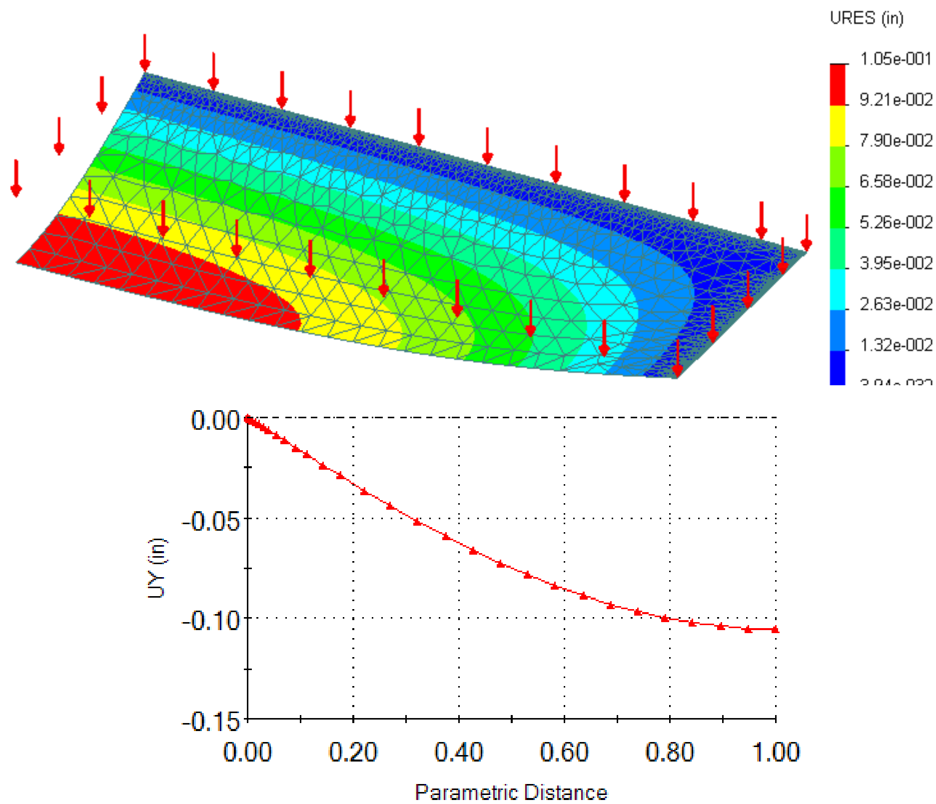


Figure 8-5 Deflections of the quarter symmetry plate, and its short edge

Another insight to the displacement is obtained by graphing its value along lines from the supported corner point. Note in Figure 8-6 that the trend shows a reverse curvature. The deflection is restrained to zero, by an external corner force, but the trend is to a lift up at the corner.

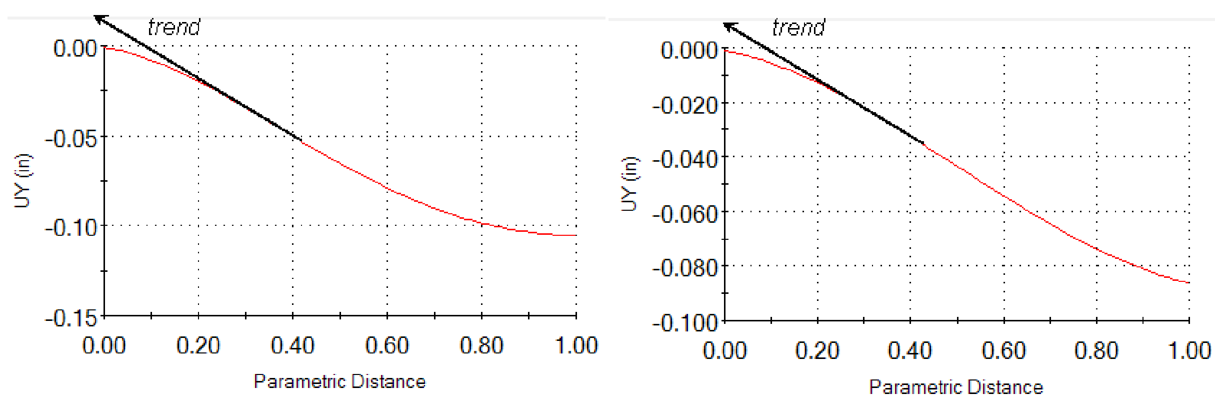


Figure 8-6 Deflection along from corner to corner (left) and to mid-side

8.3.2 Plate stresses

Since plates and shells can be subjected to both bending and membrane (in-plane) stresses the stress results should be checked on the top, bottom, and middle surfaces. Here the membrane stress is zero (*for small deflections*). At a point on the plate the stress will be in tension on one side and have an equal amount of compression on the other. That is important when the material has different strengths in tension and compression (like concrete).

The von Mises effective stress is proportional to the square root of the sum of the squares of the differences in the principal stresses, so it is always positive. The contour, and short symmetry edge, values of the von Mises stress are given in Figure 8-7. Note that the peak values exceed the yield stress, and the factor of safety (FOS) with respect to material failure is less than unity.

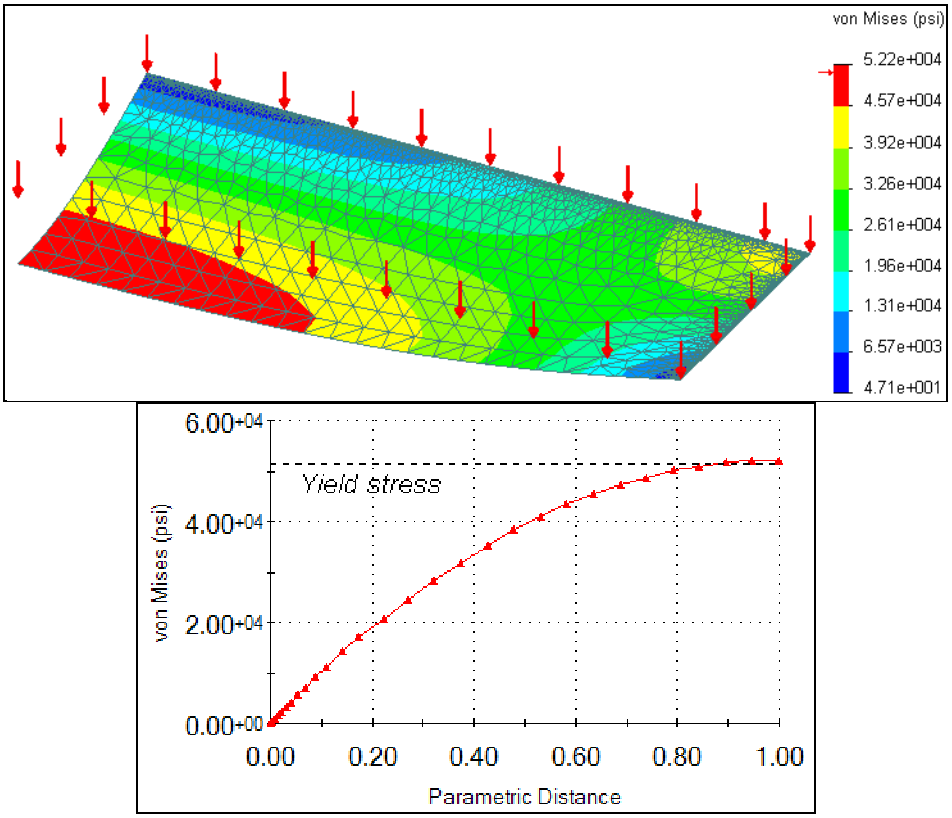


Figure 8-7 Von Mises stress in quarter symmetry plate and its short symmetry side

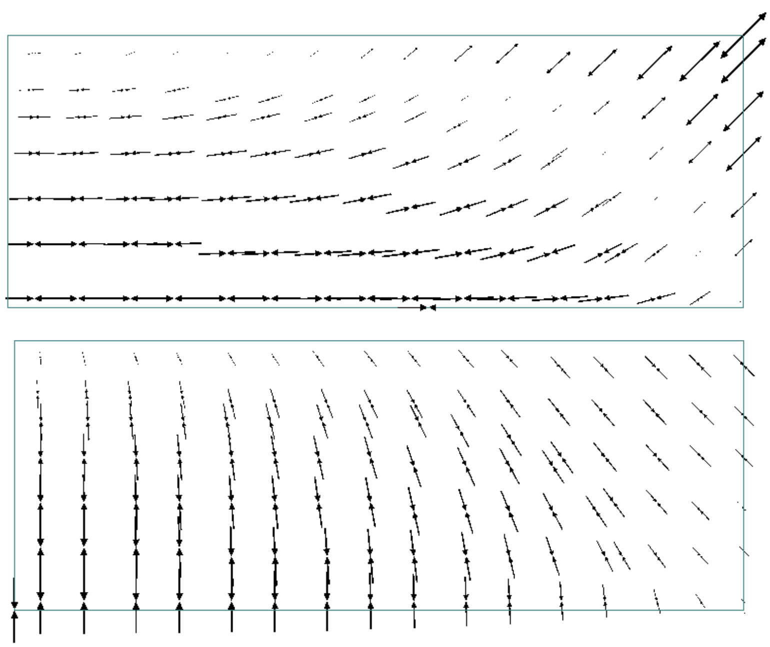


Figure 8-8 The principal stresses P1 and P3 on the top of the quarter symmetry plate

8.3.3 Design insight

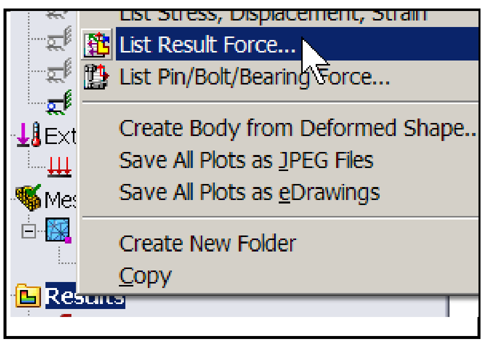
The above displacement graphs suggested that the simply supported corner has a large effect on the plate. That concept is reinforced by the plot of the material most active in the load path, plotted in Figure 8-9.

Figure 8-9 Main material load path region of the plate

8.3.4 Surprising corner reactions

The reactions can be recovered in at least two ways. The approach using a free body diagram calculation is shown here. First the transverse reaction force on the full two supporting edges are recovered and found to be equal and opposite to the resultant applied force from the pressure.

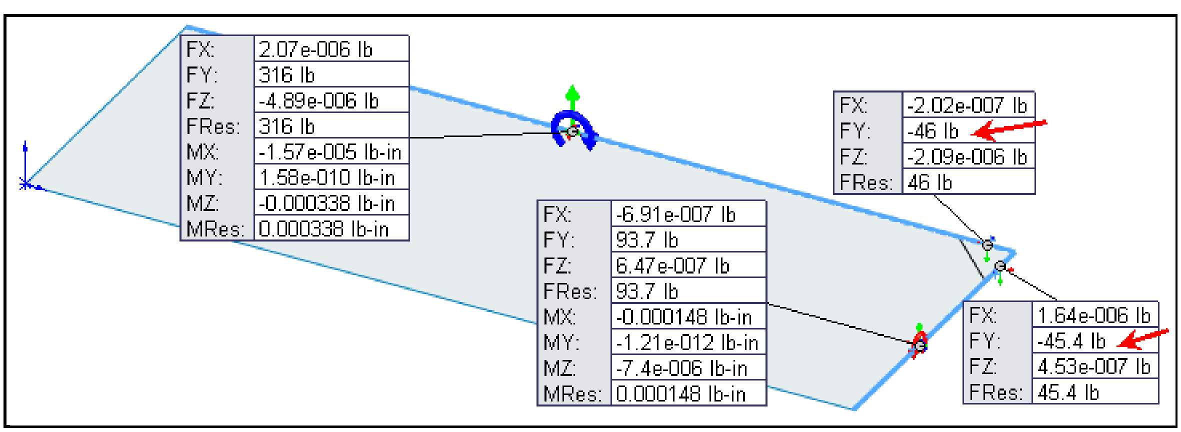
1. Right click on **Results→List Free Body Force** to open the **Reaction Force panel.**



Reaction Force (lb)

Component	Selection	Entire Model
Sum X:	2.3323E-006	3.0732E-005
Sum Y:	370.44	370.36
Sum Z:	-5.763E-006	6.8369E-005
Resultant:	370.44	370.36

2. First select all four supporting edges to get the total reaction forces. The total (above) is about *371 lb.*, which is equal and opposite to the applied resultant force. Note however, that the corner reaction forces have a negative sign. That is, they act downward in the same direction of the pressure. The total corner reaction force has a value of about 20% of the applied force from the pressure (the corner node point force was counted twice).



The resultants on a free body diagram come from the integral of the reaction force per unit length of the edge restraints. The reaction per unit length is not constant and will vary in a complicated fashion. Knowing this, the support edges were split to introduce shorter edges at the corner. That lets you find the portion of the reactions coming from the small corner segments.

8.3.5 Corner reaction discussion

Now for the related side story: A large analysis group had run the above problem to test a new finite element system that they had recently installed. I was called in as a consultant to fix an "error" they had found. Specifically, when a pressure load was applied downward to a flat plate some of its reactions were also found to be acting downward, just as noted above. That seemed to them to be physically impossible. I stated that the software was giving the proper type of response since elementary plate and shell theory shows that the edge reactions per unit length must behave in that fashion. To help understand why, I had them plot the two non-zero top principal stresses as well as the deflection and maximum (top) stress along the diagonal from the center point to the support corner, like Figure 8-6.

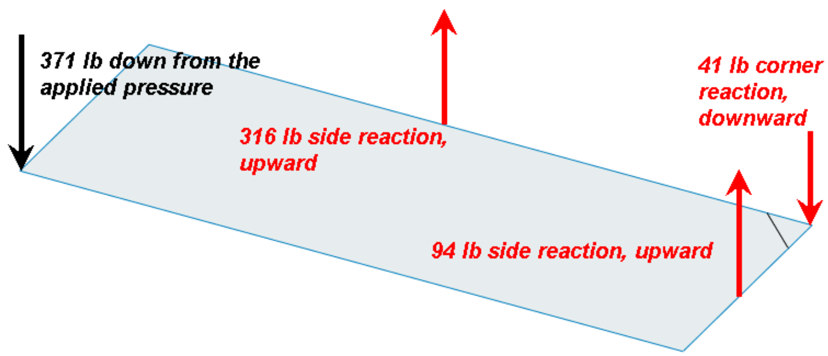


Figure 8-10 Approximate resultant applied force and reaction forces on the plate

The deflection plot in Figure 8-6 shows that the deflection curvature reverses its sign as it approaches the corner. The corner would lift up, but the *assumed* edge restraint requires that it not move. Therefore, tension forces must develop in the corner reactions to pull it down in the *assumed* restrained position. If the material along the restraint edge is capable of developing a resisting downward force, then you have the correct solution to the actual problem. Otherwise, you have the solution to the *assumed* problem. Unfortunately, many finite element studies give results for the assumed part behavior instead of the actual part behavior. Then, the plots are pretty, but wrong.

If the edges of the plate are simply sitting on top of to walls, then the wall could not pull down on the corner. An air gap would open; the corner would lift up off the wall, and all line reactions would be in compression where the plate remains on the wall. Sometimes you can actually see this corner lift off behavior in thin acoustical ceiling tiles. How much of the corner actually lifts off the wall must be computed from an iterative contact analysis.

8.3.6 Closure

If you did not have a contact analysis capability you could still get a reasonable answer to the lift off analysis. To do that you could introduce a split line on each edge near the corner (with parametric dimensions). Let the short end of each corner line be unsupported, solve the problem and check the reactions. If any negative reaction forces appear, then move the split line away from the corner and repeat the process. It may be a slow procedure, but it can lead you to the correct lift off regions.

